Comparing Object Correlation Metrics for Effective Space Traffic Management

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Motivation

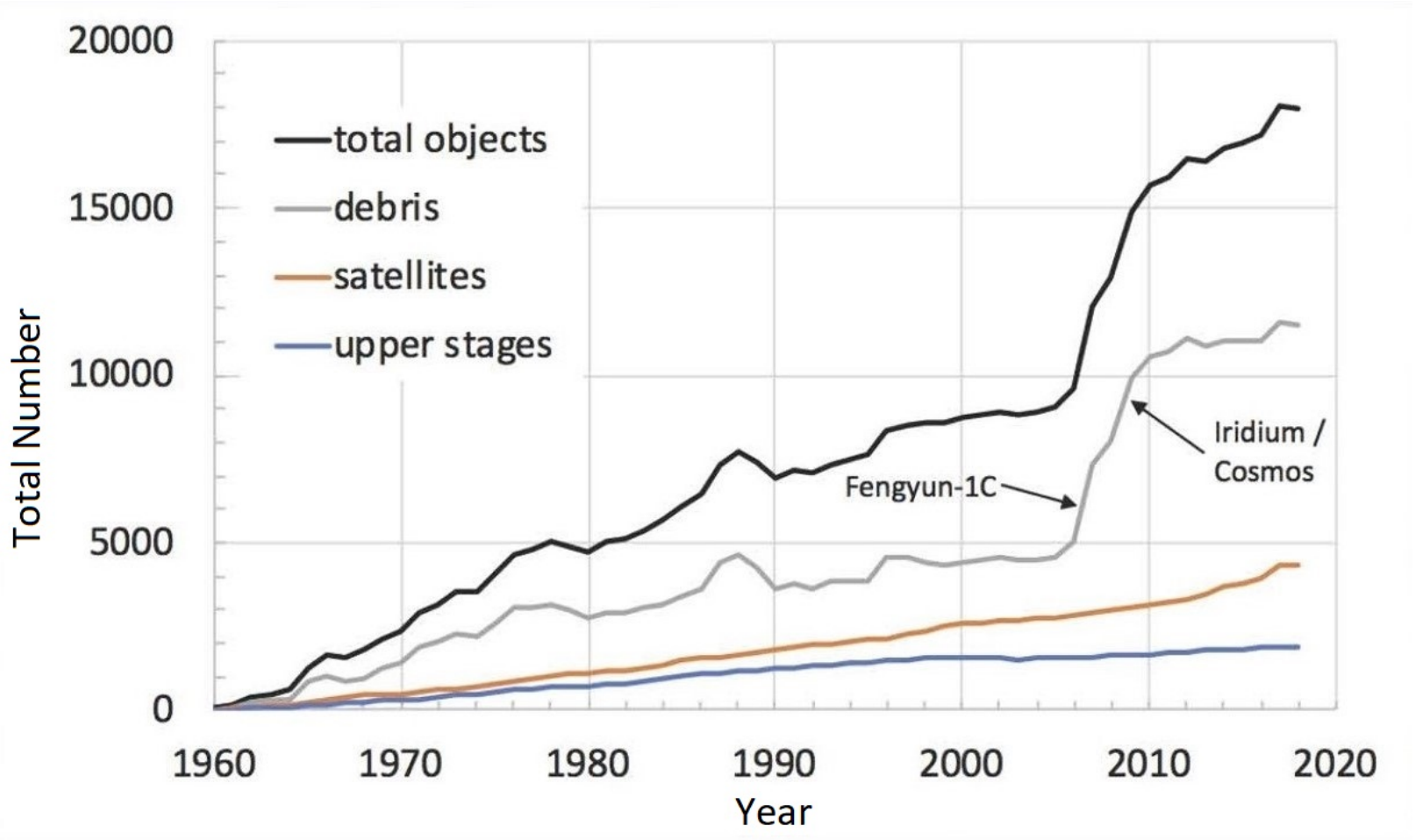

Figure 1. Catalogued Objects in Space Surveillance Network

Overview

- Using a simulation framework, estimate the state of non-cooperative objects in orbit given a priori knowledge

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- Given sensor data with measurement errors, optimally assign these measurements to known objects using a certain distance metric
- Comprehensively compare these likelihood-of-coincidence metrics

Simulation Framework-Satellites

- Simulate the state (position and velocity) and orbits of satellite
  - Orbit found through numerically solving the defining ODE in orbital mechanics
  - Modeled forces such as oblateness of the Earth, atmospheric drag, solar radiation pressure, and third-body gravity along with 2-body gravity

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- We cannot determine the exact state of a satellite, only the distribution, which is assumed to be multivariate Gaussian: $(\mu^{(t_0)}, \Sigma^{(t_0)})$ at time t_0.
- Use the orbital dynamics to determine what the distribution will be at a future time $t_1 > t_0$, which is assumed to still be Gaussian

Simulation Framework-Satellites

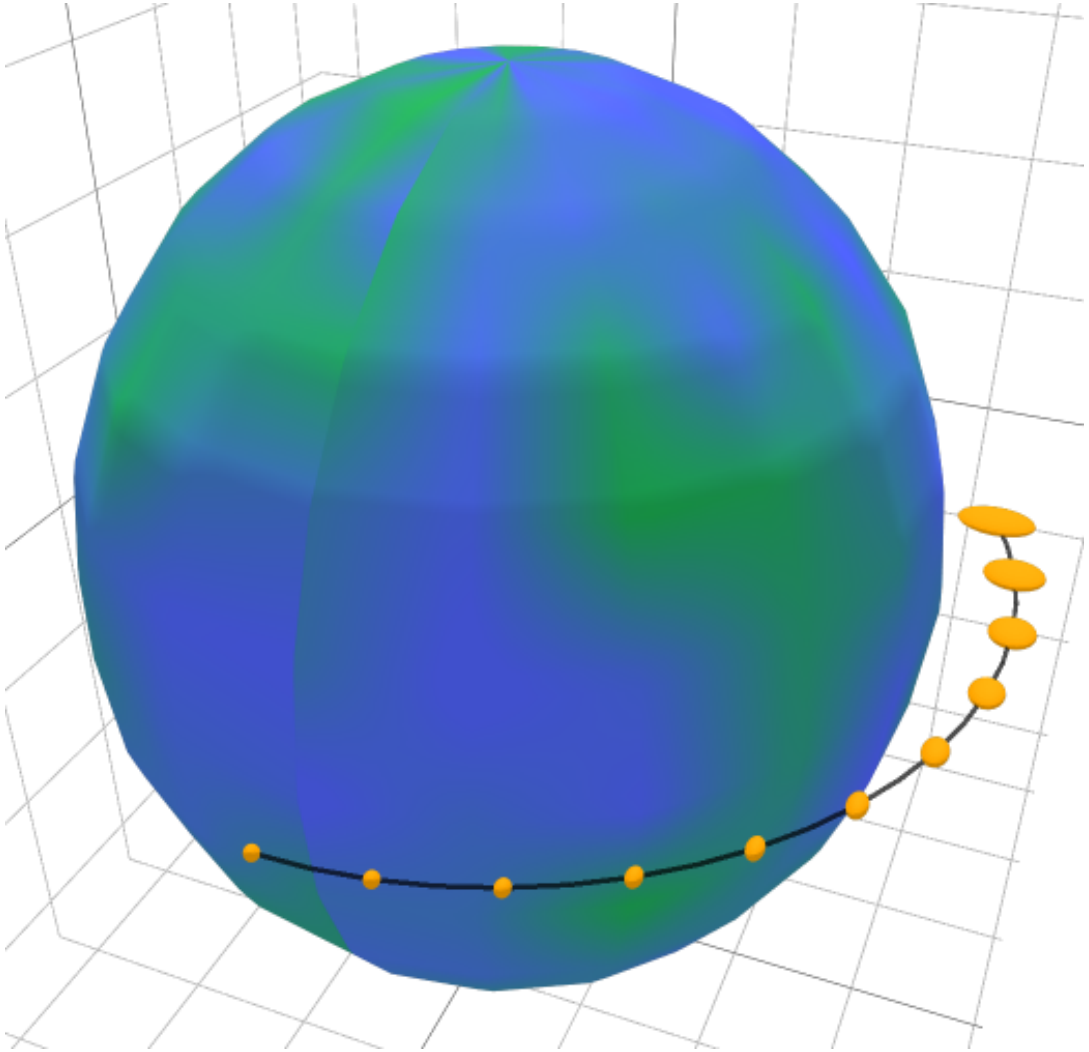

Figure 2. Propagation of Mean and Covariance

Simulation Framework-Sensors

- Model ground-based (move only as Earth rotates) and space-based (move like a satellite) sensors that take measurements
  - Range: The Euclidean distance from the sensor to the satellite
  - Angle: Represented in azimuth and elevation (also called altitude)

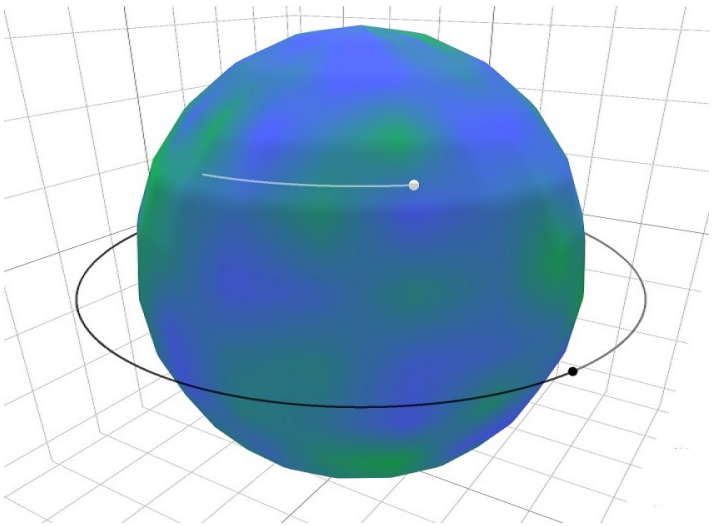

Figure 3. Sensor Movement

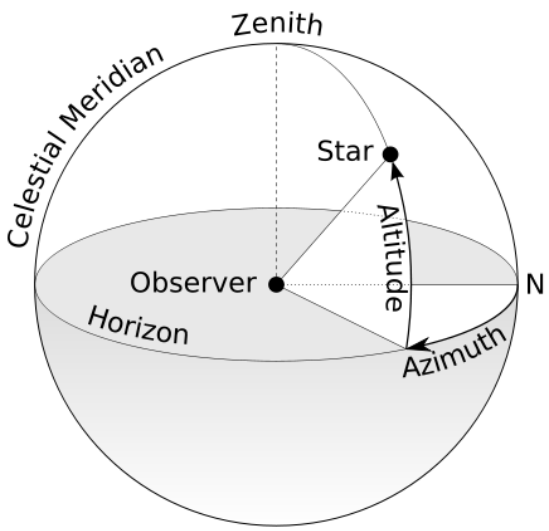

Figure 4. Angle Diagram

Object Correlation

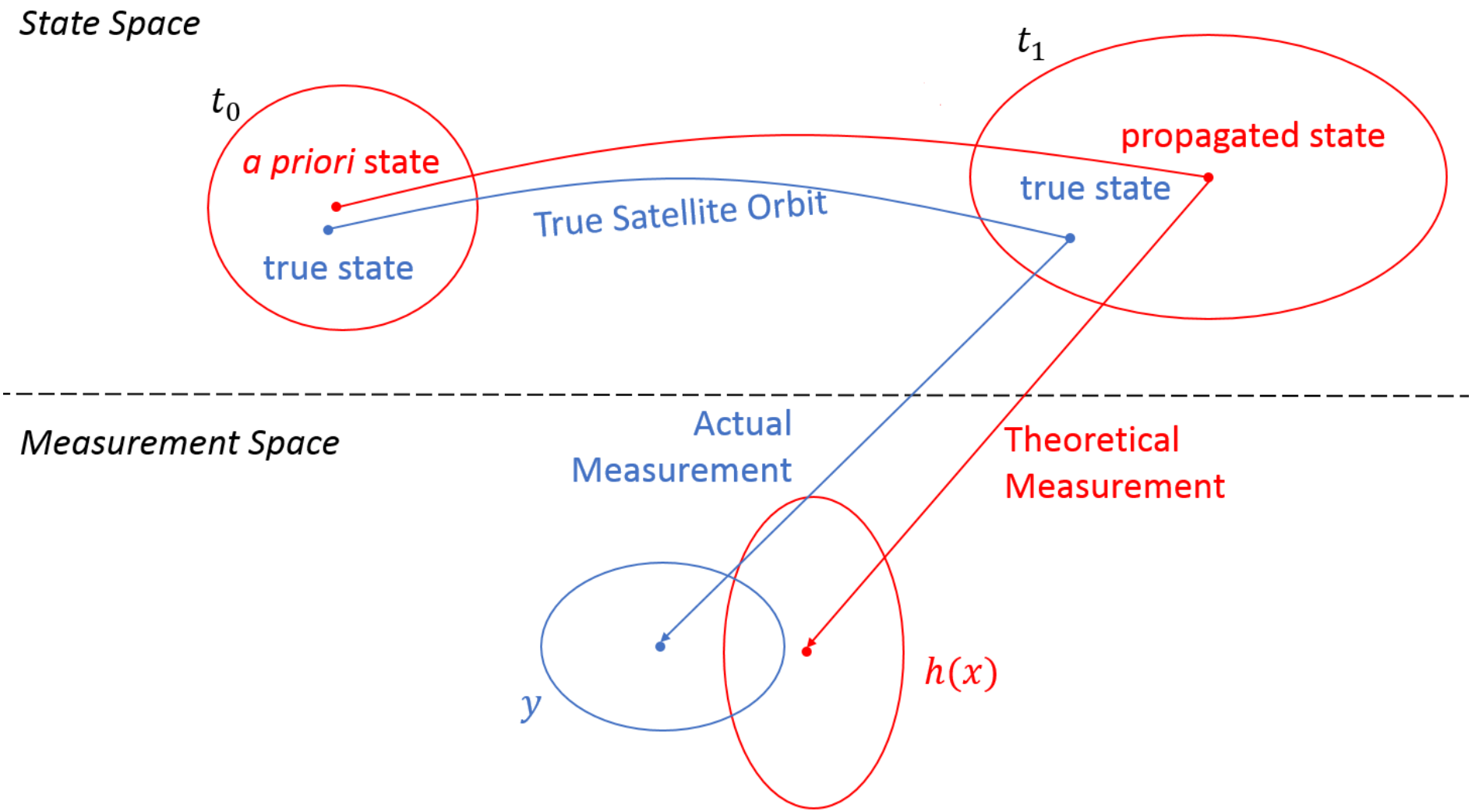

Figure 5. Object Correlation in Measurement Space

Distance Metrics

Let $D_1 \stackrel{d}{=} N(\mu_1, \Sigma_1)$ and independent $D_2 \stackrel{d}{=} N(\mu_2, \Sigma_2)$, with dimension k.

Mahalanobis:

$$d_M(D_1, D_2) = (\mu_2 - \mu_1)^T (\Sigma_1 + \Sigma_2)^{-1} (\mu_2 - \mu_1)$$

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Bhattacharyya:

$$\begin{aligned} d_B(D_1, D_2) = & \frac{1}{4}(\mu_2 - \mu_1)^T (\Sigma_1 + \Sigma_2)^{-1} (\mu_2 - \mu_1) \\ & + \frac{1}{2} \log \left(\frac{\det(\Sigma_1 + \Sigma_2)}{\sqrt{\det \Sigma_1 \det \Sigma_2}} \right) - \frac{k}{2} \log 2 \end{aligned}$$

Distance Metrics

Kullback-Liebler divergence is a nonsymmetric function, so we define two versions of it.

KL1:

$$d_{KL}(D_1, D_2) = \frac{1}{2}(\mu_2 - \mu_1)^T \Sigma_2^{-1}(\mu_2 - \mu_1) + \frac{1}{2}\log\left(\frac{\det \Sigma_2}{\det \Sigma_1}\right) + \frac{1}{2}\mathsf{Tr}(\Sigma_2^{-1}\Sigma_1) - \frac{k}{2}$$

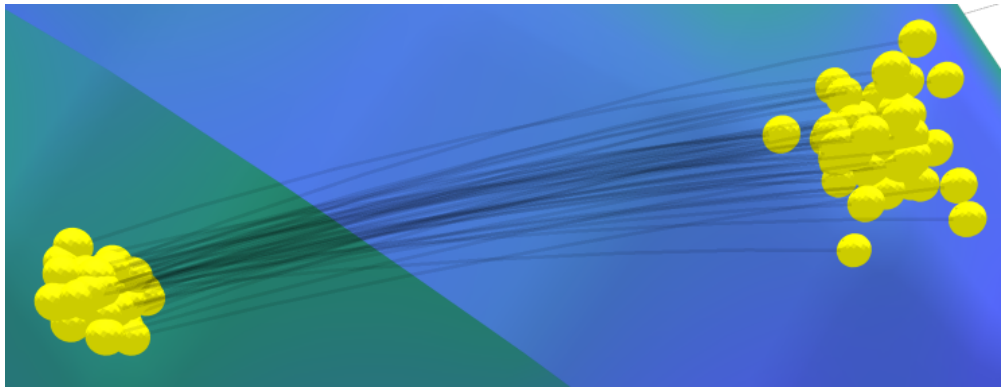
KL2:

$$d_{KL}(D_2, D_1)$$

Experiment Design

How do the metrics perform on clusters of satellites?

- `CLUSTER-OUT`: Satellites begin in a cluster at time t_0 and disperse as they are propagated to a later time t_1



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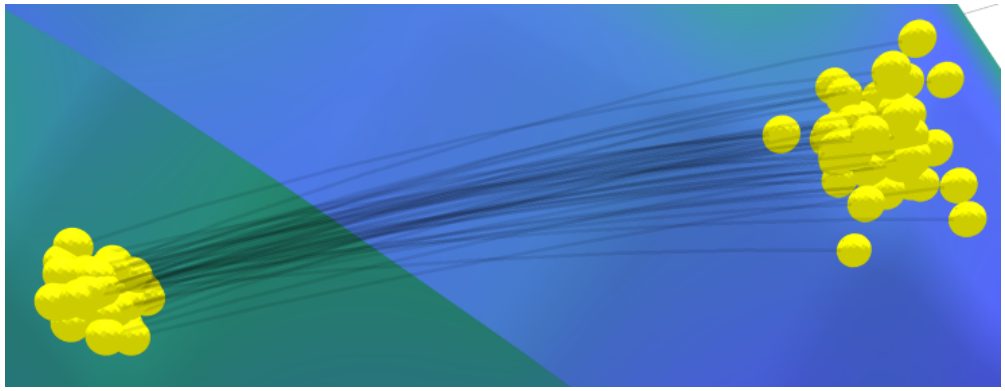
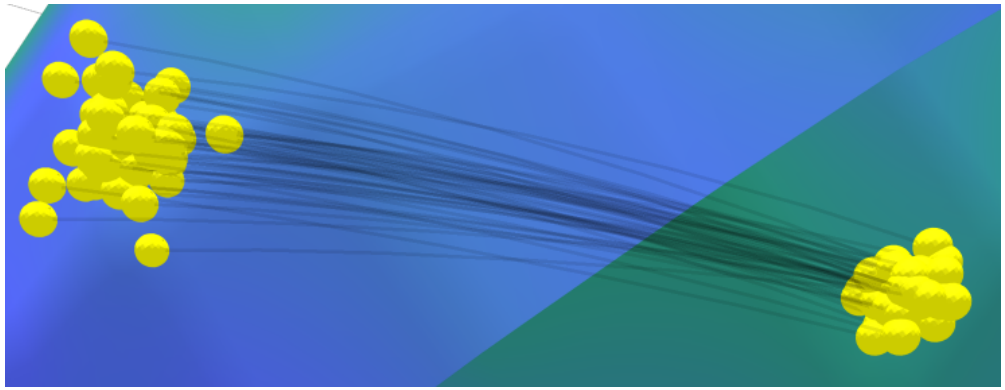


- `CLUSTER-IN`: Satellites begin dispersed at time t_0 and become clustered as they are propagated to a later time t_1



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  - 15 clusters as in `CLUSTER-OUT`
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- Generate 30 clusters of 25 satellites
  - 15 clusters as in `CLUSTER-OUT`
  - 15 clusters as in `CLUSTER-IN`
- Generate 250 single satellites with randomly selected elliptical orbits for a total of 1000 satellites
- Run 30 trials of this test, averaging the success rates across trials for each metric and modality

Parameter Values

- Observation Gap: 3600 seconds
- Altitude: 750 km - approximate height of the cluster
- Dispersal: 10^8 m^2 - a measure of cluster tightness
- Satellite *a priori* uncertainty: 10^2 m^2
- Sensor measurement error: 10^2 m^2

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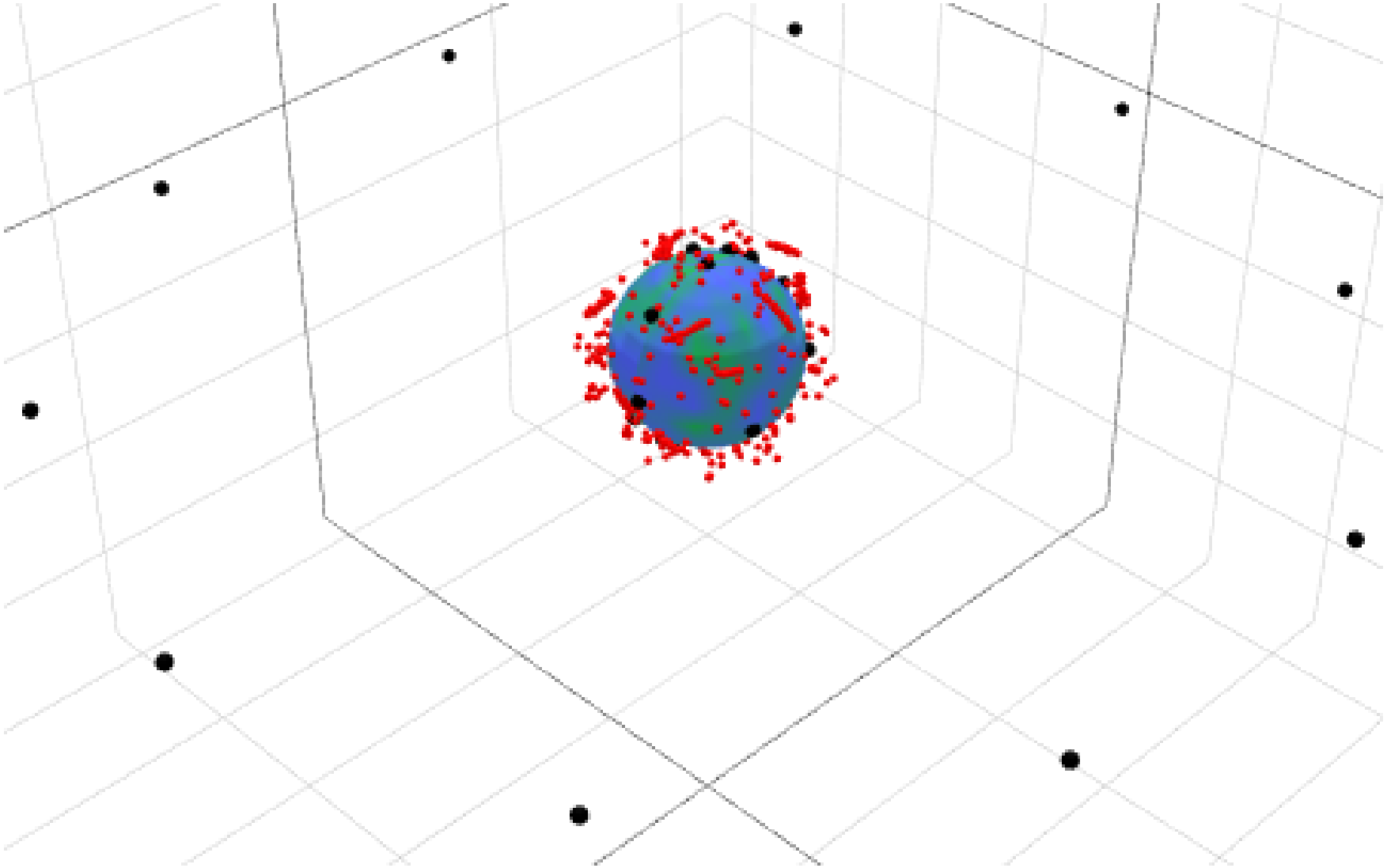
We ran also tests of 30 trials with 50 satellites in 1 cluster.

- Varied the 5 parameters listed above and calculated average success rate for each metric and modality
- For the top 2 performing metrics, used a t-test with $\alpha = 0.05$ to determine which metric performed significantly better than the others

Experiment Visual



SIMULATING-LEO: Results

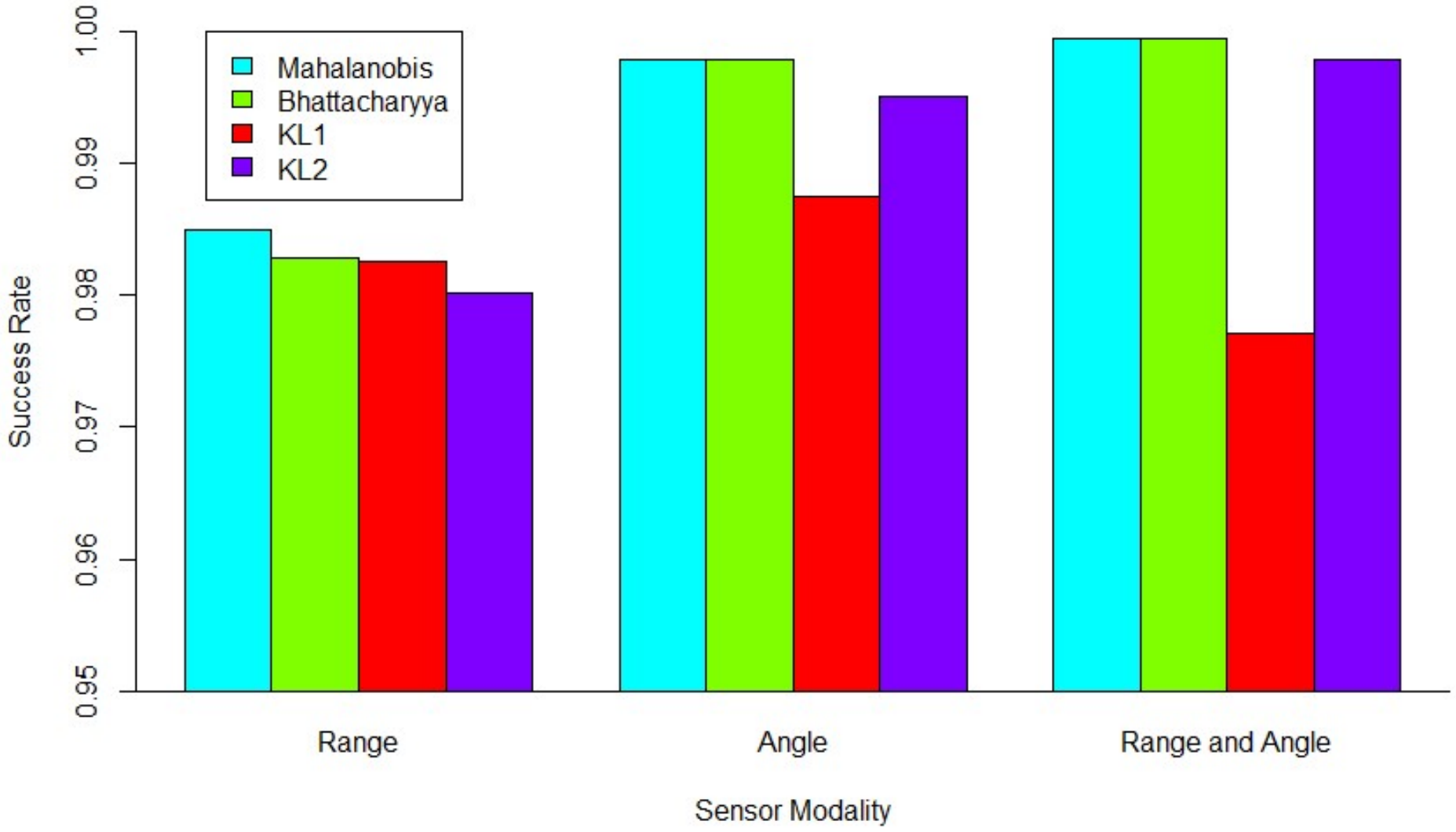


Figure 9. SIMULATING-LEO

General Results

- Single Cluster Tests
  - Mahalanobis performs best for range measurements, and significantly better when the sensor noise is low
  - KL2 performs best for angle and range-and-angle measurements, and has significantly higher accuracy in cases of tighter clusters and higher sensor noise

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- Single Cluster Tests
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- `SIMULATING-LEO`
  - Mahalanobis is consistently the best metric
  - Success rates for range measurements are overall lower than those of angle and range-and-angle measurements by $\approx 1\%$

Future Work

- Testing more existing metrics and experimenting with new metrics
- More realistic dynamical system
- Range and angle rate sensors

Acknowledgements

- Thank you to IPAM, the Aerospace Corporation, and UCLA for all their support during the RIPS Program!
- Industry Mentors: Daniel Lubey, James Gidney, and Andrew Binder of the Aerospace Corporation
- Academic Mentors: Minh Pham of UCLA, Susana Serna of IPAM

Questions?

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- Set to these values when varying:
  - Observation gap $t_1 - t_0$ (seconds): $300, 1800, 3600, 7200$
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  - Dispersal σ_r^2 (m^2): $10^6, 10^8, 10^{10}$
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- Set to nominal values when not varying:
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- If we pick 2 parameters to vary, we can vary them in higher resolution

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- If we pick 2 parameters to vary, we can vary them in higher resolution
- Use same nominal values when fixed
- Vary across these ranges in 20 equal increments:
  - Observation gap: $[300, 7200]$ s
  - Altitude: $[350, 2000]$ km
  - Dispersal: $[10^4, 10^{10}]$ m^2 (log scale)
  - Satellite *a priori* uncertainty: $[10^0, 10^4]$ m^2 (log scale)
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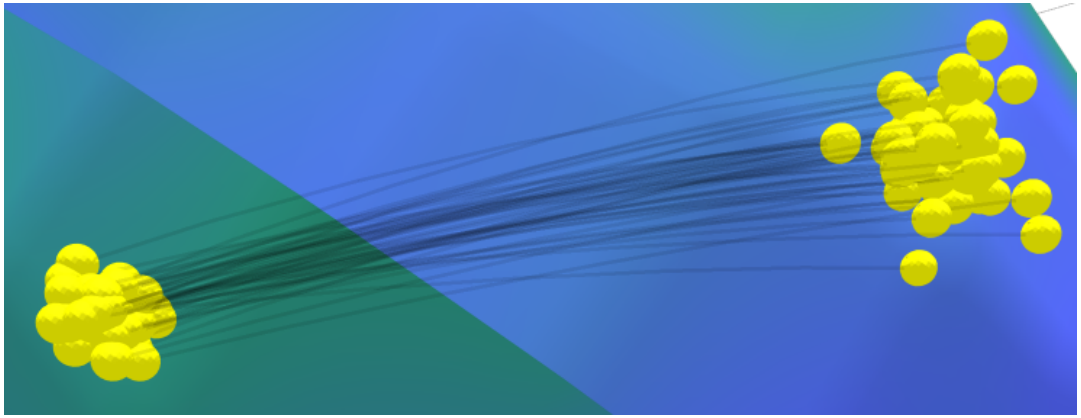
CLUSTER-OUT: Determining Impactful Variables



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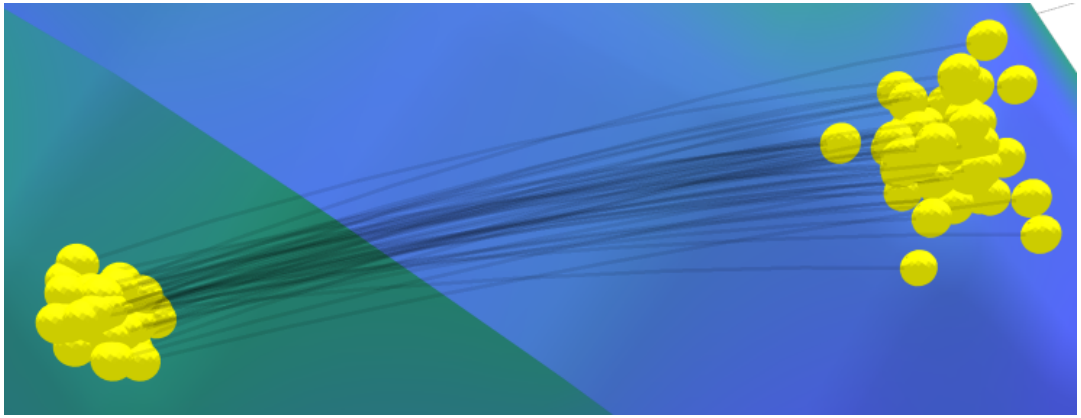

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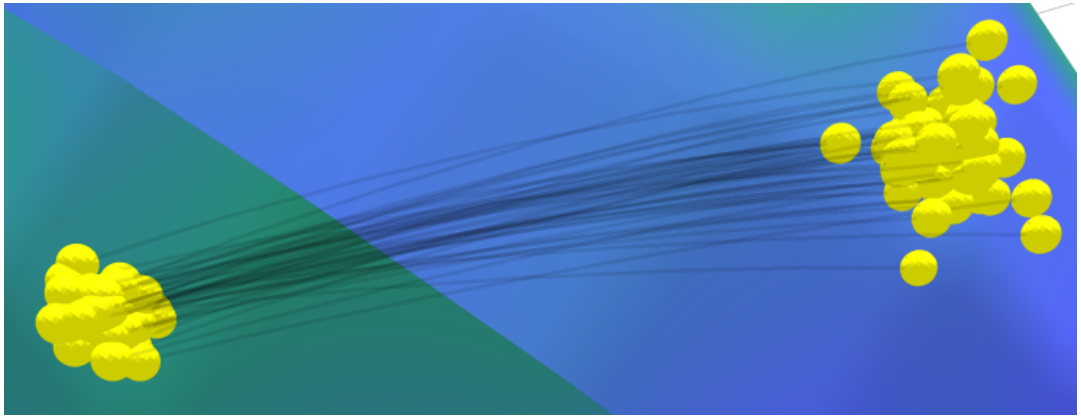

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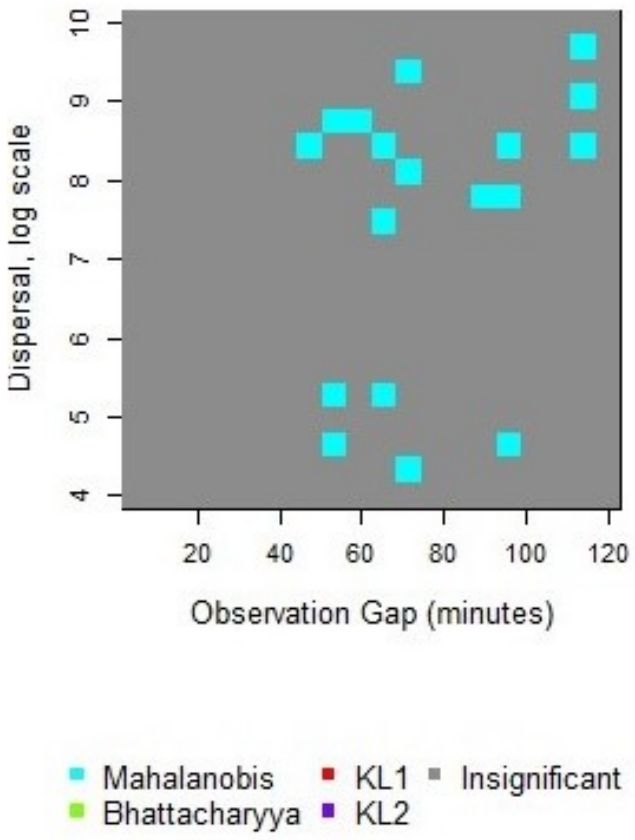
- If the null hypothesis is rejected, color the cell according to the metric
- Else, color the cell gray since the win is insignificant



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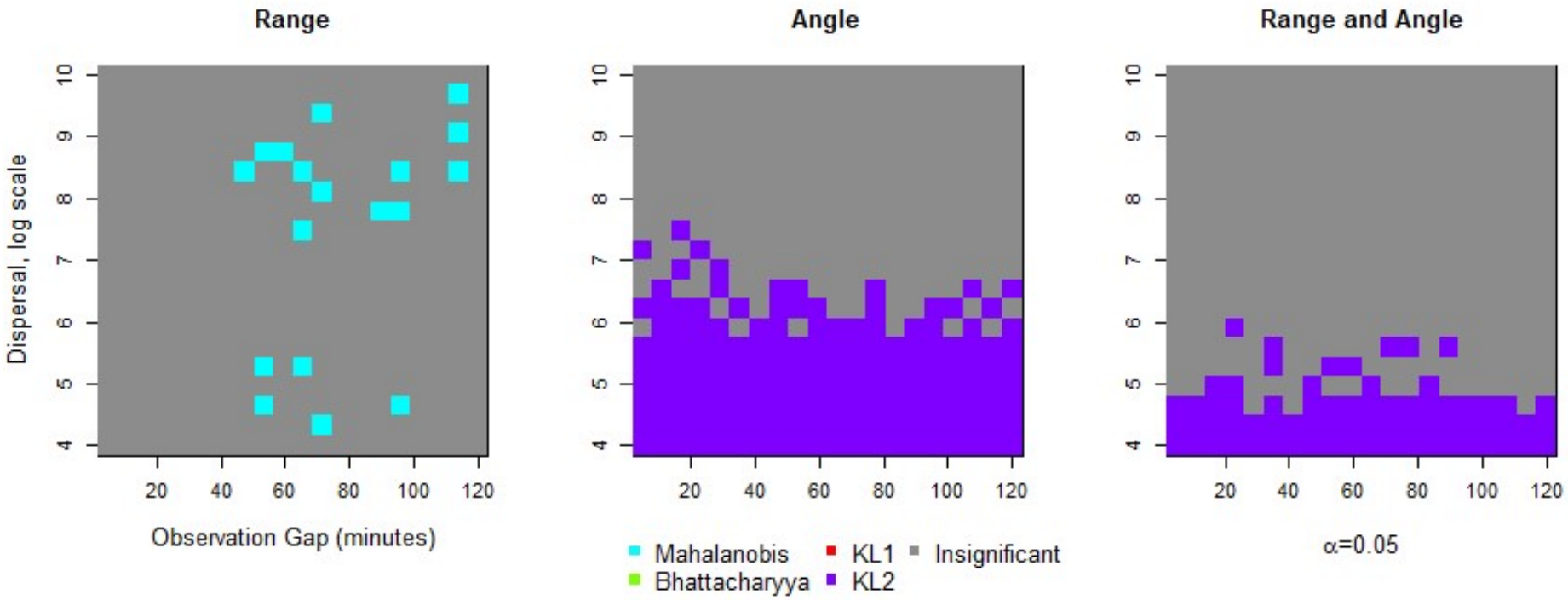

Figure 9. CLUSTER-OUT Test: Observation Gap vs Cluster Dispersal

CLUSTER-OUT: Results

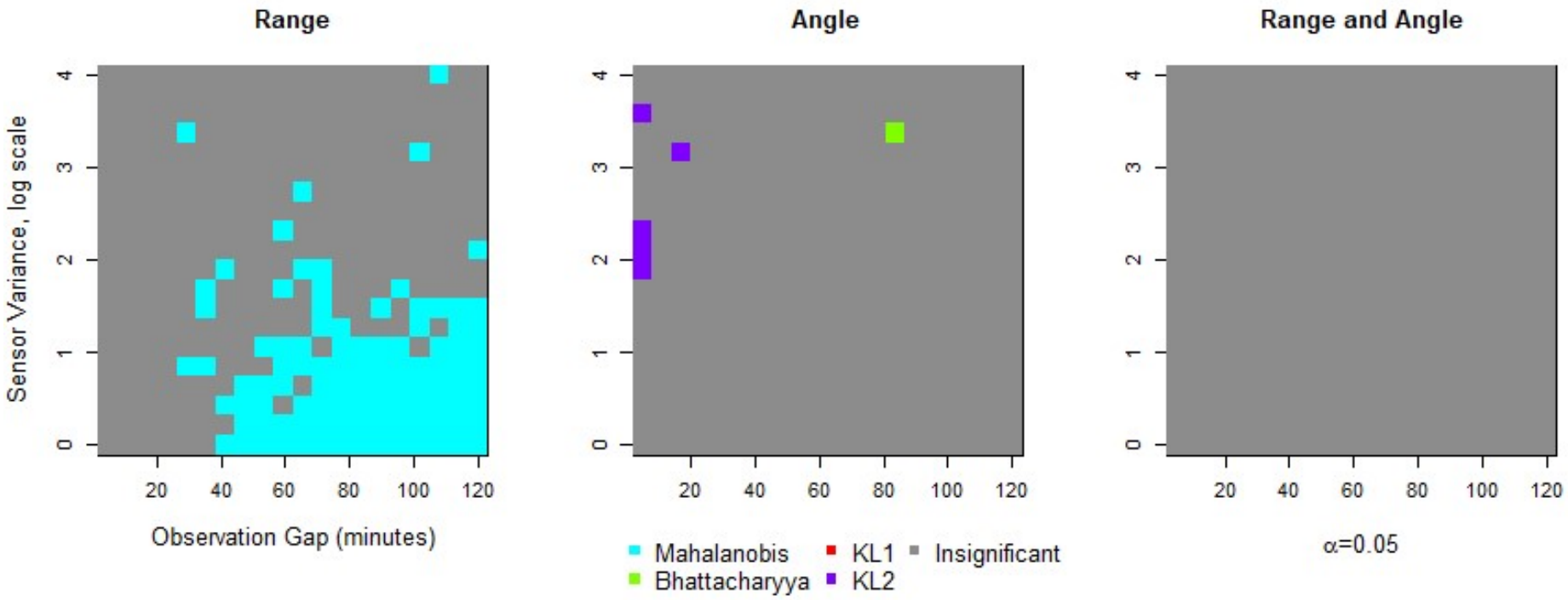

Figure 10. CLUSTER-OUT: Observation Gap vs Sensor Variance

CLUSTER-OUT: Results

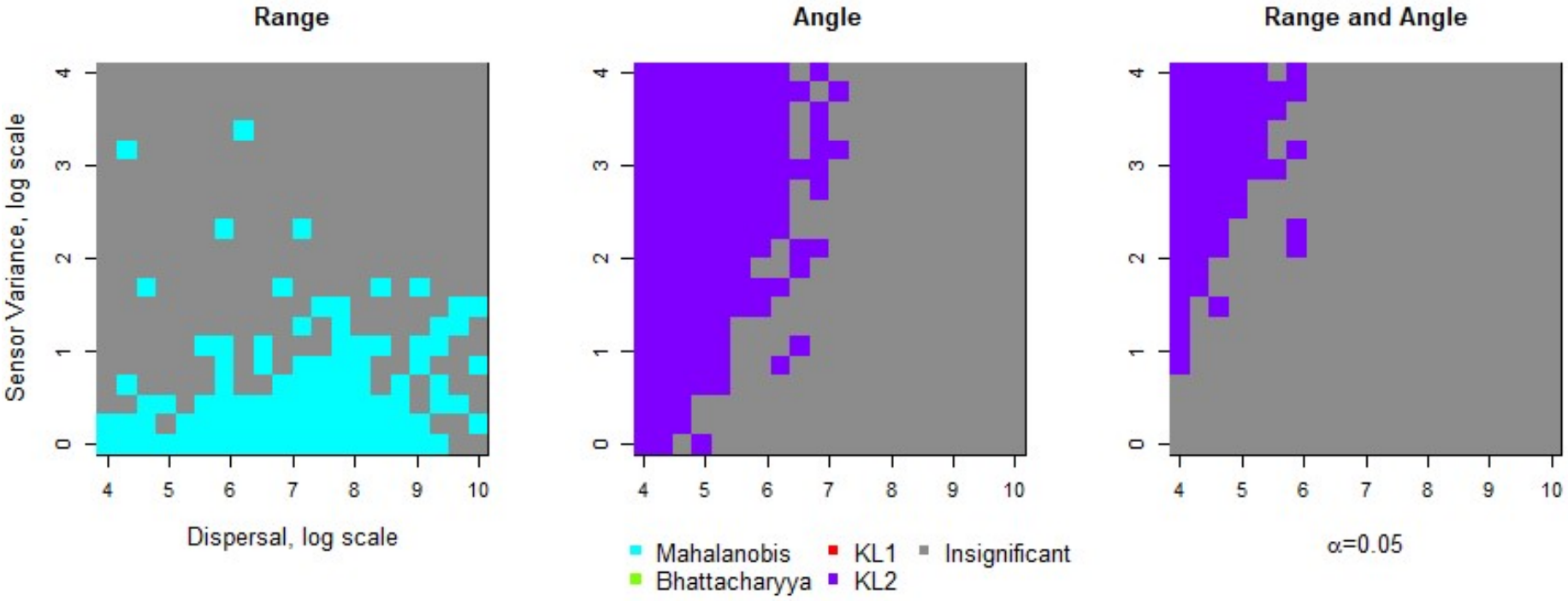

Figure 7. CLUSTER-OUT: Cluster Dispersal vs Sensor Variance

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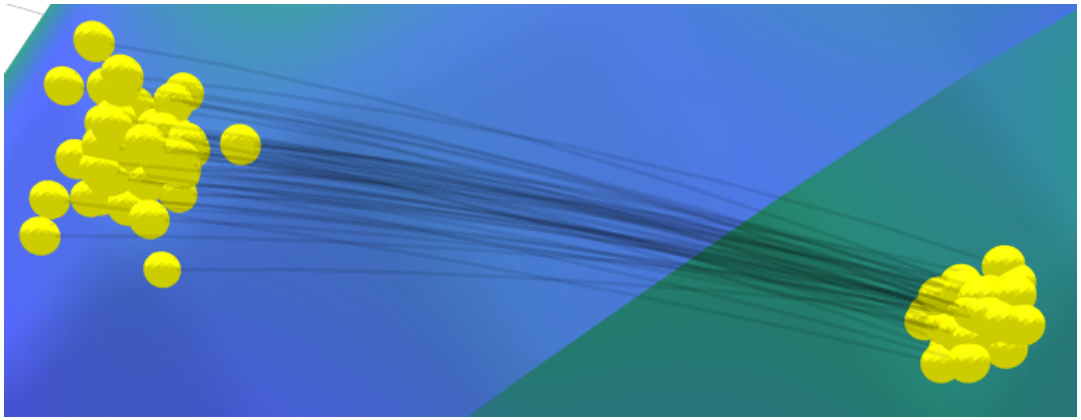
- Range and angle measurements: KL2 wins for dispersal $\log(\sigma^2_r) < 5.5$ and generally higher sensor variance

CLUSTER-IN: Determining Impactful Variables



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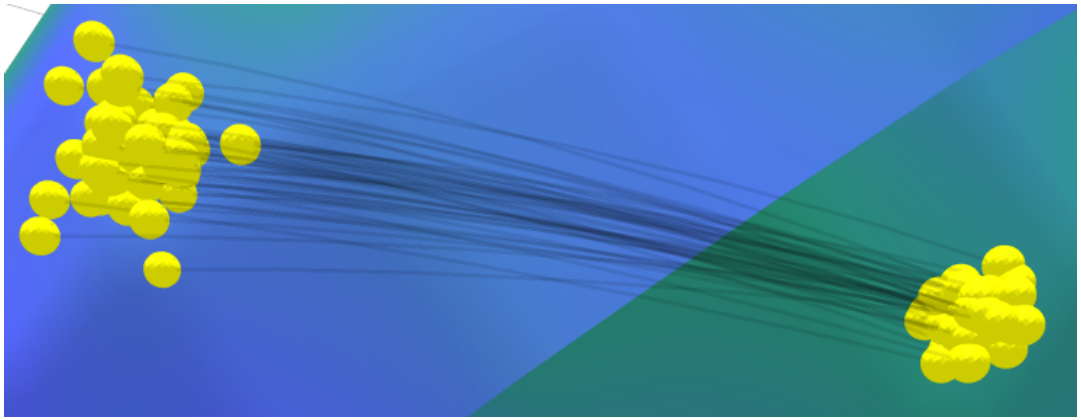

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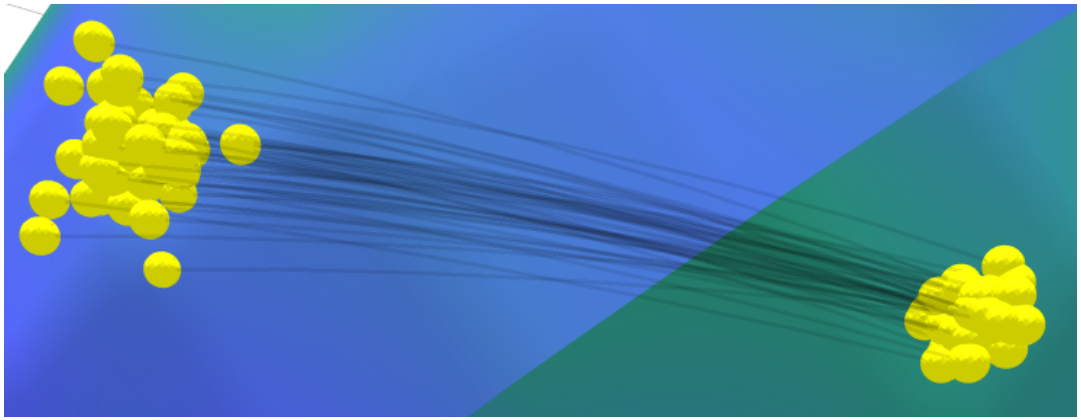

- Run 30 trials, with 50 satellites going towards a cluster
- For each metric and modality, use LASSO to determine 3 most impactful parameters
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- Altitude appears 7 out of the 12 times, with observation gap appearing 5 times

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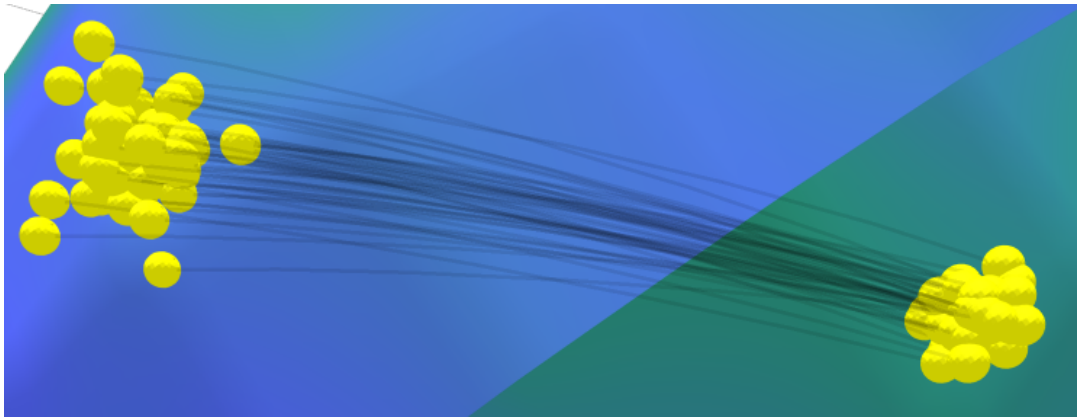

- Run 30 trials, with 50 satellites going towards a cluster
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- Will run finer pairwise investigation on altitude, dispersal, and sensor variance

CLUSTER-IN: Results

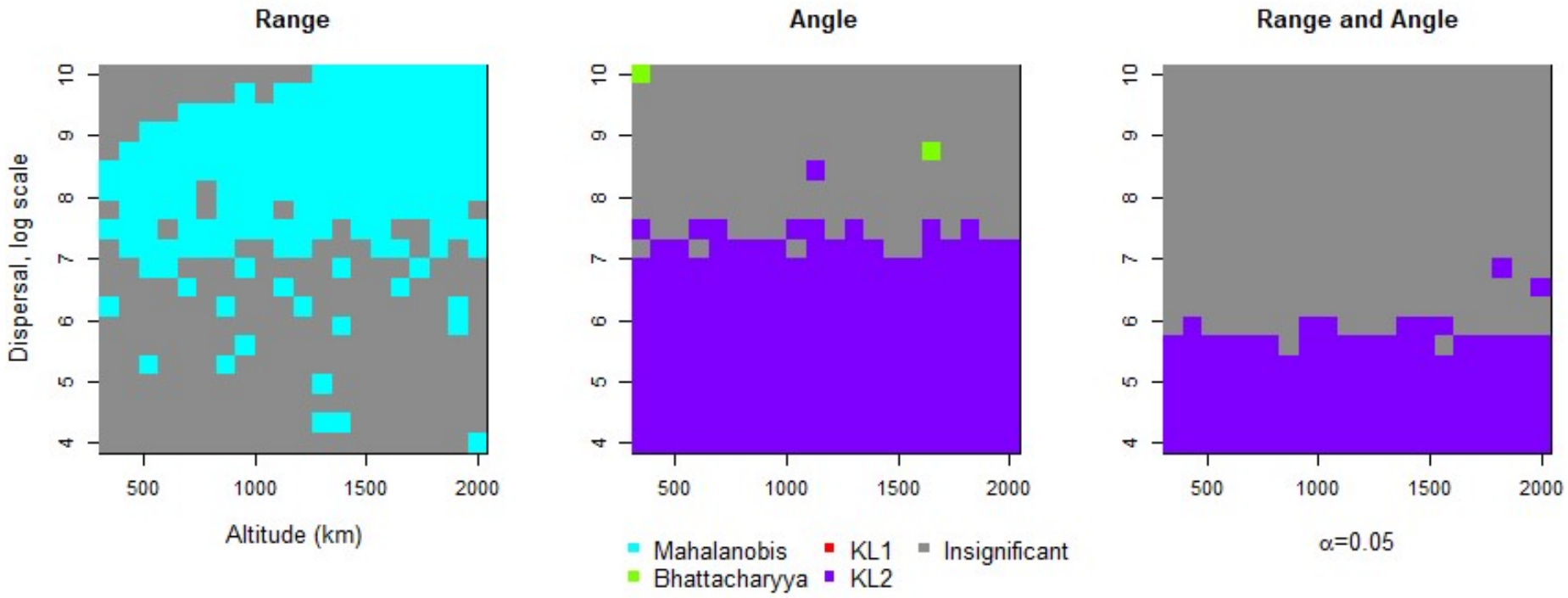

Figure 12. CLUSTER-IN: Altitude vs Cluster Dispersal

CLUSTER-IN: Results

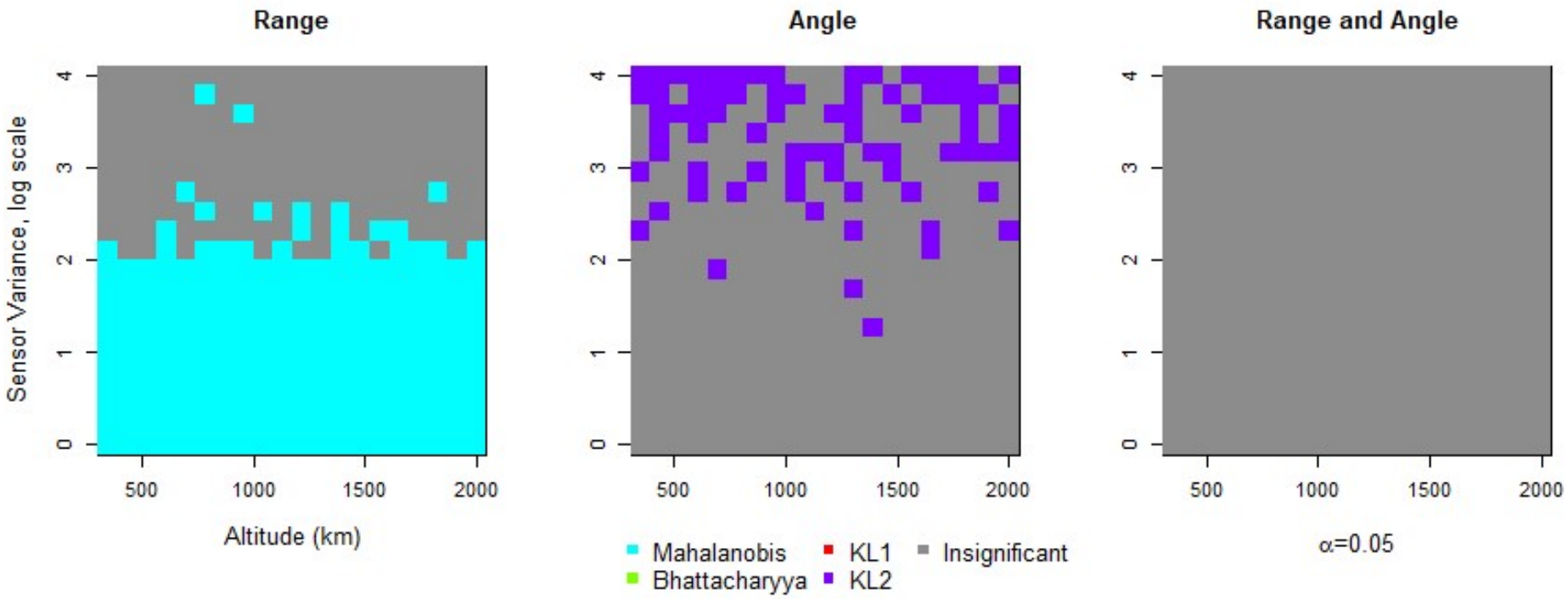

Figure 13. CLUSTER-IN: Altitude vs Sensor Variance

CLUSTER-IN: Results

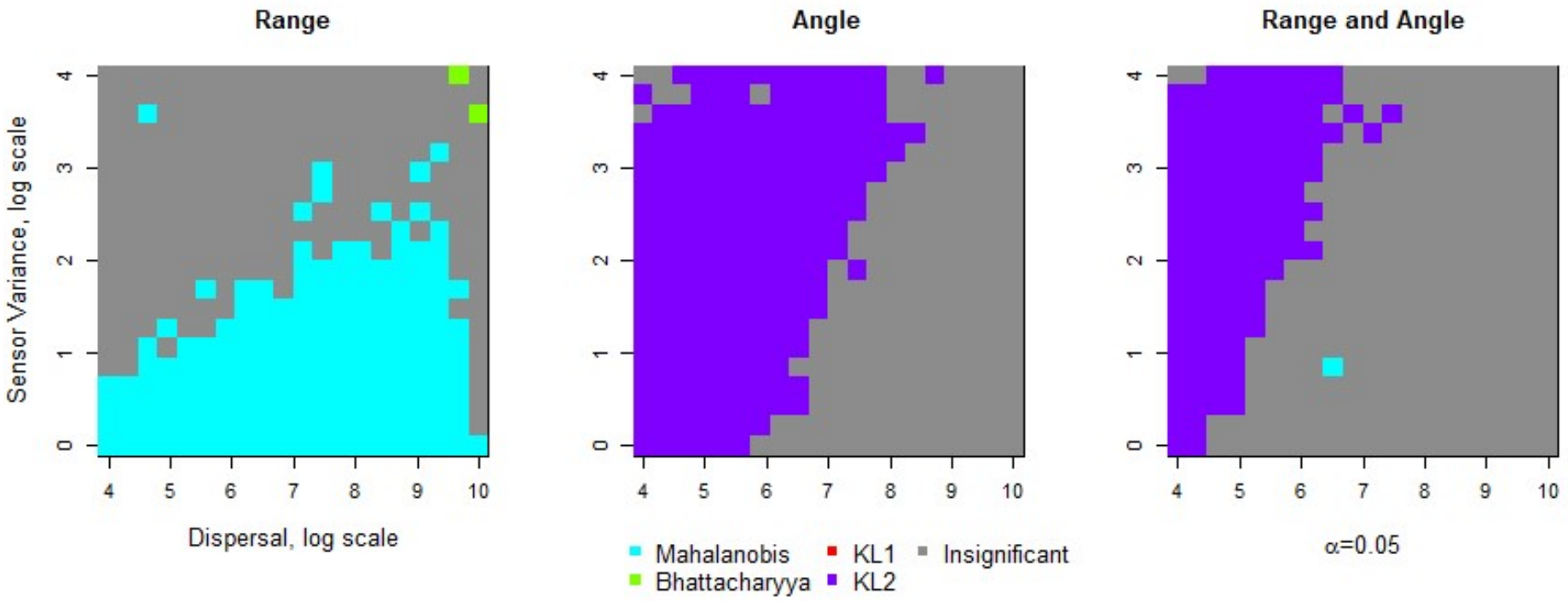

Figure 8. CLUSTER-IN: Cluster Dispersal vs Sensor Variance

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- Range and angle measurements: KL2 wins for dispersal $\log(\sigma_r^2) < 6$ and any altitude

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Figures

Figure 1. Catalogued Objects in Space Surveillance Network. G. Peterson, M. Sorge, and W. Ailor, *Space Traffic Management in the Age of New Space*, tech. rep., The Aerospace Corporation, April 2018.

Figure 4 Joshua Cesa (https://commons.wikimedia.org/wiki/File:Azimut_altitude.svg), "Azimut altitude", Text, https://creativecommons.org/licenses/by/3.0/legalcode

Figures 2 and 5-9 were generated by our own simulation framework, using Plot.ly and R.