Exploring the garden of Petaluma knots

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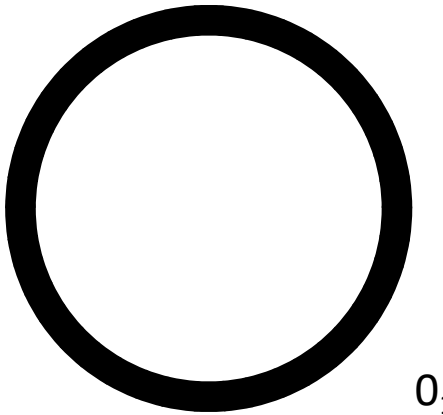
Knots and Knot Diagrams

- A closed curve in 3-dimensional space
- Typically visualized using knot diagrams
  - 2-D regular projection
- Knots are classified by *Crossing Number*



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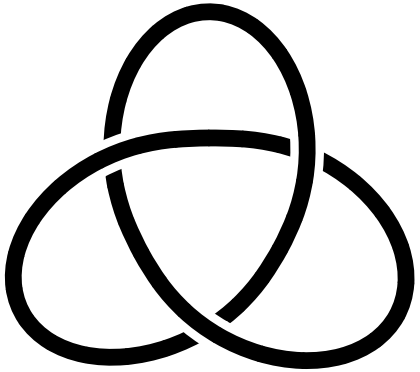

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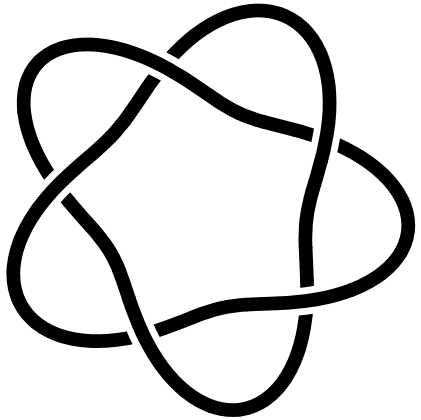

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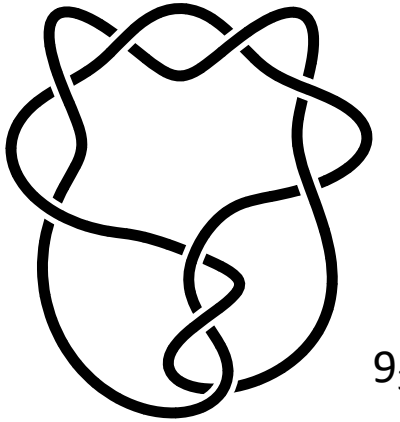

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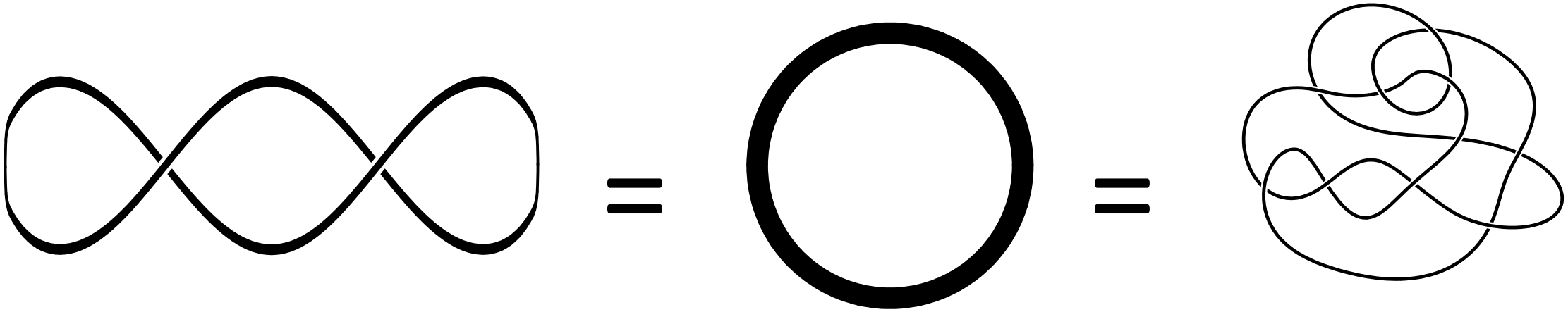
Knot Equivalence

- Knots are malleable
  - Not all knot diagrams are nice



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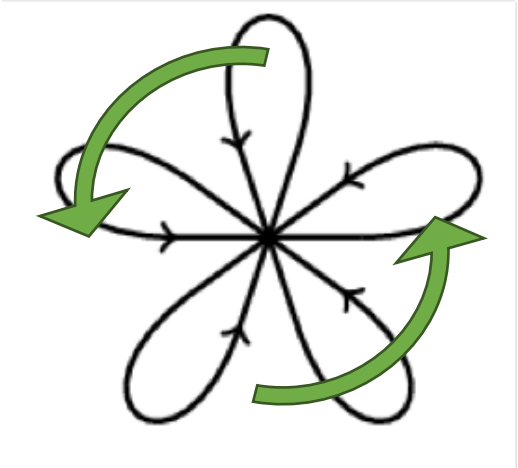
  - That is why crossing number is the least amount of crossing of any diagram

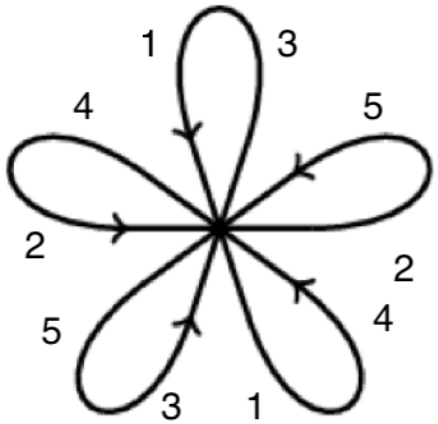
Petaluma Model

- All crossings in the center
- Classified by number of “petals”
- Impossible to tell which strands go over and under
- Heights assigned by a string of numbers (1,2,…,n)

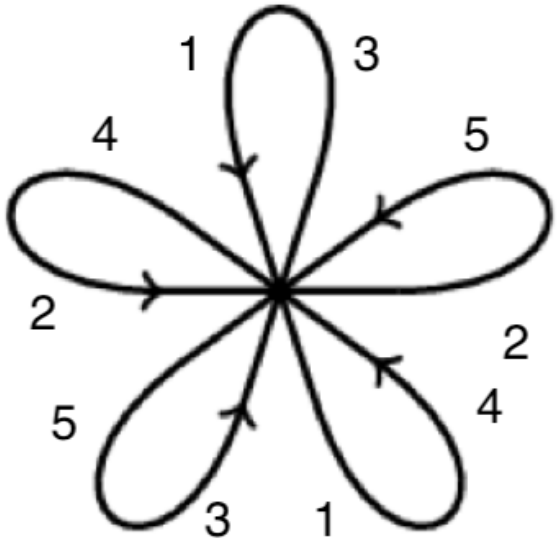
How it works

- Generate a string of numbers 1 to (# of petals)
  - Ex. For a 5-petal knot, (1,4,2,5,3)
- Assign a counterclockwise orientation
- Assign heights using the string of numbers
  - 1 is the highest (on top)
  - n is the lowest (on bottom)
- See what type of knot we get





Example: 5 petals with (1,4,2,5,3) heights

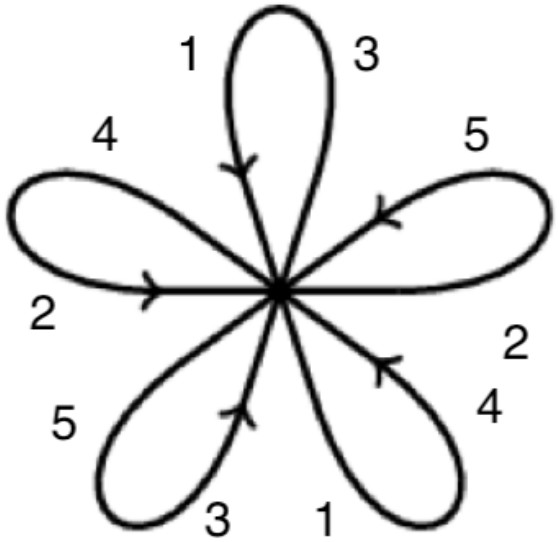


Assign Heights Counter-Clockwise

Top View

Side View

Example: 5 petals with (1,4,2,5,3) heights



Assign Heights Counter-Clockwise

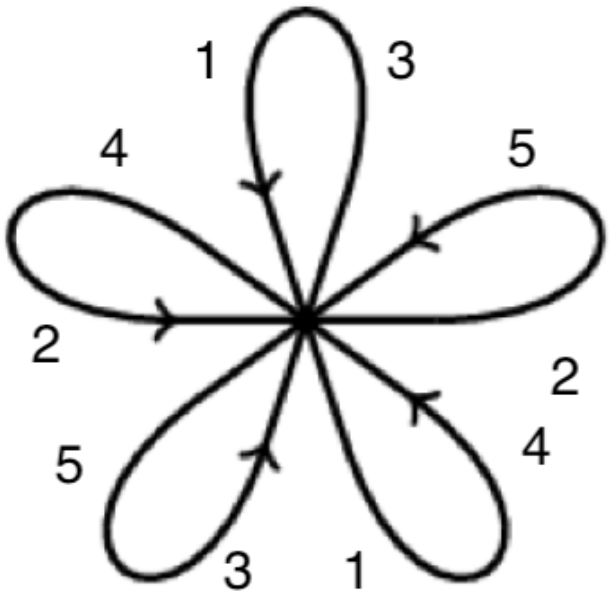
Top View

Side View

Petaluma Properties

- Cyclic invariance
  - Ex. 14253=42531=25314 etc.
  - This allows us to start every height at 1 (Heights ={1,2,…,n})

- For a diagram with n petals there are (n-1)! petal diagrams we can generate



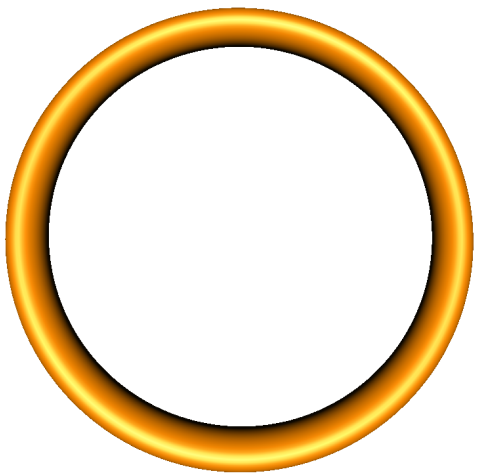
Petal Number	3	5	7	9	11	13
# of Petal Diagrams	2	24	720	40,320	3,628,800	479,001,600

Project Goals

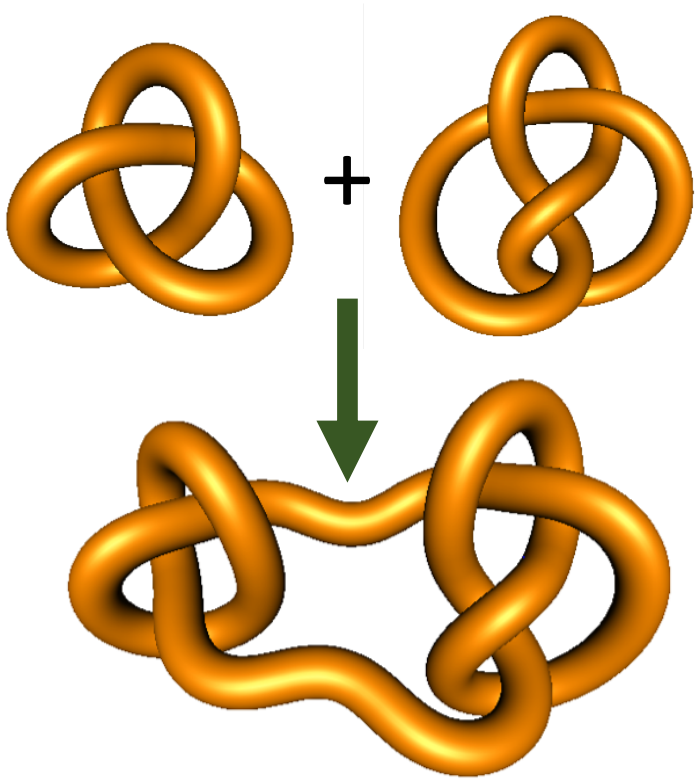
- Classify all 5, 7, 9, 11, and 13 petal knots
- Separate the knots into 5 categories
  - Unknots
  - Alternating
  - Non-Alternating
  - Composite
  - Unknown

- Study relationships within a petal number
- Study relationships across petal numbers

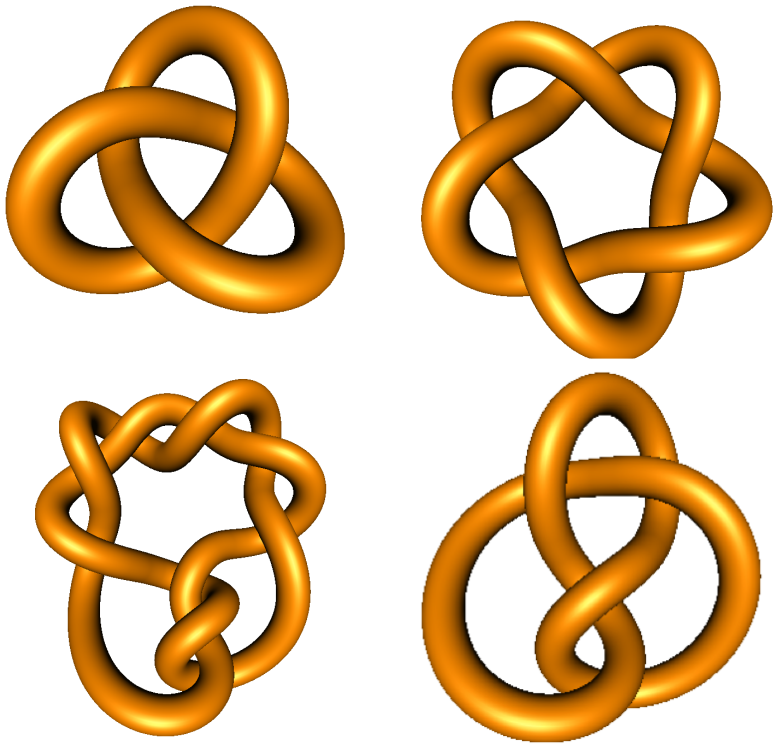
Knot Categories



Unknot

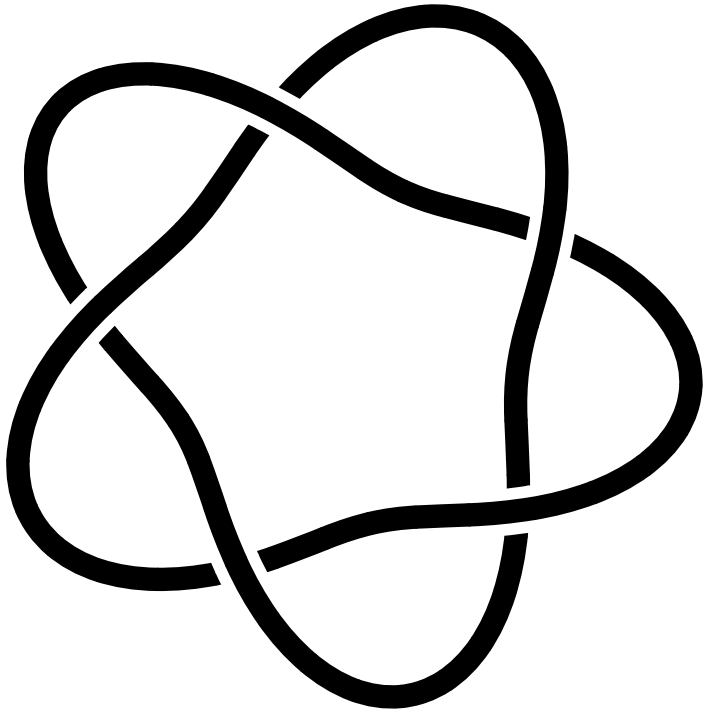


Composite Knots



Prime Knots

Alternating vs. Non-Alternating



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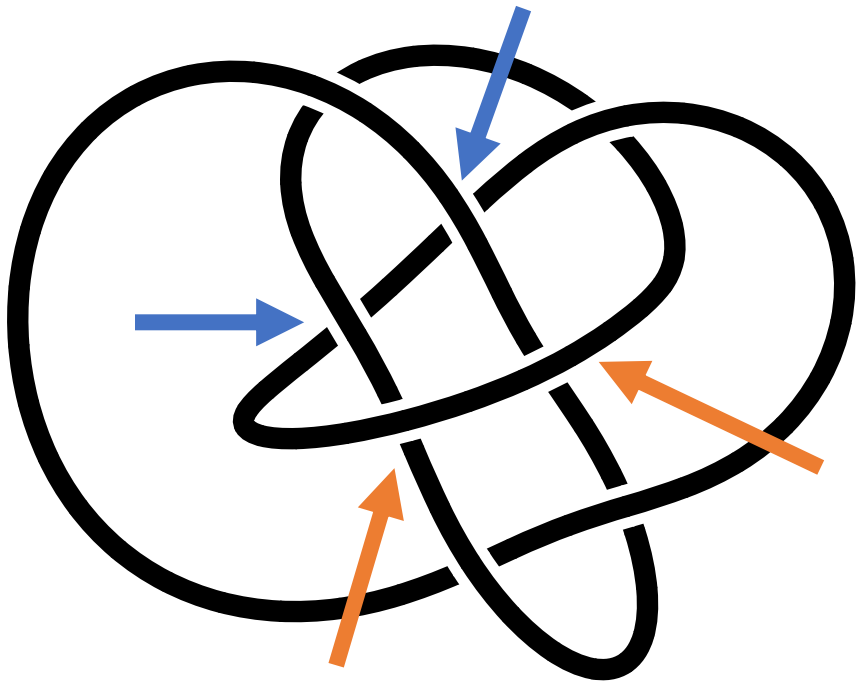

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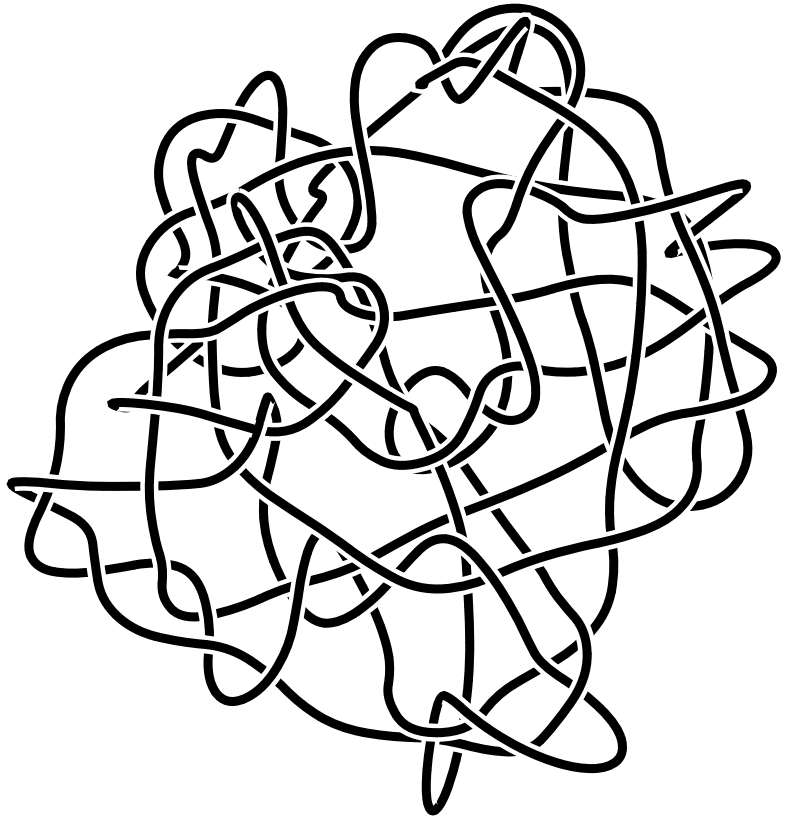
Unknowns

- All knots have been classified through 16 crossings
- Unknowns have 17 or more

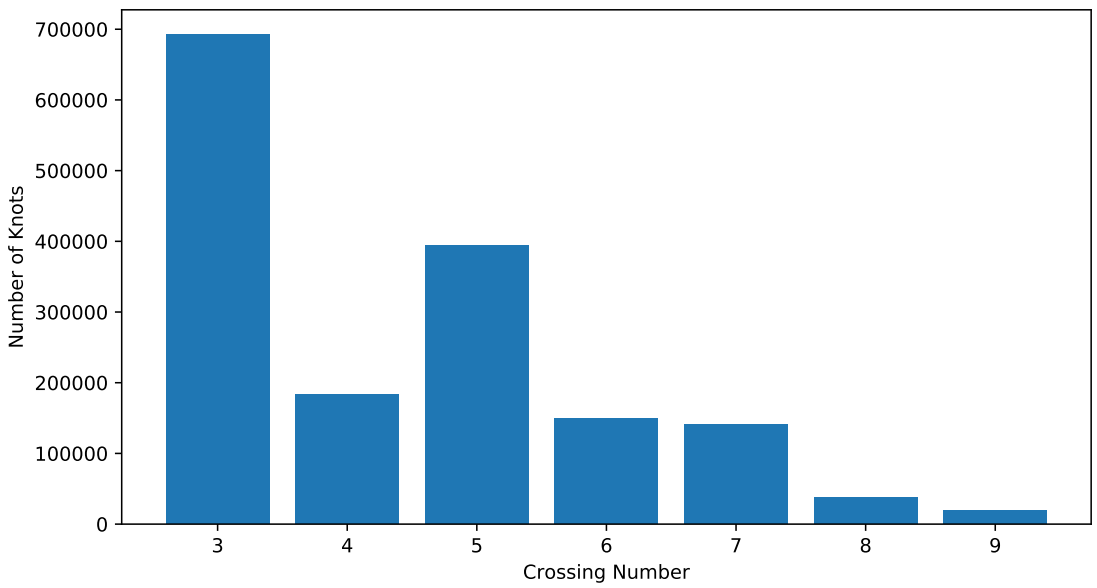
- So in our data:
  - Unknots have crossing number 0
  - Alternating have crossing number ≥3
  - Composite have crossing number ≥6
  - Non-alternating have crossing number ≥8
  - Unknowns have crossing number ≥17



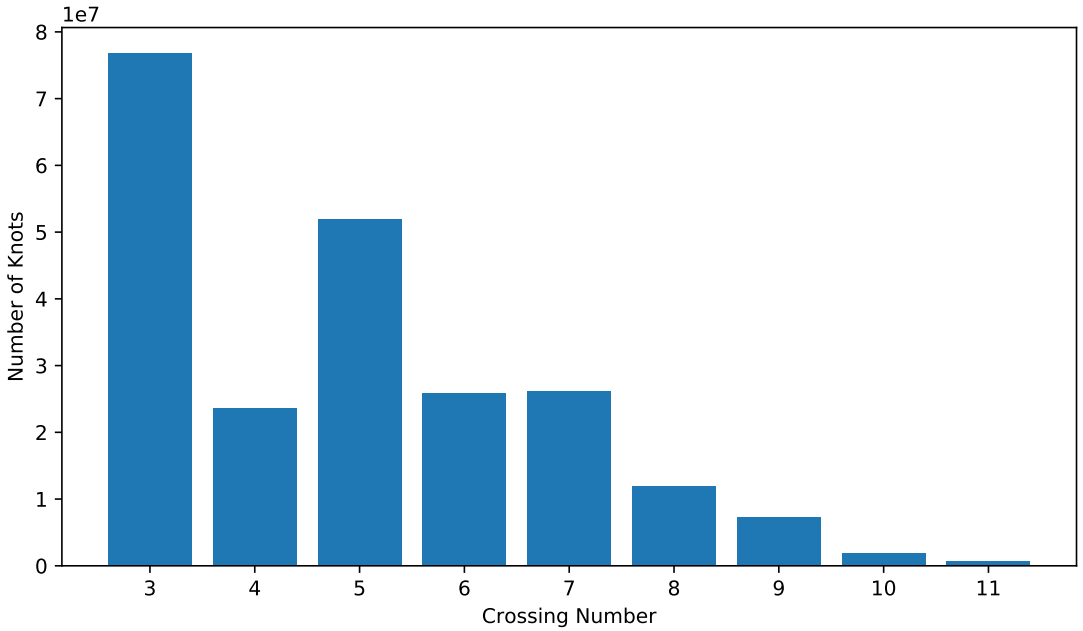
Alternating Knots by Crossing Number

- Alternating knots favor odd crossing numbers

11 Petals

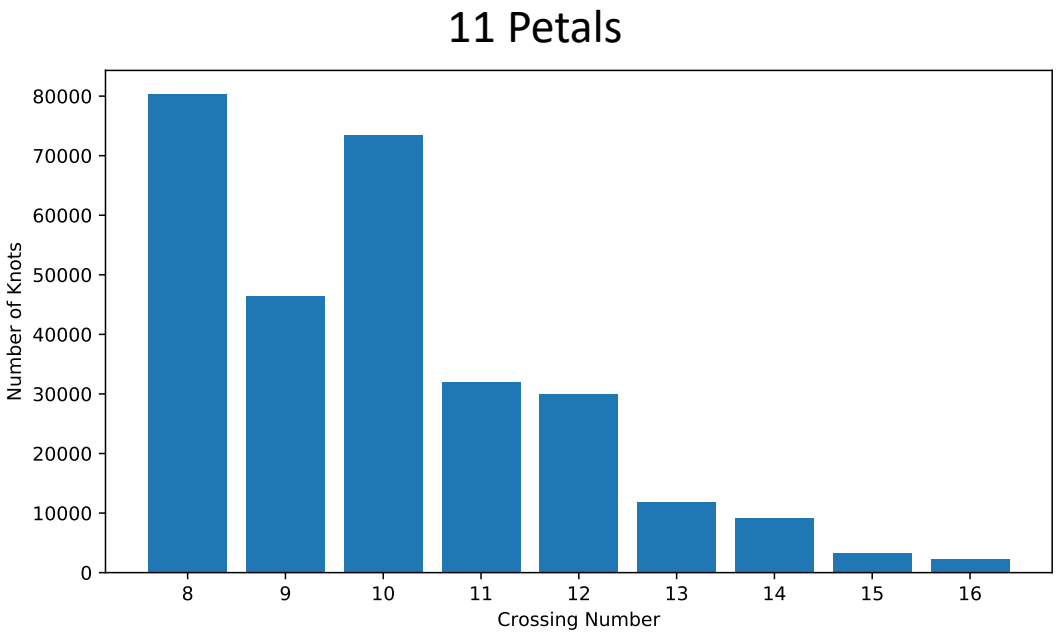


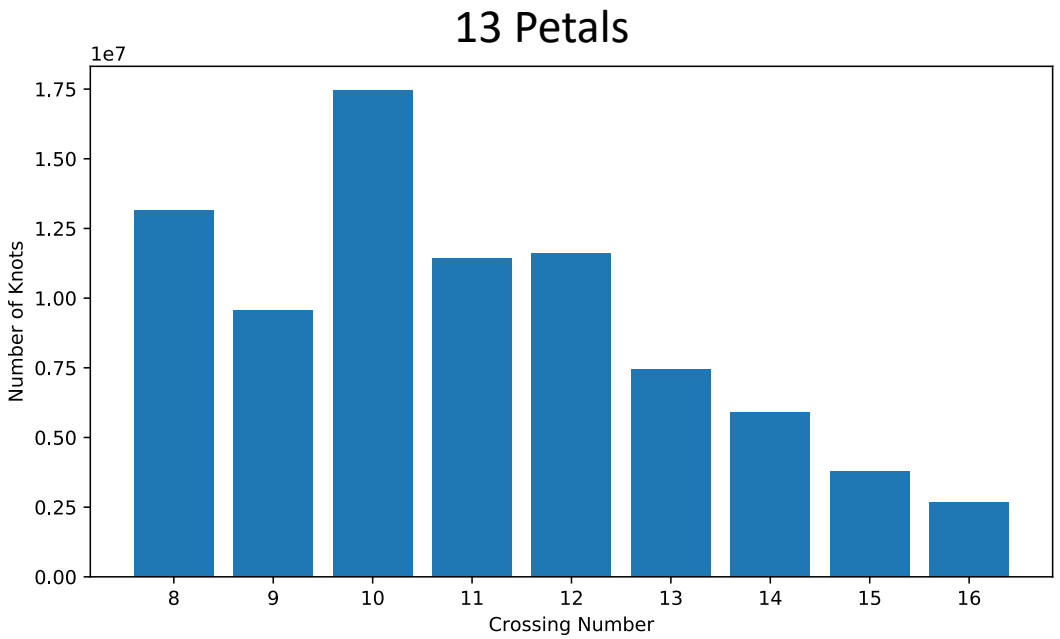
13 Petals



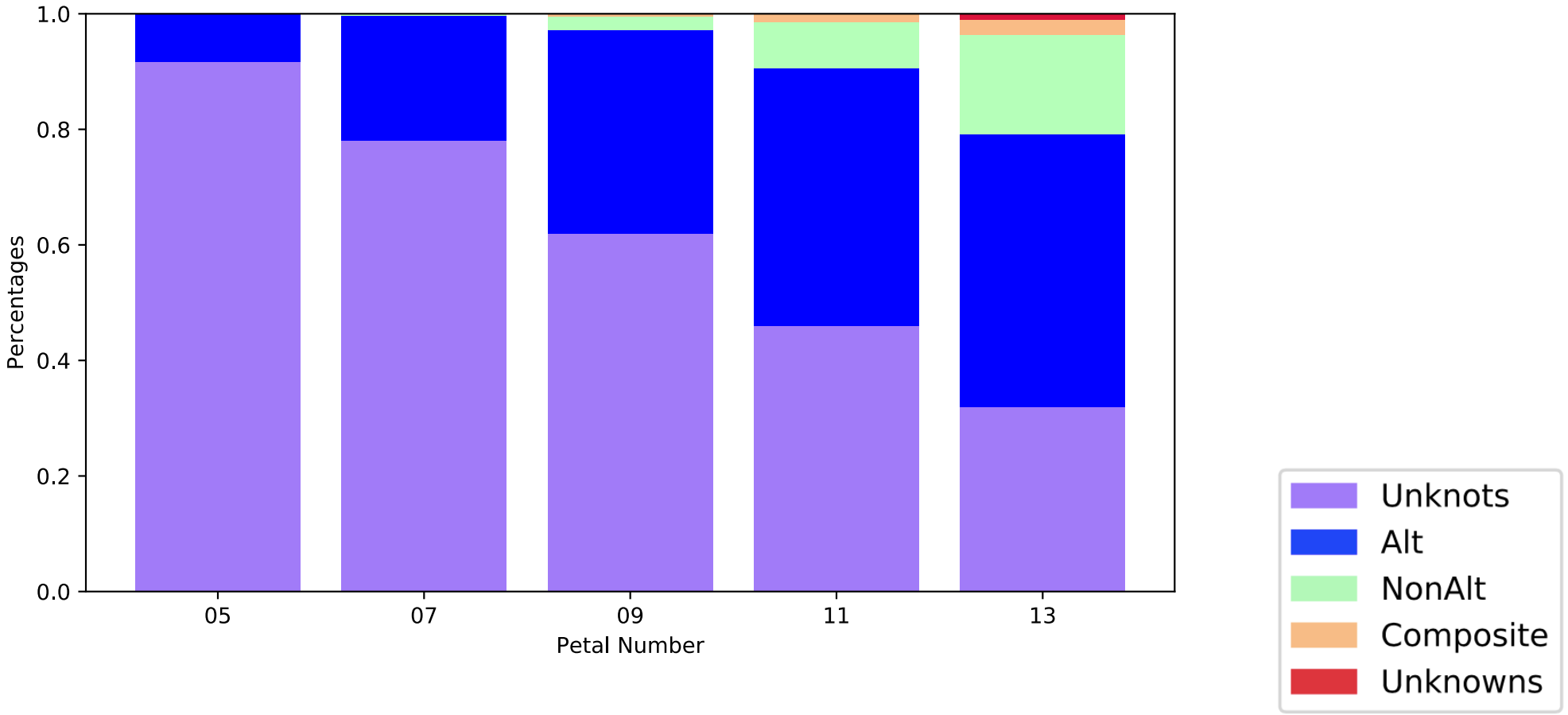
Non-Alternating Knots by Crossing Number

- Non-Alternating knots favor even crossing numbers

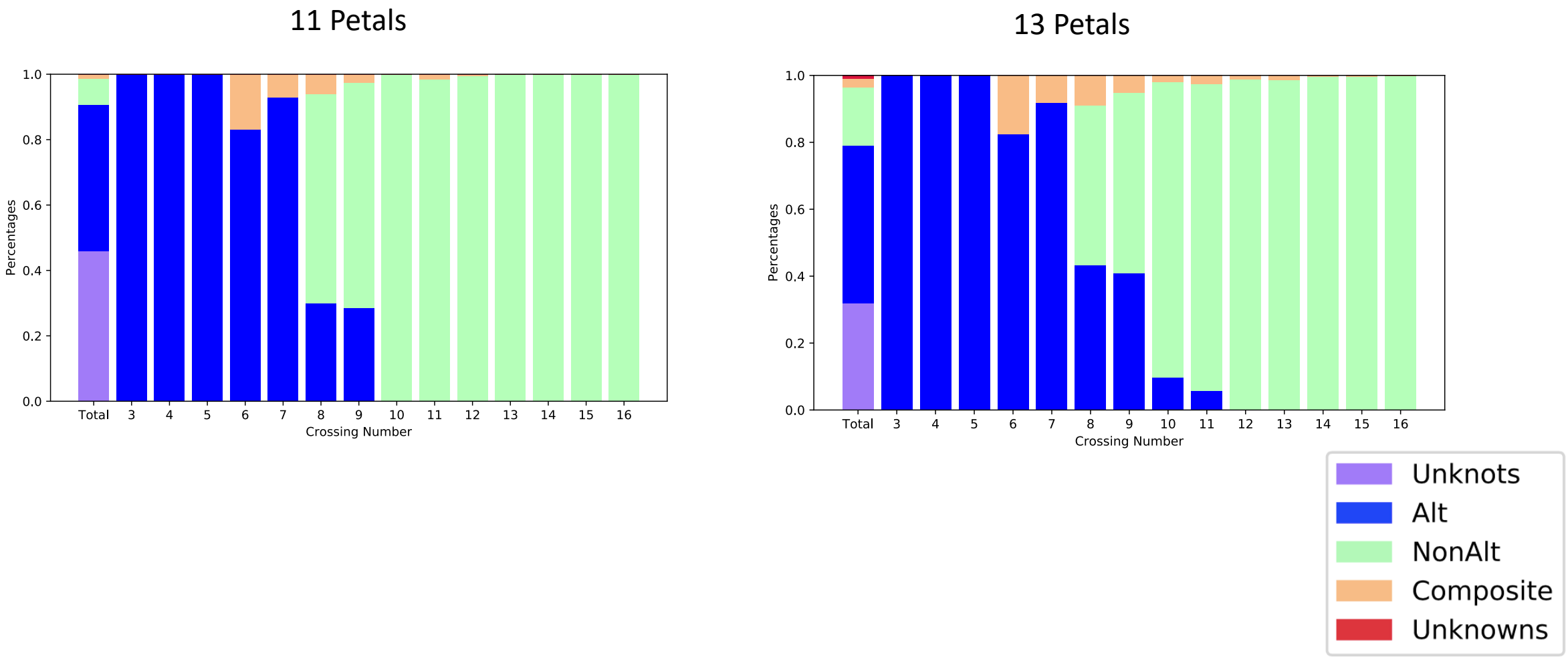




Totals for All Petal Numbers



Category Percentages by Crossing Number



Future Work

- Tabulate 15 Petals
  - Computationally heavy – use properties to thin out how many need to be generated

Petal Number	3	5	7	9	11	13	15
Number of knots	2	24	720	40,320	3,628,800	479,001,600	87,178,291,200

- Study/create other knot models

Acknowledgments

- Dr. Eric Rawdon
- Dr. Jason Parsley and Grace Yao (Wake Forest University)
- Brandon Tran and Elizabeth Whalen
- National Science Foundation
- KnotPlot by Rob Scharein (Hypnagogic Software)

- Even-Zohar et al., The distribution of knots in the Petaluma model, Algebraic & Geometric Topology 18 (2018) 3647–3667.

Thank You