Effects of Mars' Chaotic Obliquity on Ice Cover: A Budyko Approach

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Planetary Climate

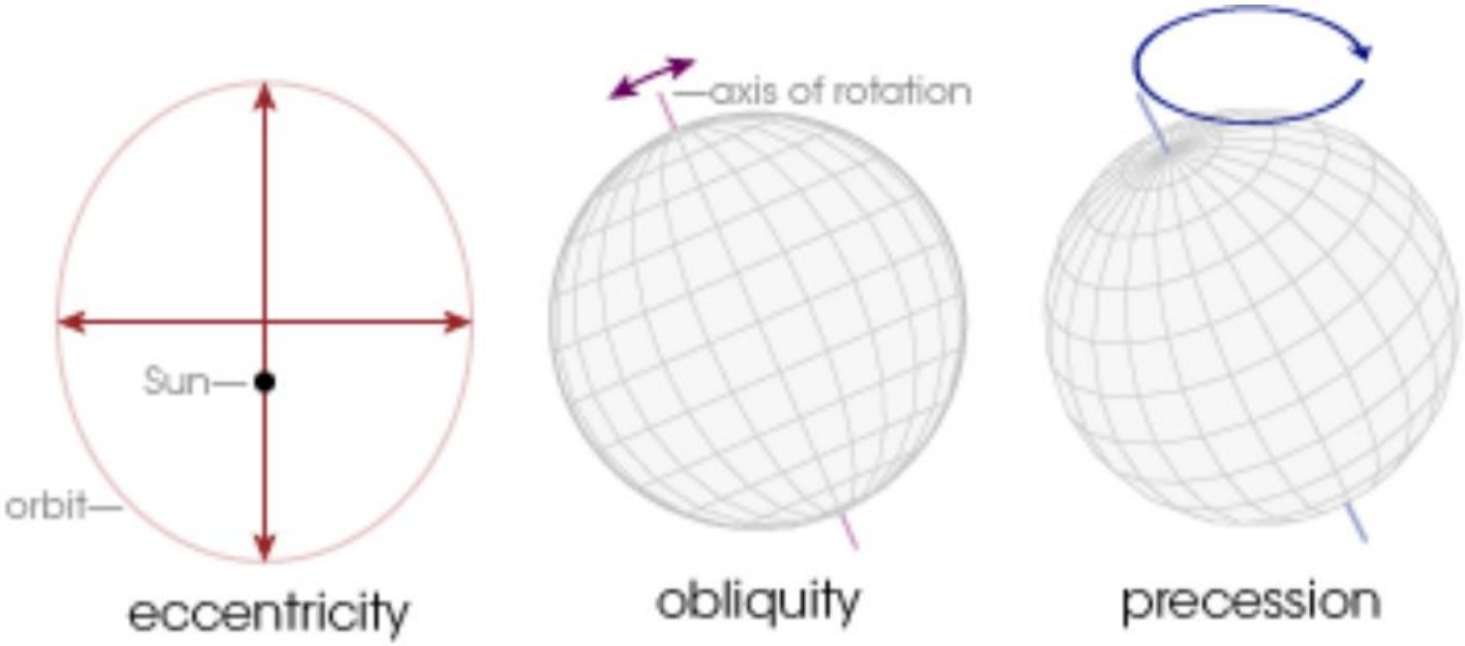

Figure: Variables of A Planet's Orbit [https://earthobservatory.nasa.gov]

Mars' Chaotic Obliquity

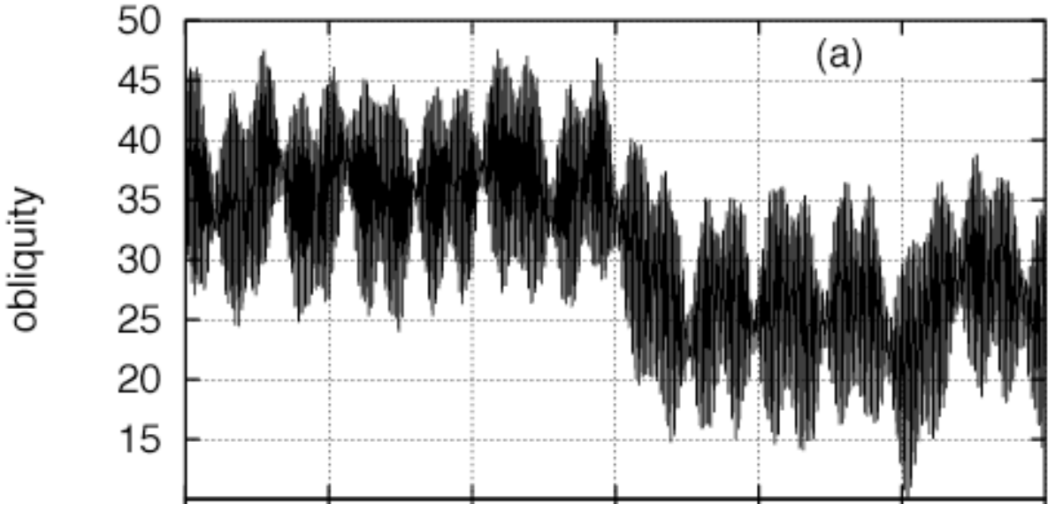

Figure: Obliquity at the north pole surface of Mars at the summer solstice. Time Axis: -20Myr to 10Myr in increments of 5Myr [7]

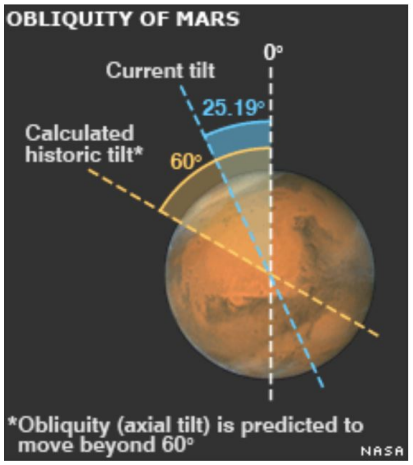

Figure: Change in Mars' Obliquity

Energy Balance: Earth as an Example

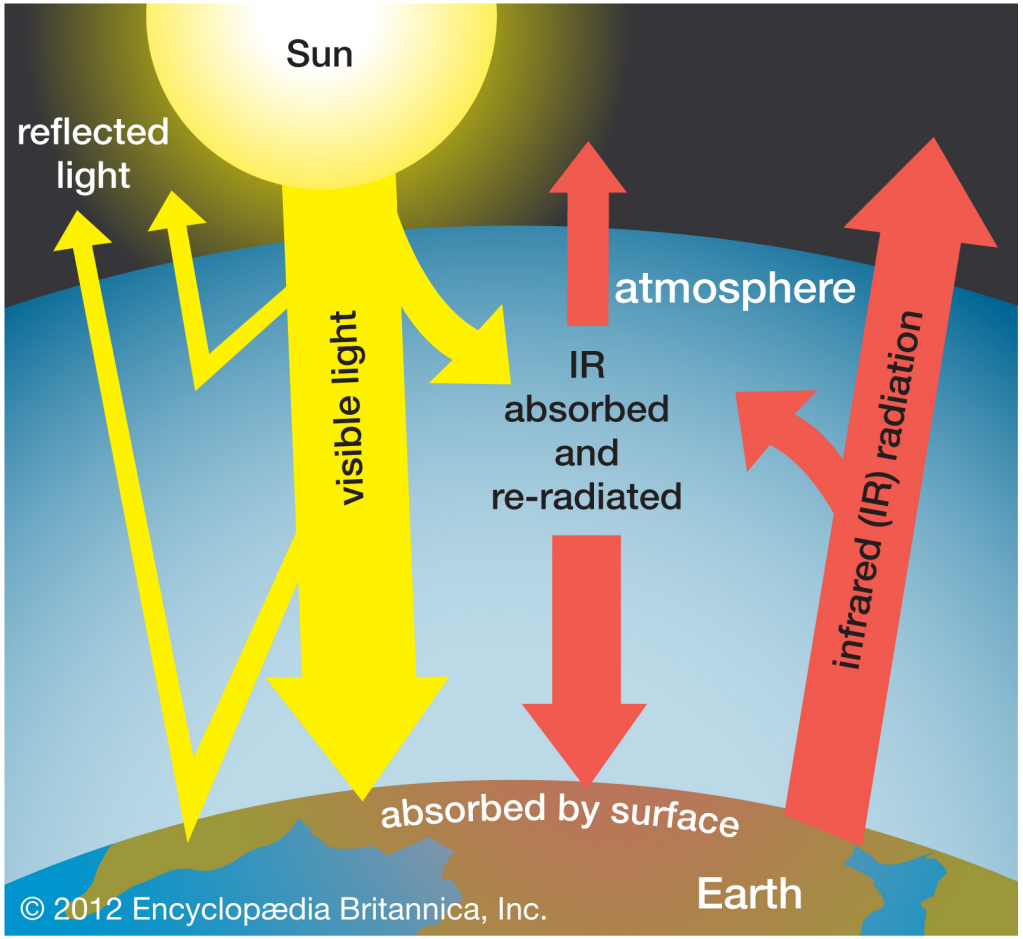

Figure: A: Energy Balance Diagram on Earth

Temperature Change = Energy In - Energy Out

Budyko Equation Model

$$R\frac{\partial T}{\partial t} = \underbrace{Qs(y)(1-\alpha(y,\eta))}_{\text{absorbed insolation}} - \overbrace{(A+BT(y,t))}^{\text{emitted longwave radiation}} - \underbrace{C\left(T(y,t)-\overline{T}(t)\right)}_{\text{energy transport across latitudes}} \tag{1}$$

Energy Balance: Temperature change = energy in - energy out
R denotes heat capacity of surface layer

Absorbed Insolation: Finding the Annual Average Q and Distribution

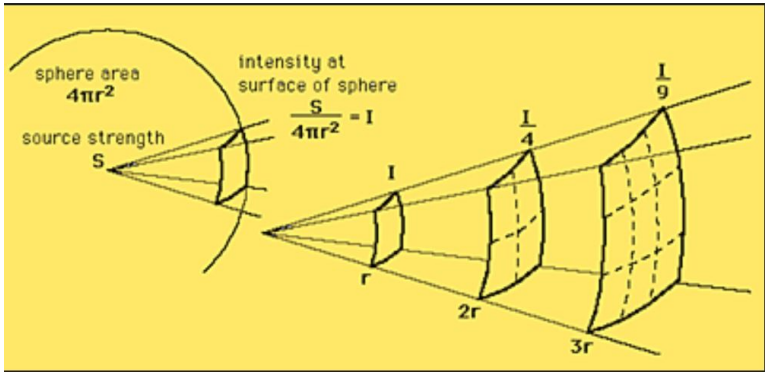

Figure: Finding Solar Constant (I) (Irradiance per Area) for a Planet [nasa.gov]

$$Qs(y)(1-\alpha(y,\eta))$$

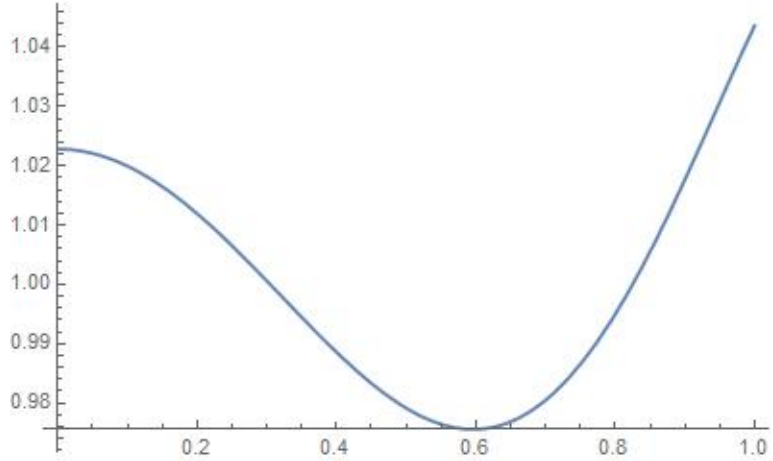

Figure: S-plot representing the distribution function of Q across the sine of latitude.

Albedo

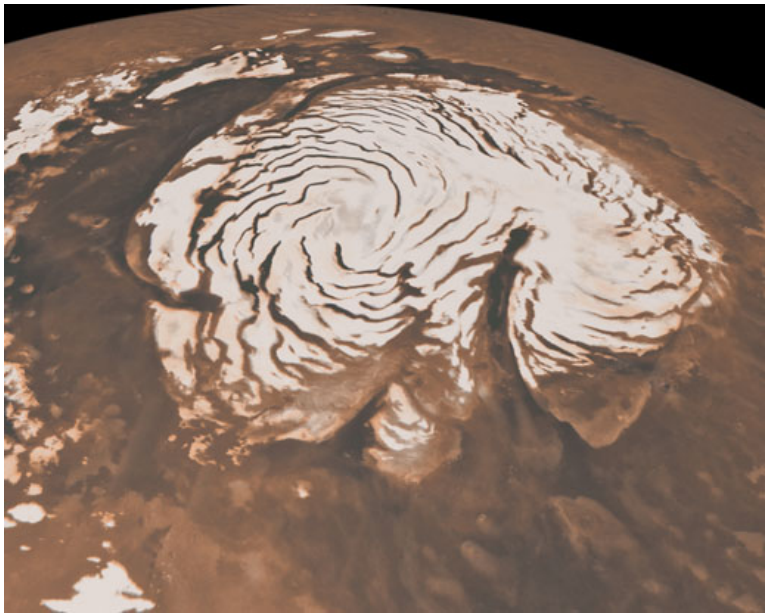

Figure: Image produced by combining data from instruments from NASA's Mars Global Surveyor depicting the Martian north polar ice cap.[https://mars.nasa.gov/]

$$Qs(y)(1-\alpha(y,\eta))$$

Figure: Super-black material named Vantablack created by Surrey NanoSystems

Albedo Function

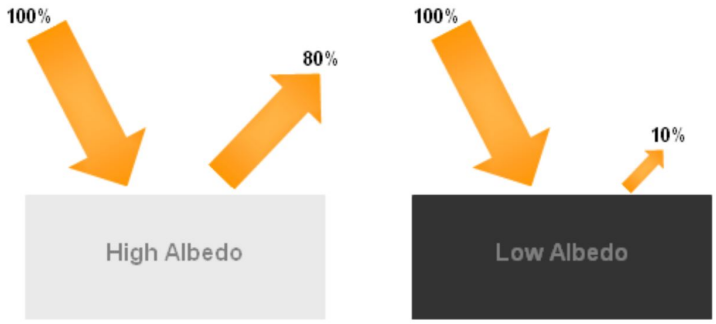

Figure: Contrasting Reflectivity levels on different colored surfaces with different albedos

$$Qs(y)(1 - \alpha(y, \eta))$$

$\alpha(y, \eta)$ represents the albedo (reflectivity) dependent on sine of latitude and the position of ice line

Emitted Longwave Radiation and Latitudinal Transport

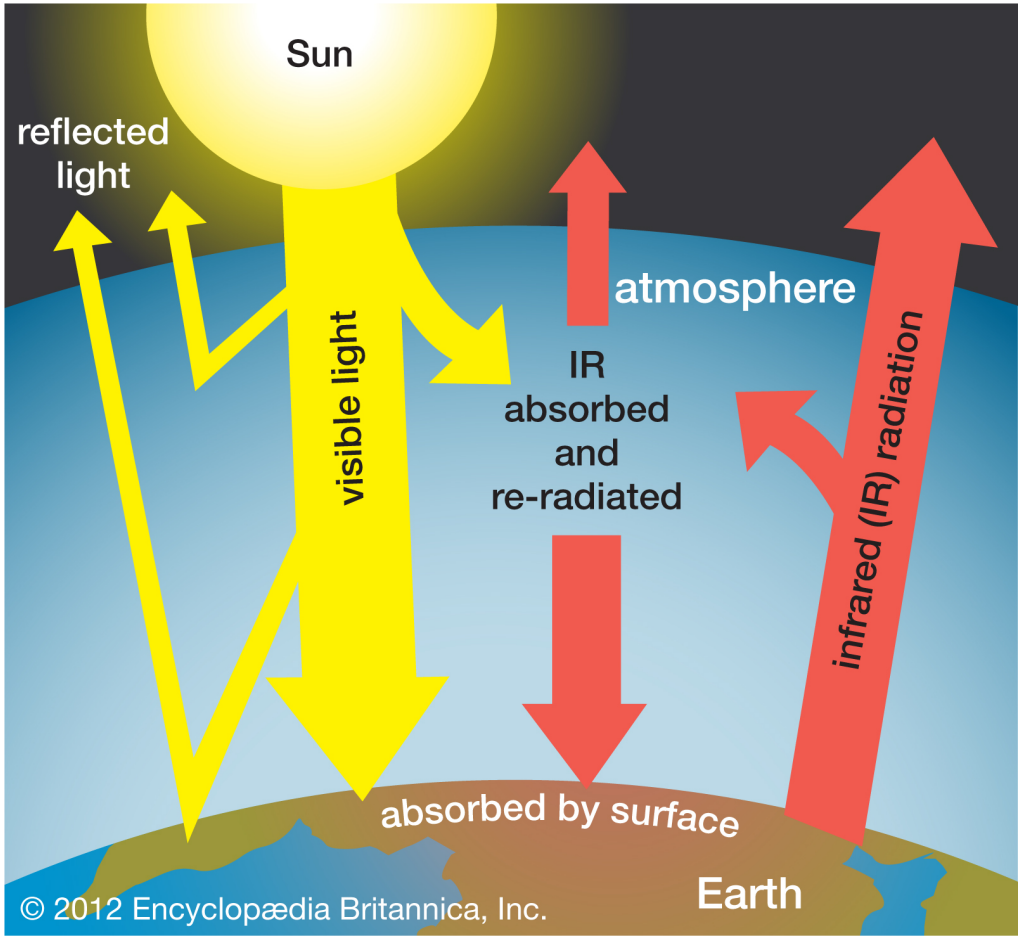

Figure: A:Energy Balance Diagram of Earth

$$(A + BT(y,t))$$
$$C\left(T(y,t) - \overline{T}(t)\right)$$

Dynamic Ice Lines

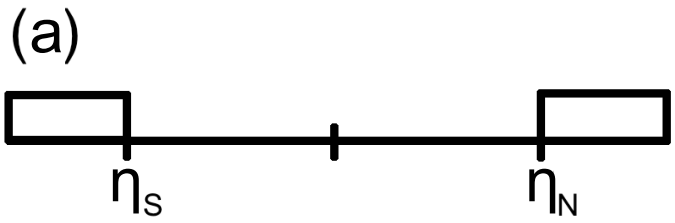

Figure: Ice Cap Diagram: η_e and η_p on the interval $[-1, 1]$ with condition that $\eta_e \leq \eta_p$.

$$\begin{aligned} \frac{d\eta_e}{dt} &= \rho(T_c - T(\eta_e, t)), \\ \frac{d\eta_p}{dt} &= \rho(T(\eta_p, t) - T_c). \end{aligned} \tag{2}$$

- T_c = highest temp. ice is present year-round
- ρ determines rate of change of ice line relative to temp. change
- $T(\eta_e, t) > T_c$ implies degradation (poleward) of η_e
  $T(\eta_e, t) < T_c$ implies formation (equatorward) of η_p

Ice Cover

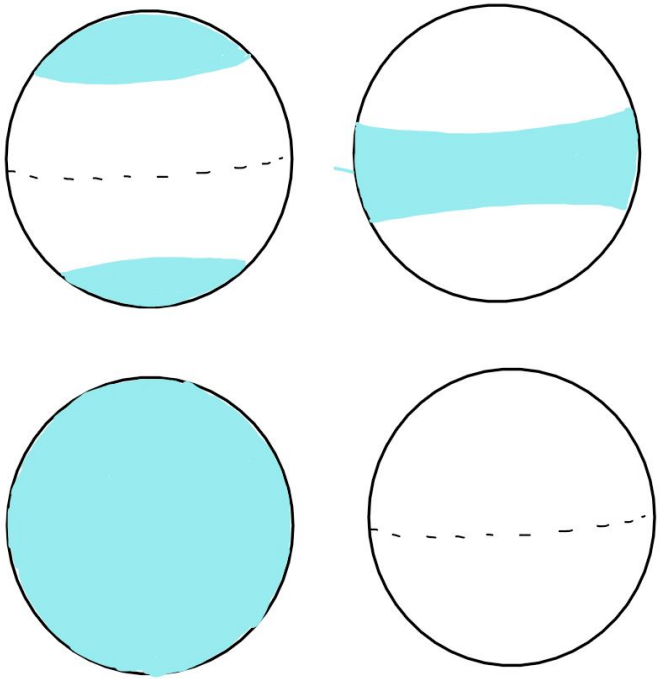

Figure: Different Ice Regime Scenarios. Top Left: Ice Belt around Equator, Top Right: Polar Ice Caps, Bottom Left: Ice Free, Bottom Right: Full Ice Cover (Snowball State)

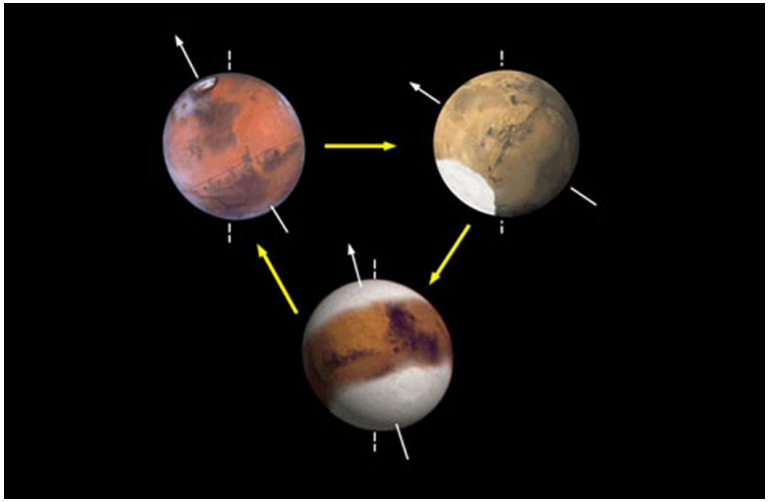

Figure: Depiction of Ice Regimes on Mars at Different Obliquities. Low Obliquity shows a Large stable Ice Cap. High Obliquity shows one Ice Cap.

Non-dimensionalized Equations

The final non-dimensionalized version of our model is as follows:

$$T^*(y) = (\frac{q}{(1+\delta)})(\sigma_6(y,\beta)(\alpha^*(y,\alpha)) + \delta(1-\alpha(S(\beta))) \tag{3}$$

where

$$\alpha^*(y,\alpha) = \begin{cases} 1, & y < \eta_e, y > \eta_p \\ 1-\alpha/2, & y = \eta_e, y = \eta_p \\ 1-\alpha, & \eta_e < y < \eta_p \end{cases} . \tag{4}$$

and

$$\frac{d\eta_e}{d\tau} = \lambda(T^*(\eta_e) - 1) \qquad \frac{d\eta_p}{d\tau} = -\lambda(T^*(\eta_p) - 1) \tag{5}$$

Fillipov System

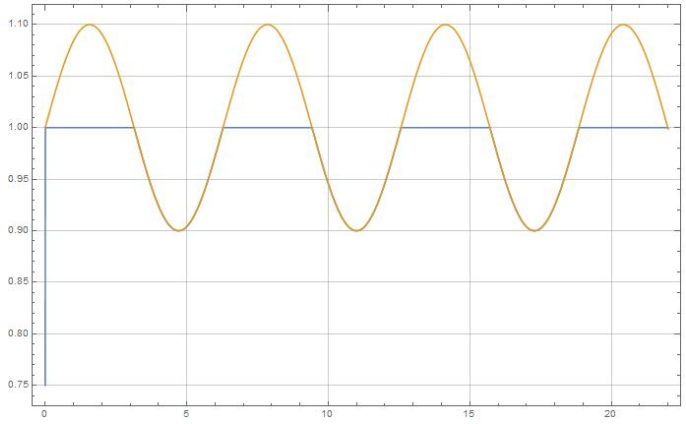

Figure: Simple Fillipov System (blue) with a sine function (yellow)

$$\frac{dx}{dt} = -(x + 1/2) * (x - 1/2) * (x - (1 - .1 * \sin t))$$

Nondimensionalized Parameters

Table: Non dimensionalized Parameter Values for Mars

Parameter	Value	Formula	Earth Value
$\delta_1, \delta_2, \delta_3$	1.19, 0.28, 0.03	C/B	1.6
q	1.05	$Q * (1 - \alpha_1)/(A + B * T_c)$	1.27
α	0.46	$(\alpha_1 + \alpha_2)/2$	0.44

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- α: contrast between albedos and the extent of the ice albedo feedback
- q: radiative forcing of the planet
- δ: efficiency of the horizontal heat transport term

Observations: Looking at α

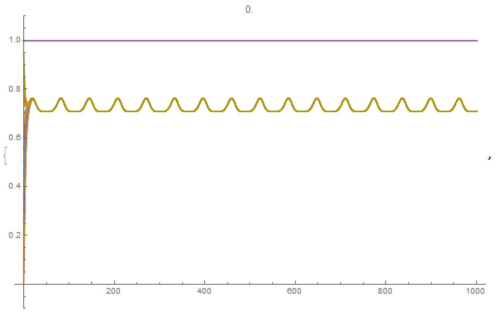


Figure: Stable Ice Cap at $\alpha = 0$ for δ_1 using the sine function

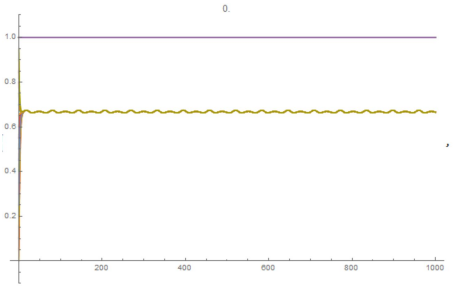


Figure: Stable Ice Cap at $\alpha = 0$ for δ_2 using the sine function

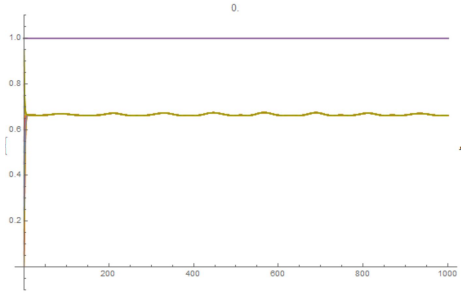
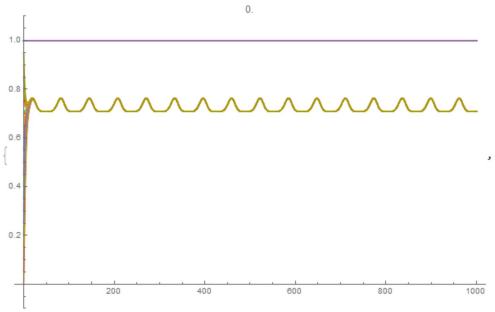


Figure: Stable Ice Cap at $\alpha = 0$ for δ_2 using Laskar data

Observations: Looking at α



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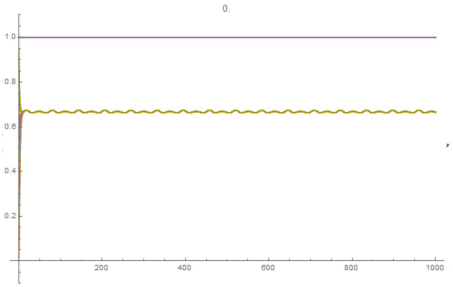


Figure: Stable Ice Cap at $\alpha = 0$ for δ_2 using the sine function

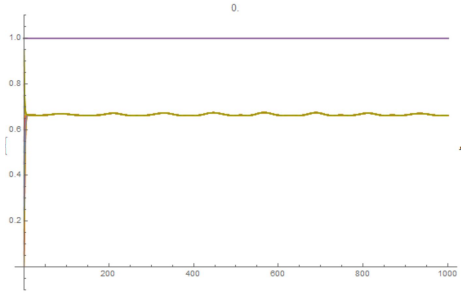


Figure: Stable Ice Cap at $\alpha = 0$ for δ_2 using Laskar data

Constant ice cap with small values of albedo contrast (α) with oscillations

Observations: Comparing Laskar Data and the sine Function for q

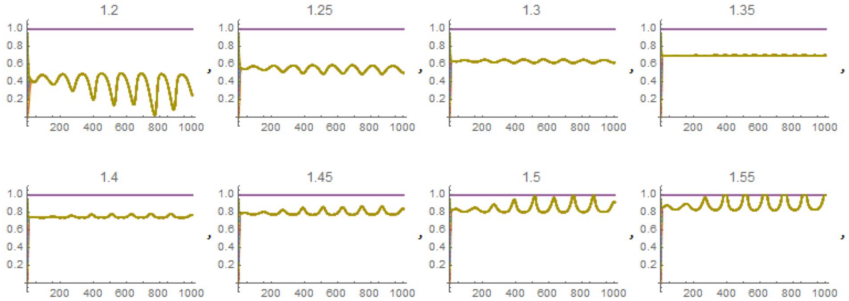


Figure: Ice Cap Regimes for $q = [1.2,1.55]$ for δ_2 using Laskar Data

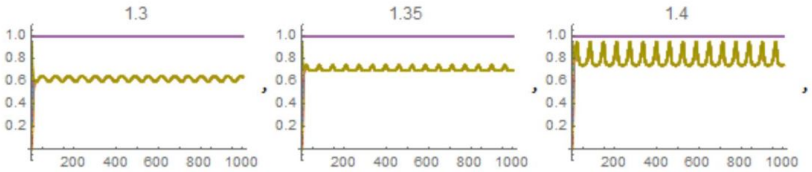


Figure: Ice Cap Regimes for $q = [1.3,1.4]$ for δ_2 using the sine Function

Observations: Comparing δ_2 and δ_3 for q

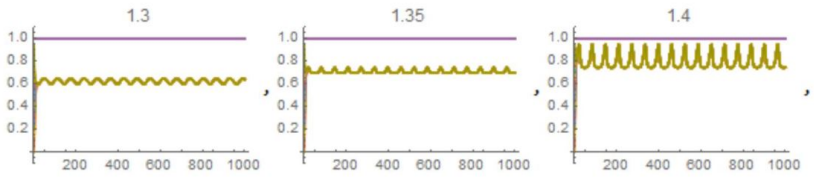


Figure: Ice Cap Regimes for $q = [1.3,1.4]$ for a larger heat efficiency (δ_2) using the sine Function

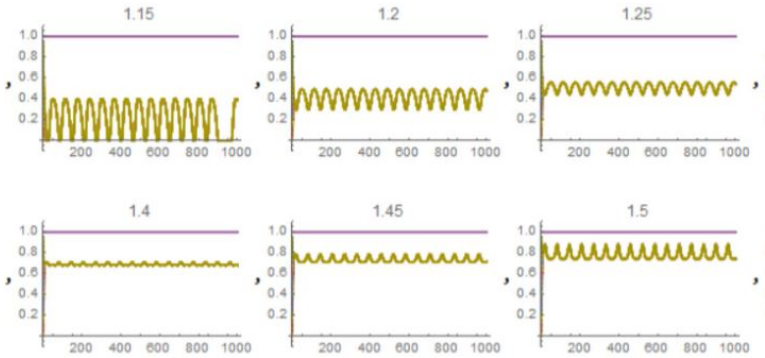


Figure: Ice Cap Regimes for $q = [1.15,1.5]$ for a lower heat efficiency (δ_3) using the sine Function

Observations: Looking at $q = [1.6\text{-}1.95]$

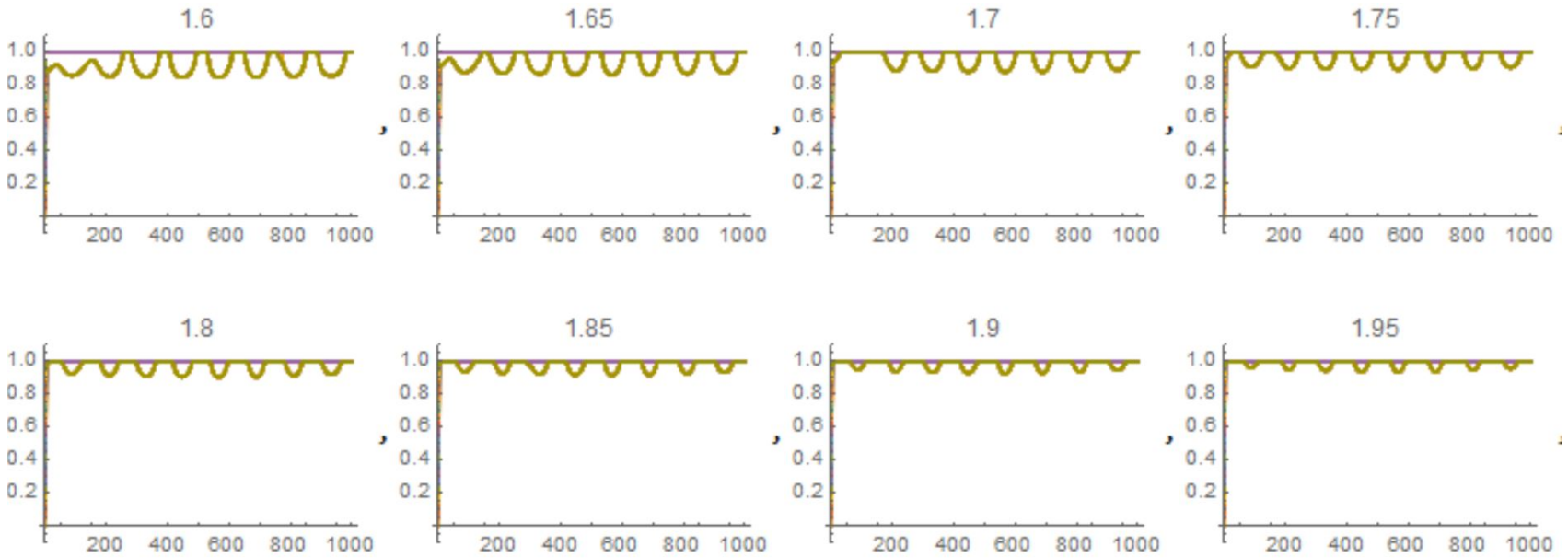


Figure: Ice Cap Regimes for $q = [1.15,1.5]$. Oscillations between Stable Partial Ice Cover and Ice Free Regimes

Observations: Looking at $q = [1.6\text{-}1.95]$

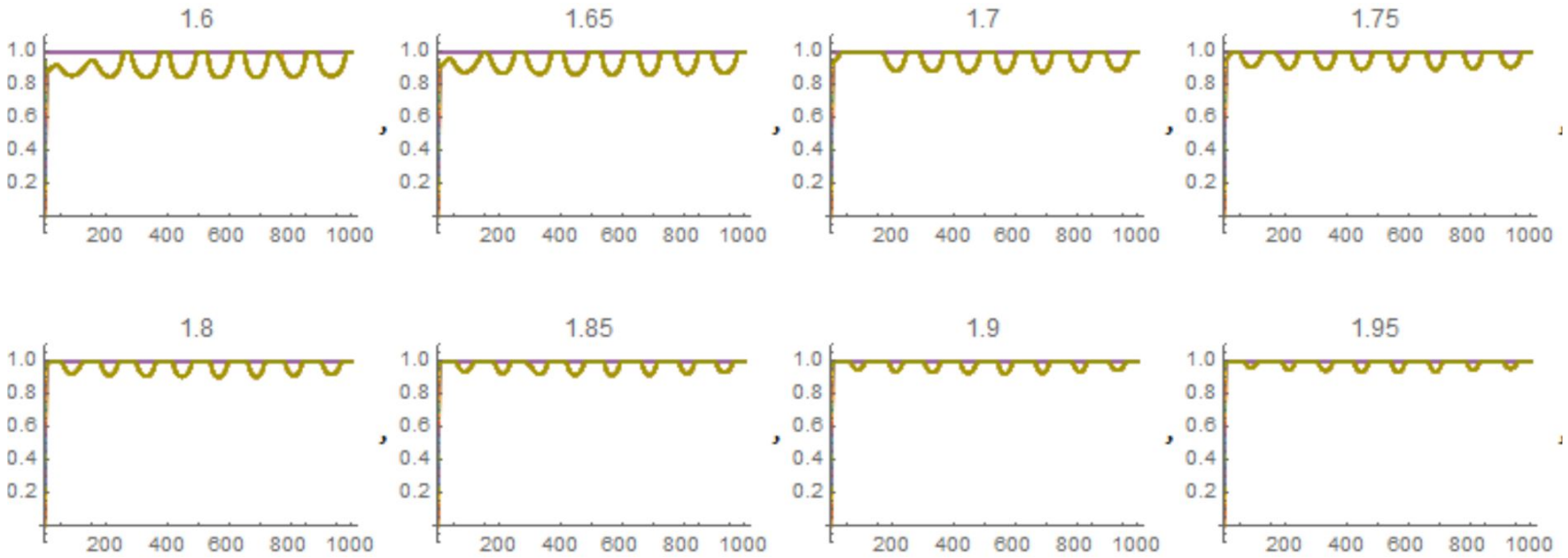


Figure: Ice Cap Regimes for $q = [1.15,1.5]$. Oscillations between Stable Partial Ice Cover and Ice Free Regimes

Presence of oscillations between stable ice cover and ice free regimes.

Albedo Contrast and Horizontal Heat Efficiency

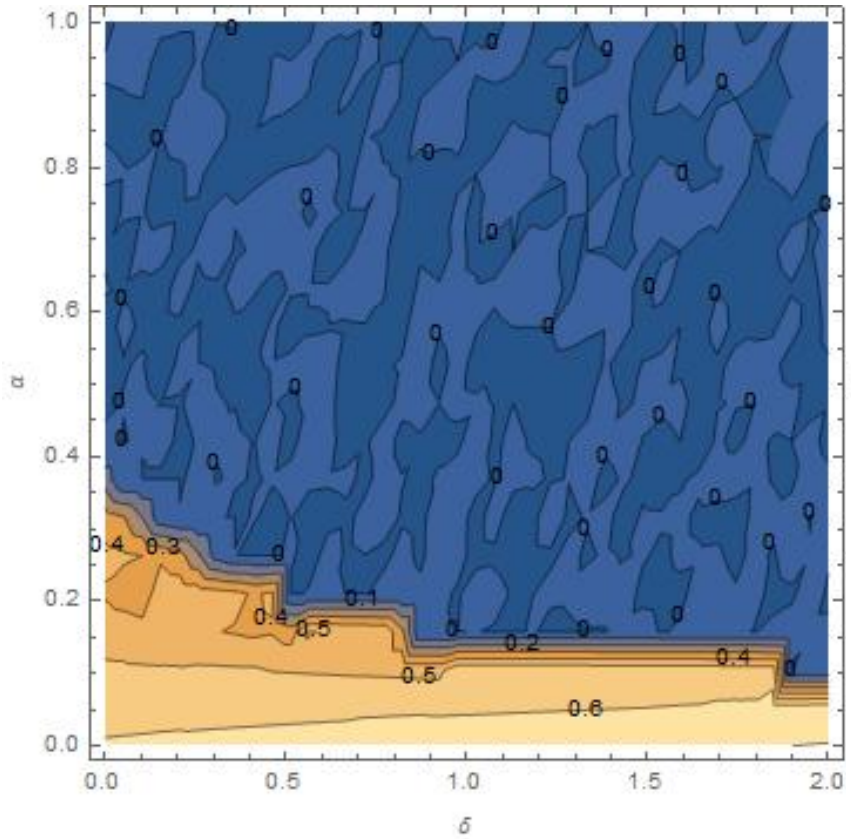
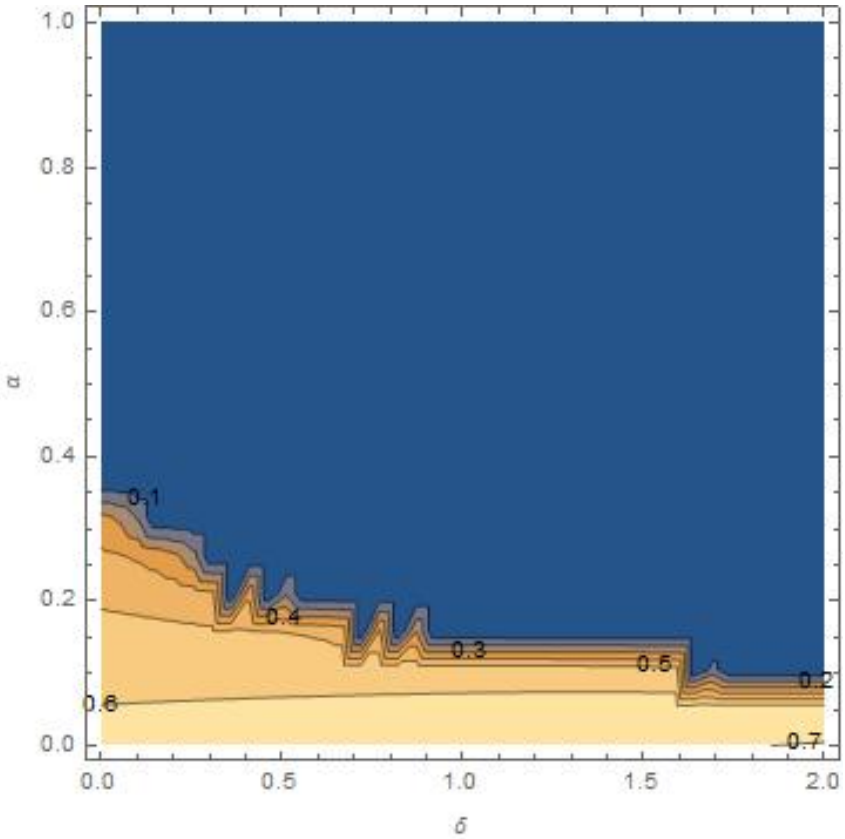


Figure: Plot of Local Minimum Point Values of Solutions



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Overall Discussion Ideas

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- Constant ice cap with oscillations with small values of α (albedo contrast)

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- Lower magnitudes of δ (efficiency of horizontal heat transport) show more instances of stable ice caps

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- Constant ice cap with oscillations with small values of α (albedo contrast)
- Specific interval in the q (radiative forcing) parameter space where stable oscillations are seen. Oscillations between stable ice cover and ice free regimes seen with higher values of q.
- Lower magnitudes of δ (efficiency of horizontal heat transport) show more instances of stable ice caps
- No presence of stable ice belt (poles ice free)

Next Steps

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- Looking at longer time scale data of 5Gyr

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- Accounting for diffusion of solutions in our sinusoidal function. How mean obliquity and standard deviation increase over 5Gyr

References

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Thank You!