The Twenty-Fourth Annual Nebraska Conference for Undergraduate Women in Mathematics

January 21 – January 23, 2022

TALK ABSTRACTS

PLENARY TALKS

Dr. Joan Ferrini-Mundy
President, University of Maine
Vice Chancellor for Research and Innovation for University of Maine System
Filling bathtubs, algebraic topology, and the NSF: Pathway to a presidency

Mathematics – my curiosity about how it is learned, my awe of its elegance, and my experience in supporting others to succeed in it – has been central for me for a long time. My pathway to my current position has been paved with opportunities to teach mathematics to learners of many ages, to do mathematics, to study how it is learned, and to work on state and national policy vital to the future of mathematics. Even now I can still lose myself and find focus in working on a problem in mathematics or thinking about a question in mathematics learning. And yes, there is a connection between algebraic topology and my daily work as a university president. I look forward to talking with you about how mathematics can be a key to fulfilling, and sometimes unexpected, career possibilities.

Dr. Pamela E. Harris
Associate Professor, Williams College
Cohost of "Mathematically Uncensored" Podcast
Co-Founder of Lathisms
Parking Functions: Choose your own Adventure

Consider a parking lot consisting of n consecutive parking spots along a one-way street labeled 1 to n. Suppose n cars want to park one at a time in the parking lot and each car has a preferred parking spot. Each car coming into the lot initially tries to park in its preferred spot. However, if a car's preferred spot is already occupied, then it will proceed forward in the street parking in the next available spot. Since the parking lot is along a one-way street, it is not guaranteed that every car will be able to park before driving past the parking lot. If we let a_i denote the preference of car i and all of the cars are able to park under these conditions, then the preference list $(a_1, a_2, \ldots, a_n)$ is called a **parking function** (of length n). For example, $(1, 2, 4, 2, 2)$ is a parking function, but $(1, 2, 2, 5, 5)$ is not (you should convince yourself of this!). In this talk we provide an answer to the question of how many parking functions of length n there are and we consider many new avenues for research stemming from this enumerative question.

Dr. Talithia Williams
Associate Professor of Mathematics, Harvey Mudd College
Author of "Power in Numbers: The Rebel Women of Mathematics"
Power in Numbers: The Rebel Women of Mathematics

The movie "Hidden Figures" brought visibility to the lives of African American women who served as NASA "human computers" in the 1960s, women who dreamed the impossible in a field where their presence was lacking. When it comes to inspiring the future productivity and innovation of our nation, women mathematicians are on the front lines. In this talk, I'll discuss my personal journey as a woman of color in mathematics and share ways we can excite public interest in mathematics, building upon the rich legacy of the Hidden Figures that have come before us. As we shift the fixed mindset around mathematics ability, we can begin conversations that improve public perception of STEM and bring people from all backgrounds into this important work.

Talks by Undergraduate Students

Madelyn Andersen, Harvey Mudd College
Finite Rank Kernel Varieties: A Variant Of Hilbert's Nullstellensatz For Graphons

Graphons are symmetric measurable functions that arise from a sequence of graphs. A graphon variety is the set of all graphons defined by a condition of the form $t(g, W) = 0$ for a fixed quantum graph g, where $t(\cdot, \cdot)$ is the homomorphism density and a quantum graph is a formal linear combination of multigraphs. Using a method of representing graphs as polynomials, we construct a surjective homomorphism from the space of quantum graphs to a subring of the complex polynomial ring that is invariant under permutations of variables. When graphons are of finite rank, we demonstrate that the paralleled Algebraic Geometry "ideal" inverse of varieties is an ideal in our polynomial representation. Defining a kernel algebraic set using kernel varieties, we demonstrate that we can call such sets closed under the Zariski Topology. We determine several ties to Algebraic Geometry as a result of utilizing finite rank kernels and discover that a weaker version of Hilbert's Nullstellensatz applies to kernel zero-sets with respect to homomorphism density. Throughout, we examine the connection between Algebraic Geometry and Graphon Theory.

Isidora Bailly-Hall, Grinnell College
Decomposing Energy Currents of Higher Spin Fields

Control of energies plays an important role in the study of the long-time evolution of physical fields. For the simplest scalar wave equation, anisotropic control can be obtained through two methods: using weighted energies or using hyperboloidal slicing. Our work focuses on understanding how these strategies can be implemented for higher spin fields that occur in electromagnetism, gravity, and string theories. In this presentation we begin with background on energies and its connection to divergence integrals. We then introduce the spin 1, 2 and "2×0" fields. For each field we discuss properties and how T (unweighted) and A (weighted) energies of these fields are determined, with respect to both the flat and hyperboloidal slicings. From there, we show the sum-of-squares decompositions we found and use them to illustrate the differences between isotropic and anisotropic control. Work on this project was completed in conjunction with Zach Gauronski and overseen by Professor Willie WY Wong of Michigan State University; many thanks to both of them.

Cara Bennett, Georgia Institute of Technology
Knot Colorability and Maximum Knot Determinants

Knot theory is the study of embeddings of circles in $\mathbb{R}^3$, where such embeddings are called knots and links. Invariants are a means of differentiating knots and links of different types. We focus on two closely related such invariants, called p-colorability and the knot determinant. Grid diagrams are rectilinear representations of knots and links which provide novel methods for understanding and computing these invariants. The grid diagram perspective on determinants raises the natural question of determining the maximal knot or link determinant amongst links represented by grids of a fixed size. We connect this problem to the long-standing Hadamard's maximal determinant problem regarding determinants of matrices for which the entries are restricted to 0 or 1. After exploring these connections, we present our findings on lower and upper bounds for the maximal determinants of knots and links on grid diagrams of a fixed size.

Brianna Bradley, Francis Marion University
Interconnectedness of Math and Art: Exploration of the Fibonacci Sequence

As a future mathematics educator, it is inevitable I will be asked the question "why do I need to know this?" Individuals like myself are being taught to find the relevance and purpose in each of their lessons. It is my suggestion, mathematics educators expose their students to the diversity of math as it can be found in nature, music, arts, and sports, to name a few. I believe when math can be found in every aspect of a student's life, it will inspire them to take more of an interest in mathematics. I will be dissecting the interconnectedness between mathematics and art forms through the exploration of the Fibonacci sequence. I will discuss the origins of the Fibonacci sequence, the golden ratio, and the appearance of the sequence in paintings, drawings, music, architecture, and photography.

Ja'Nya Breeden, Francis Marion University
A One-dimensional Field Dislocation Mechanics Model Using Discontinuous Galerkin Method

A one-dimensional, time-dependent model is developed using the Discontinuous Galerkin Method and is capable of simulating dislocations by implementing a set of partial differential equations. The dispersal of plastic deformation and stress within the system is caused by the transport of dislocations over time are modeled. By implementing the Discontinuous Galerkin method, the evolution equation for the dislocation density is discretized. We are then able to determine how the discretized system is affected by the application of a non-zero applied strain.

Angela Brobson, Converse University
Detecting Malicious URLs

For this project, I worked in a team setting with a liaison from a large financial institution to detect whether or not a URL was malicious. Python was used to create programs that rate a URL's reliability, which was then used to venture down three different avenues: logistic regression, random forest, and a lexicon point based system. Through the research of preexisting programs and an analysis of data-sets containing both good and bad URLs, we narrowed our list of permissible websites to exclude social media websites and web-shortening sites. Machine learning was used with the goal of creating a program that bettered itself as it studied different malicious websites. Logistic regression analyzed data based on how well it fit the curve between a binary set of "good" and "bad", while random forests generated a set of decision trees in order to best clarify individual data points. The lexicon based point system was designed to rate URLs on a predetermined scale from the probabilities of individual static features' occurrence. One factor that increased a URL's score was based on calculating the probability of a string of letters to be a word. This was after observing the use of long, incoherent strings of randomly ordered letters in the URL's path section, as this is an easily duplicated way for hackers to hide under neutral appearing URLs. After reviewing each of the program's success rates, we concluded that random forest provides the most reliable and consistent results.

Moira Camacho, United States Naval Academy
Describing the Effects of Diurnal Forcing on the Kelvin Wave

There are various atmospheric waves that effect rainfall in the tropical atmosphere centered at the equator. One of these fast (15 m/s) eastward moving waves is the Kelvin wave. One source of uncertainty for the Kelvin wave is as it passes over the islands of the Maritime Continent in the Indian Ocean. The objective of this project is to examine and explain the effects of Diurnal Forces on the Kelvin Wave on two islands a known distance apart. A storm can receive constructive or destructive interference from these forces, so our goal is to examine this phenomenon from a mathematical perspective. We then examine a one-dimensional ODE model, paying specific attention to the impacts made by external forces on these systems. Lastly, we'll evaluate an N-dimensional linear ODE that models various tropical atmospheric waves. We show what kind of constructive or destructive interference the system will receive based on the initial conditions of the system as well as the external forcing. We then examine how the physical system will react when the Kelvin wave passes over islands various times and distances apart.

Beth Anne Castellano, Lafayette College
A Graph Theoretic Approach to Regularity of Toric Ideals

One measure of the complexity of an ideal is a quantity called its regularity, which may be difficult to compute using algebraic tools. To study the regularity of a particular class of toric ideals (prime ideals of a polynomial ring that are generated by differences of monomials), we instead take a combinatorial approach, constructing graphs from our toric ideals and examining their properties. Our motivation for this approach comes from a key theorem by Khosh-Ahang and Moradi, which gives a method of computing the regularity of a monomial ideal using only the induced matching number of a particular graph associated to it. However, application of this theorem requires that these graphs are induced 5-cycle-free and pure vertex decomposable. We determine the circumstances under which graphs originating from toric ideals satisfy these properties, and we give a formula for their induced matching numbers.

Erica Choi, Columbia University
Dehn surgeries versus double branched covers

Dehn surgery and branched covering are important constructions that can be applied to knots in S^3 to produce 3-manifolds. Our paper considers a simple question: When is the double branched cover of a nontrivial knot K in S^3 obtained by Dehn surgery on that same knot K? This question was motivated by a particular 17-crossing knot J whose double branched cover is indeed obtained by Dehn surgery on J itself. Thus far, the only knots known to have this property are obtained as a family of simple generalizations of J. To investigate the question of whether this property is unique to this family of knots, we show that certain families of knots such as alternating knots, Montesinos knots, and torus knots cannot produce the same 3-manifolds through Dehn surgery and double branched covering. To rule this property out among knots with 15 crossings or less, we also present an algorithm that blends Casson-Walker invariants, Khovanov homology, and Heegaard Floer homology to compare 3-manifolds generated by Dehn surgery and double branched covering along a given knot.

Meredith Clayton, Stephen F. Austin State University
Superfactorials

A superfactorial is defined as $n^! = (n)!(n-1)!\dots(2)!(1)!$ where n is some positive integer. This talk will address two different research questions posed about the superfactorial function. Specifically, how to count the number of trailing zeros of $n^!$, and for what values of n can a single factorial be removed from the product of $n^!$ to obtain a perfect square.

Amelie Comtois, University of Ottawa
Universal Algebra and its Generalization via Category Theory

Universal algebra, dating back to at least 1898, is the study of algebraic structures, which are defined in terms of operations and axioms. Groups, rings, monoids, lattices, and many other algebraic structures can be studied from this viewpoint. I will start by presenting some of the classical results in the subject, such as the compactness theorem. With the more recent invention of category theory came the possibility of studying universal algebra in a whole new way to obtain new results in the field. The most important innovation in universal algebra that category theory allows is the possibility of studying structures with infinitary operations. An example of a setting where one requires infinitary operations is in the theory of complete lattices. (No knowledge of category theory is required for this talk.)

Harper Crosson, Grinnell College
The Development of Vector Fields: Grounding Educational Technology in a Historical Context

The discovery of electromagnetic (EM) phenomena in the nineteenth century confounded scientists, as force fields could not be fully described with Newtonian physics. Through attempts to represent EM phenomena mathematically, more cumbersome and incomplete tools such as complex geometry, quaternions, and Newtonian mechanics collectively evolved to produce modern vector analysis. My work contributed to the development of an interactive virtual reality experience created to enrich mathematics students' understanding of vector fields. In order to ground this pedagogical tool in a historical understanding of the concepts we aimed to convey, I explored previous mathematical methods of conceptualizing fields, representing space, and understanding forces. In fact, the search for an elegant mathematical description of force fields changed the way that math was viewed per se: it became a "representational" tool in addition to its traditional role as a computational tool. Furthermore, I investigated larger trends within nineteenth century science, for which EM research was surprisingly influential. In this era, science witnessed immense professionalization, "mathematization", and the rising exclusion of women. Not only is the evolution of vector analysis fertile for investigation of mathematical epistemology, but the concomitant social and scientific developments provide insight into many facets of modern mathematical education and higher education in general.

Helen Dai, Harvard University
Annemily Hoganson, Carleton College
Zoe Markman, Swarthmore College
Minimizing the discrete Alt-Caffarelli functional on path graphs: uniqueness and applications

Our research is on free boundary problems, a type of differential equation that varies over different domains. We study a discrete version of a specific free boundary problem, the Alt-Caffarelli functional, which is often used to model water movement. We first study the structure of minimizers of this functional and the question of their uniqueness on path graphs. While minimizers of the continuous functional on an interval are unique for most boundary values, minimizers of the discretized functional on path graphs are nonunique for infinitely many boundary values. We then study this problem on more complex graphs via a splitting technique, which allows us to reduce to the path graph case.

Nyah Davis, University of Iowa
Lattice Models & Puzzles

We constructed a solvable lattice model for the dual weak symmetric Grothendieck polynomials given by Pylyavskyy and Lam in hopes of using such a model to prove related properties of these polynomials, including Cauchy identities and branching rules. We also considered a similar lattice model construction for the weak symmetric Grothendieck polynomials in hopes of proving a Cauchy identity given by Yeliussizov, concluding with a negative result. Moreover, we expand on the work done by Pylyavskyy and Yang and by Zinn-Justin by giving boundary conditions for a proposed lattice model for the Littlewood Richardson coefficients of the dual weak symmetric Grothendieck polynomials, via an MS puzzle construction.

Elise Dettling, Grand Valley State University
Trajectory Planning with Space-Filling Curves

Unmanned Aerial Vehicles (UAVs) have become widely used for many different applications. It is now possible to install sensing equipment (cameras of different kinds, radar devices, lidar devices) onto UAVs to collect data. A question that is faced by many organizations using UAVs is: What should be the trajectory for a flight of an UAV? Given an area where data needs to be collected, what flight path should the UAV take to cover that entire area with minimal costs? Our proposed research would produce a prototype algorithm to demonstrate an optimized trajectory of a UAV using a triangular curve called the Devries Curve.

Emma Dobson, Cedarville University
The Ternary Golay Code and Mathieu Groups M11 and M12

Throughout history intellectuals have wondered at the complex and interwoven nature of mathematics. Each branch of mathematics seems to appear in every other mathematical bough creating a vast network ripe for discovery. An excellent example of interplaying mathematical topic can be seen between the mathematical branch of coding and the mathematical branch of finite simple groups. We specifically will be dwelling on the Ternary Golay Code and its connections to the Mathieu groups M11 and M12. Specifically, we will show the automorphism group of the $[12, 6, 6]$ Ternary Golay Code is the Double Cover of M12. In order to achieve this, explore some of the connections between M12 and M11 using group theory. After that we will concentrate on building our own Ternary Golay Code and examining the vectors within that code. By exploring the automorphism group of our constructed Ternary Golay Code, we shall end with showing that the order of the automorphism group of our Ternary Golay Code is the same as two times the order of M12.

Mallory Dolorfino, Kalamazoo College
Cordelia Horch, Occidental College
On Good Infinite Families of Toric Codes or the Lack Thereof

Toric codes were first introduced by Johan P. Hansen to extend the Reed-Solomon code as a k-dimensional subspace of $\mathbb{F}_q^n$. A toric code is determined by a toric variety or its associated integral convex polytope $P \subset [0, q-2]^n$, where k is the number of integer lattice points of P. There are two relevant parameters that determine the quality of a code: the information rate, which measures how much information is contained in a single bit of each codeword; and the relative minimum distance, which measures how many errors can be corrected relative to how many bits each codeword has. Soprunov and Soprunova defined a good infinite family of codes to be a sequence of codes corresponding to polytopes such that neither the corresponding information rates nor relative minimum distances go to 0 as the dimension of the polytopes approaches infinity. We examine different ways of constructing families of codes by considering polytope operations such as the join and direct sum. In doing so, we give conditions under which no good family can exist and strong evidence that there is no such good family of codes.

Rie Durnil, Gonzaga University
Rithmomachia: An Academic Proposal of Rules

Dr. Tomas Guardia and I propose of a set of rules for the medieval math game of Rithmomachia. The set we introduce results from the reading and interpretation of the rules from several medieval manuals. As a result of testing and studying the rules, we introduce a new rule that we coin the Innocent Delivery. No rules of Rithmomachia are definitive due to the different ways of capturing pieces as well as the game's ending. We offer this proposal a starting point for further academic discussion and testing of a future set of simplified rules in the hopes of a definitive set of rules for future Rithmomachia players.

Lina Ellis, Arizona State University
On the Nikiforov-Uvarov Method in Quantum Mechanics

This talk focuses on the so-called Nikiforov-Uvarov method of solving the Schrodinger equation in the field of quantum mechanics. Originally this approach was developed in the former Soviet Union and published in the Nikiforov and Uvarov book on special functions of mathematical physics translated into English in 1988. Since then this method became popular among people who study analytical methods of non-relativistic and relativistic quantum mechanics. We discuss implementation of this approach to various potentials, such as the Coulomb and Poschl-Teller potentials, with the help of Mathematica computer algebra system. Our approach helps to study the fundamentals of quantum mechanics. This is joint work with Christoph Koutchan and Sergei Suslov.

Madeline Fischer, United States Air Force Academy
Spreading Activation modeled in ACT-R Technology

Adaptive Components of Thought Rational, ACT-R, simulates the optimal decision making process of the human mind. Humans make decisions by retrieving pieces of information stored in memory applied to a given context. Facts stored as nodes are linked by associations, then activated when needed. Spreading activation defines the process of activation spreading amongst associations to related nodes. We investigate an implication of increased retrieval time asserted by the fan effect, which relates increased associations to increased retrieval time. ACT-R models spreading activation with a logarithmic model— the problem being that it becomes negative, physically meaning that for large fans, an association will decrease rather than strengthen activation. We examine the mathematical relationship between the current model and its framework of Bayesian statistics. Based on the deterrence, we propose a new model with a natural zero asymptote considering behavior at higher fans and the impact of initial conditions to eliminate the physical inaccuracies.

Kaylee Freudenthal, Northern Arizona University
Hannah Golab, Northern Arizona University
Ruth Perry, Northern Arizona University
New Results on Completely Trivial Graphs

A simple connected digraph is strongly connected if there exists a directed path between every pair of vertices in both directions. A random assignment of edge directions can result in a digraph not being strongly connected if a vertex has all incident edges being either in-edges (i.e., the vertex has maximal indegree) or out-edges (i.e., the vertex has minimal indegree). We say a digraph is completely trivial if, assuming no vertex has maximum or minimum indegree, any choice of edge orientations results in a strongly connected digraph. In this talk, we present new results on families of digraphs that are always completely trivial or never completely trivial. These families include wheel graphs, n-chord graphs, complete and complete bipartite graphs, and chordal graphs with nonzero crossing numbers.

Parneet Gill, California State University, Fresno
Pamela Vargas, Smith College
Preferential and k-Zone Parking Functions

Parking functions are vectors that describe the parking preferences of n cars that enter a one-way street containing n parking spots numbered 1 through n. A list of each car's preferences is compiled into vectors in which we denote as $(a_1, \ldots, a_n)$, such that a_i is the parking preference for car i. The classical parking rule allows cars to enter the street one at a time going to their preferred parking spot and parking if that space is unoccupied. If it is occupied, they then proceed down the one-way street and park in the first available parking spot. If all cars can park, we say the vector $(a_1, \ldots, a_n)$ is a parking function.

In our research, we introduce new variants of parking function rules with backward movement called k-Zone, preferential, and inverse preferential functions. We study the relationship between k-Zone parking functions and k-Naples parking functions and count the number of parking functions under these new parking rules which allow cars that find their preferred spot occupied to back up a certain parameter. One of our main results establishes that the set of non-increasing preference vectors are k-Naples if and only if they are k-Zone. For one of our findings we provide a table of values enumerating these new combinatorial objects in which we discover a unique relationship to the order of the alternating group A_{n+1}, numbers of Hamiltonian cycles on the complete graph, K_n, and the number of necklaces with n distinct beads for $n!$ bead permutations.

Annie Giokas, University of Utah
The Noetherian Property of Invariant Rings

A Noetherian ring is a ring that has the ascending chain condition, which means that any ascending chain of ideals of the ring must stabilize after a finite number of steps. The concept of Noetherian rings came to be after the German mathematician, Emmy Noether, discovered that primary decomposition of ideals is a consequence of the ascending chain condition in 1921. It is known that for a graded ring over a field, the Noetherian property is equivalent to the ring being finitely generated over the field. Noether proved in 1926 that the ring of invariant for the action of a finite group via k-algebra automorphisms of finitely generated algebras over a field k are in fact, Noetherian. In a related direction, a Noetherian ring containing the field of rational numbers, will have also a Noetherian invariant ring under a finite group action. The proof depends on the ring containing a characteristic 0 field, because it uses an especially useful formula, the Reynold's operator, that involves the inverse of the order of the given finite group to define the map between the ring and its invariant ring that turns out to be a projection. This however is not necessarily true in other cases. We constructed a class of rings of characteristic p for each prime integer p such that each ring in the class is Noetherian with a finite group G acting on it such that the ring of invariants under this group action is not Noetherian. This class of rings is generalized from the characteristic 2 counterexample due to Nagarajan, in 1968.

Hannah Golab, Northern Arizona University
see **Kaylee Freudenthal**

Clare Hamblen, University of Oregon
Chase-escape on Finite Graphs with Fixed Degree Sequences

Chase-escape is a stochastic spatial growth process taking place on a graph populated by red and blue particles, modeling an ecological process of predator-prey spread. Red (modeling prey) transitions from red sites to unoccupied sites (thus coloring them red) at average rate λ, and blue (modeling predator) transitions from blue sites to red sites (thus coloring them blue) at average rate 1. A key question concerns the critical rate λ_c. If $\lambda > \lambda_c$, then red can reach a large fraction of the vertices of the graph with positive probability, but if $\lambda < \lambda_c$, red is fully consumed by blue and ceases to spread throughout the graph. Previous work has shown that on some types of infinite graphs, $\lambda_c < 1$, meaning that red can be slower than blue and still survive. I studied the setting of finite graphs in which less is known, and determined a sufficient condition for red to survive on finite graphs with fixed degree sequences. By using percolation processes, it can be shown that a sufficient condition for red's survival is the existence of a giant component of open vertices (vertices where red will always escape blue). The Molloy-Reed condition is used to prove the existence of the giant component.

Anna Heger, Saint Mary's College
Total Coloring Games on Graphs

Based on the ideas of the game chromatic number and total coloring, we define a competitive game of coloring the vertices and edges of a graph. In particular, we consider a two-player game in which players A and B, starting with A, take turns coloring an element of a graph G from a set of k colors, so that adjacent vertices, incident edges, and an edge and its incident vertices receive different colors. Player A wins if all elements of the graph are colored, while B wins if there are uncolored elements but no moves remain. We define the total game chromatic number of a graph G to be the smallest k for which A has a winning strategy. We find the total game chromatic number for large cycles, stars, friendship graphs, and trees with large maximum degree.

Annemily Hoganson, Carleton College
see **Helen Dai**

Cordelia Horch, Occidental College
see **Mallory Dolorfino**

Sidra Jawaid, University of Central Oklahoma
Identifying Critical Higher-order Interactions in Complex Networks

Diffusion on networks is an important concept in network science observed in many situations such as information spreading and rumor controlling in social networks, disease contagion between individuals, and cascading failures in power grids. The critical interactions in networks play critical roles in diffusion and primarily affect network structure and functions. While interactions can occur between two nodes as pairwise interactions, i.e., edges, they can also occur between three or more nodes, which are described as higher-order interactions. This report presents a novel method to identify critical higher-order interactions in complex networks. We propose two new Laplacians to generalize standard graph centrality measures for higher-order interactions. We then compare the performances of the generalized centrality measures using the size of giant component and the Susceptible-Infected-Recovered (SIR) simulation model to show the effectiveness of using higher-order interactions. We further compare them with the first-order interactions (i.e., edges). Experimental results suggest that higher-order interactions play more critical roles than edges based on both the size of giant component and SIR, and the proposed methods are promising in identifying critical higher-order interactions.

Christina Jones, Farmingdale State College
Coupled Solutions to a Discrete Nabla Caputo Fractional Boundary Value Problem

Discrete fractional calculus is a relatively new area of mathematics that seeks to develop discrete analogues for the continuous fractional calculus. The nabla Caputo fractional difference is defined in terms of a fractional integral and a whole-order nabla difference, where the fractional integral allows us to integrate by any positive real number order. We consider the existence, uniqueness, and Hyers-Ulam stability of coupled solutions to the following nabla Caputo fractional boundary value problem:

$$\begin{cases} -\nabla_{a*}^{\nu_1} x_1(t) = f_1(t, x_1, x_2), & t \in \mathbb{N}_{a+1}^{b}, \\ -\nabla_{a*}^{\nu_2} x_2(t) = f_2(t, x_1, x_2), & t \in \mathbb{N}_{a+1}^{b}, \\ \nabla^i x_1(a-1) = \nabla^i x_2(a-1) = 0, & i \in \mathbb{N}_0^{N-2}, \\ \nabla^j x_1(b) = \nabla^j x_2(b) = 0, & \text{where } j = 0 \text{ or } j = 1. \end{cases}$$

Using the Banach Fixed Point Theorem we establish sufficient conditions for the nonlinear nonhomogeneous functions f_1 and f_2 that yield the existence and uniqueness of coupled solutions. Using a method from Urs (2013) we also prove the Hyers-Ulam stability of those solutions.

Ariel Liu, University of Wisconsin-Eau Claire
Spear and Shield: Coding to Thwart Adversarial Aggression

In coding theory, we study methods to send information more effectively, for example, in telecommunications. Authentication is important when sending information to detect any potential interference. Our research is built on the recent results on authentication with multiple users. We explore some limitations for constructing good codebooks with certain achievable rates for partial correction in a two-user multiple access channel with or without adversary participation. We also discuss the circumstances under which a special kind of code, called a linear code, can be successful or unsuccessful.

Xufei Liu, Georgia Institute of Technology
On the Girth of Assignment Graphs Generated from Digraphs

Graph pebbling is a mathematical game played on a graph G with no loops or multiple edges. A standard pebbling move consists of removing two pebbles from a vertex and adding one pebble to an adjacent vertex. An assignment graph is a Hasse diagram derived from each sequence of possible pebbling moves. In this presentation, we focus on digraphs G with no bidirectional edges. We investigate assignment graphs and analyze what properties yield certain girths.

Chloe Makdad, Butler University
On Selecting an Algorithm for Counting y-smooth Integers up to x

Let $x, y \in \mathbb{Z}$ such that $x \geq y > 0$. We say $n \in \mathbb{Z}$ is y-smooth if for every prime divisor p of n, we have $p \leq y$. Our work examines the number of y-smooth integers less than x, denoted $\Psi(x, y)$. The computation of $\Psi(x, y)$ is required for applications like optimizing the running time of integer factoring algorithms and the post-quantum key exchange protocols. We can compute $\Psi(x, y)$ with Buchstab's identity or approximate it using methods based on a theorem from Hildebrand and Tenenbaum. However, other algorithms are advantageous for certain (x, y)-pairs. First, we examine the fastest approximation, $\Psi(x, y) \approx x \cdot \rho(u)$, where $u = \log x / \log y$ and $\rho(u)$ is the Dickman function. This method is only valid for y greater than a cutoff, $L(x)$. Assuming the extended Riemann hypothesis, Hildebrand gives $L(x) = (\log x)^{2+\varepsilon}$. Comparing the Dickman ρ method to the Hildebrand-Tenenbaum estimates, we propose a practical value for ε to determine $L(x)$. Then, we present a new algorithm based on a theorem from Ennola, proven to produce a valid estimate for $2 \leq y \leq (\log x)^{\theta}$ with $0 < \theta < 3/4$. By comparing estimates generated by this algorithm to Buchstab's identity and the Hildebrand-Tenenbaum estimates, we show that the algorithm outperforms its theoretical guarantees. With these results, we propose guidelines for choosing an algorithm to compute $\Psi(x, y)$ given an (x, y)-pair.

Zoe Markman, Swarthmore College
see **Helen Dai**

Jovana Mitic, Converse University
Data-Driven Retention and Advertising for Camp Kippewa

This project is focused on participant data from Camp Kippewa, a residential 3.5 week camp for girls ages 7 to 15. The primary focus is to use historical camper data to predict whether a particular camper will become a repeat customer. Python is used to create a tree trained on select data for this prediction. An evaluation survey was used to collect additional data from 2021 campers to inform advertising based on campers' preferences and on frequently enjoyed aspects of camp. Other data analysis including clustering and marketing tools that Camp Kippewa should use to attract more campers and to keep returning ones will also be presented.

Leah Mork, Concordia College, Moorhead
Generating b-Prime Fully Augmented Links

In an effort to enumerate fully augmented links that are not belted-sum decomposable (b-prime), this work will present an operation, called complete augmentation, on prime links that will produce all b-prime fully augmented links. A lemma proven by Jorge Calvo in 1985 will be vital in order to prove all completely augmented prime links will result in b-prime fully augmented links.

Anushka Murthy, Columbia University
Near-Tight Bounds on Near-Sunflower Variants

The Erdös-Szemerédi Sunflower Conjecture is a long-studied open problem in Extremal Set Theory, and we study variants of the conjecture. In particular, we say that a family of r binary vectors of length n is an "*(r,Q)-system*" if the number of 1s in every column is an element of Q when the family is viewed as a $r \times n$ binary matrix. Define an "*(r,Q)-free*" family to be a family of binary vectors such that no r vectors form an (r, Q)-system, and ask what bounds exist on the maximum size of an (r, Q)-free family; the original Sunflower Conjecture problem is the special case where $Q = \{0, 1, r\}$. For $Q = \{0, d, d+1, \ldots, r\}$, we establish nearly tight upper and lower bounds on this maximum size. Our bounds generalize results found by Alon, Fachini, and Körner on "r-thin" families, which are a special case of our result with $d = 2$. We also study a "focal" version of our variants, which additionally requires that the sunflower has a vector x such that at least d of the other $r-1$ vectors agree with x at each coordinate. We again establish nearly tight upper and lower bounds on the maximum size of a "focal-free" family; this generalizes a result of Alon and Holtzman, which focused on the specific case where $d = r - 2$. At their core, our upper bound proofs rely on several applications of the generalized pigeonhole principle to find many collections of vectors with similar behavior. We take a probabilistic approach for our lower bounds, using the "random choice with alterations" method.

Viridiana Neri, Columbia University
Izah Tahir, Georgia Institute of Technology
Quantitative Properties of 1-bridge Braids

Low-dimensional topology, a subfield of pure mathematics, studies 1-, 2-, 3-, and 4-dimensional spaces. One way to study these spaces is by using knots and braids, which are 1-dimensional objects. In general, it is difficult to show that two knots are the same. In order to tackle this task, topologists often use knot invariants to instead decide when two knots are different. One example of a knot invariant is the braid index, the minimal number of strands required to represent a knot as the closure of a braid on that many strands. We determine the braid index for a special family of knots known as 1-bridge braids, denoted $K(w, b, t)$. Our main theorem gives a formula for the braid index of 1-bridge braids in terms of their defining parameters, w, b, and t. We also present some directions for future work.

Thu Nguyen, University of Central Oklahoma
Homology Preserving Graph Compression

Recently, topological data analysis (TDA) that studies the shape of data by extracting its topological features has become popular in applied network science. Although recent methods show promising performance for various applications, enormous sizes of real-world networks make the existing TDA solutions for graph mining problems hard to adapt with the high computation and space costs. This talk presents a graph compression method to reduce the size of the graph while preserving homology and persistent homology, which are the popular tools in TDA. The experimental studies in real-world large-scale graphs validate the efficiency of the proposed compression method.

Ruth Perry, Northern Arizona University
see **Kaylee Freudenthal**

Natalie Petruzelli, St. John Fisher College
Analyzing Epidemic Thresholds on Dynamic Network Structures

In early 2020 and onward, COVID-19 case numbers in parts of the U.S. have shown unforeseen shifts from exponential to linear growth in the number of daily new cases. This research explores the use of a network-based epidemic model that interpolates between lattice-like and configuration model networks, while keeping the degree distribution and reproduction number (R_0) constant to show these dynamics. This model gives nodes (representing people) locations and connects them to their nearest neighbors, rearranging a proportion p of the edges in a configuration model subnetwork. As p increases, we examine a shift from linear to exponential growth. Real human contact networks have a great deal of local interactions and fewer long-distance ones, so social distancing affects the effective reproduction number R_t and the proportion of long-distance connections. While the impact of changes in R_t is understood, less is known about the effect of subtle changes in network structure. Our analysis finds that the threshold between dynamics occurs with a low percentage of reconfigured edges. Furthermore, the total infections in an epidemic substantially increase around the threshold even with a constant R_0. This study reveals that enacting and relaxing social distancing restrictions has more complex and dramatic effects on epidemic dynamics than previously understood.

Michaela Polley, Carleton College
The 334-Triangle Graph of $SL(3, \mathbb{Z})$

The 334-triangle group, $T = \langle a, b \mid a^3 = b^3 = (ab)^4 = e \rangle$, is important in the study of thin groups because its representations in $SL_3(\mathbb{Z})$ provide examples of thin group candidates. We introduce a graph on the order three elements of a group G which visualizes the representations of T in G. For any group G, the 334-triangle graph of G, which we denote by $\Delta 334(G)$, is the graph whose vertices are the elements $a \in G$ such that $a^3 = e$, in which there is an edge between two vertices a and b if and only if $(ab)^4 = e$. In this talk we will prove a number of properties of 334-triangle graphs in general before narrowing our focus to $\Delta 334(SL_3(\mathbb{Z}))$, $\Delta 334(SL_3(\mathbb{Z}/2\mathbb{Z}))$, and $\Delta 334(SL_3(\mathbb{Z}/3\mathbb{Z}))$. We will use information about $\Delta 334(SL_3(\mathbb{Z}/2\mathbb{Z}))$ to show that the chromatic number of $\Delta 334(SL_3(\mathbb{Z}))$ is at most eight. By generating a portion of $\Delta 334(SL_3(\mathbb{Z}))$ we show its chromatic number is at least four; we conjecture it is equal to four.

Jessica Radford, Institute for Pure and Applied Mathematics
Predicting Start-Up Behavior of Vapor Chambers and Heat Pipes from the Frozen State

Heat pipes are devices that transfer heat via phase changes in electronic devices. They can be found in laptops, workstations, and satellites. The device is made of an outer metal shell lined with a porous wick and filled with a working fluid. Heat pipe behavior is predictable and reliable at normal temperatures. However, if the heat pipe is exposed to very low temperatures, the working fluid may freeze and change the behavior of the start-up. In particular, if the ingoing heat flux is too large, the heat pipe may overheat, which can result in permanent damage to the pipe and the device in which it is used. In this project, we attempted to numerically simulate the start-up behavior of a conventional heat pipe from a frozen state using transient finite element method within the MOOSE framework. Specifically, we successfully modeled liquid-phase fluid flow and solid-liquid phase change of the working fluid. Additionally, we explored potential methods for liquid-vapor phase change and created a heat pipe mesh that consists of all the parts common for a conventional heat pipe.

Mackenzie Ray, Augsburg University
Perspective 3D Modeling of a 4D Hypercube for Education Purposes

Various figures of four-dimensional hypercubes have been created using computer programming and 3D printing, many of which showcase static cube-first stereographic projections of the hypercube in 3D space. In this project, we created a 3D model of a hypercube that has moveable parts which showcases a rotation about the xz-plane, for the purposes of allowing a broader range of students accessibility to four-dimensional concepts. Based on the research done in this project, adaptations to previous models of hypercubes were made with the intention to have the new model have parts that students can manipulate for the goal of not only showing the main geometric characteristics of hypercubes, but some possible 4D rotations that can be seen within the 3D projection. There is an emphasis on the importance of perspective and psychological concepts of perception and learning, with the goal of creating the 3D model for educational purposes. The final model is based upon a clear understanding of student visualization, perception, and perspective, for the purposes of a complete 3D manipulative model of a 4D hypercube that can be utilized in educational contexts for student benefits.

Hannah Reid, Emory & Henry College
Integer Sequences Arising from Grids

This presentation will be discussing a combinatorial proof giving a recursive formula for $\binom{n^2}{2}$ interpreted as the number of ways to choose two squares from an $n \times n$ grid. Then we will talk about how this relates to the triangular numbers. A visual proof will be provided of why $\binom{n^2}{2}$ is always a triangular number and how this is connected to Gauss' formula for adding consecutive integers. We will also discuss using the Online Encyclopedia for Integer Sequences (OEIS) as a tool for research and include additional sequences derived from considering $n \times n$ grids.

Carly Reith, Duquesne University
3D Agent-based Model of Neural Activity in the Central Nucleus of the Amygdala During Pain

The central nucleus of the amygdala (CeA) is a region of the brain important in pain processing. Neurons within the CeA expressing protein kinase c-delta (PKC-d) or somatostatin (SOM) have opposing roles in pain modulation. A 3-dimensional computational space was created in an in-silico manner to study the role of these neuron populations and their connectivity to predict system-level measures of pain. The model consists of 5000 neurons with cell-type specific properties and behaviors estimated from laboratory data. During each model time step, neuron firing rates update based on an external stimulus, and a network of directed links sends inhibitory signals between neurons. A measure of pain is calculated as the difference in the cumulative firing rates of PKC-d and SOM neurons. Results demonstrate the model's ability to produce both spontaneous and evoked pain in response to noxious stimuli. We are continuing to enhance the ABM to include a spatial domain that accurately reflects the topology of the CeA and its subregions.

Catherine Riley, Cedarville University
An Overview of Monstrous Moonshine

The monstrous moonshine conjecture by Conway and Norton started a journey to show how the J-function and the Monster were connected. The presentation will cover a few of the components of the conjecture, specifically an overview of the J-function, and then discuss the conjecture itself. Some of the ideas that led to the solution of the conjecture will also be mentioned.

Daniela Rodriguez-Chavez, Cornell University
Understanding Sea Surface Height Variability at 10km Scales

In 2022, NASA will launch the Surface Water and Ocean Topography (SWOT) mission, which will send a satellite to collect data on sea surface height and ocean topography. However, even though the data will have a good spatial resolution, it will lack a strong temporal resolution due to the nature of the satellite's orbit; this limits our ability to distinguish between slow geostrophic eddies and fast wave motion. Thus, in 2019 a pre-launch campaign was carried out for 4.6 months to provide a foundation of robust time statistics for the future SWOT mission data. The in-situ campaign included the deployment of two moorings, placed 10 kilometers apart off the coast of California, where sea surface height (SSH) was measured every second. Using these in-situ data, we tested whether previous predictions for 10km SSH signal are consistent with the observations. We calculate a prediction based on a model from recent work for the observable frequency power spectrum of both moorings as well as the difference across the moorings and find that these predictions are generally reflected in the data. Finally, we discuss the implications of these results as they relate to the upcoming SWOT mission.

Jenna Royce, Creighton University
Establishing Distance Using Persistent Homology

Persistent homology is a tool used to study qualitative features of data over a variety of size scales, often with the goal of identifying noise within high dimensional data sets. It has various real-life applications, such as finding similarities in medical imaging methods, as well as areas such as global development and tourism. Specifically, the persistent homology process applied to a data set returns a barcode. The goal of this project is to construct and analyze a variety of possible "distance" functions on the space of all barcodes, which can be then be interpreted as providing a "distance" between data sets.

Kayla Selvig, Concordia College, Moorhead
Chi Tran, Concordia College, Moorhead
The Hiring Problem: Modifications and Win Strategies

Our research focuses on optimal stopping problems in the context of the Secretary Problem, a scenario in which a fixed number of candidates, N, are interviewed one at a time and given a unique relative rank. There is a known optimal strategy for hiring the best candidate in the classical version of the problem. We are interested in finding strategies that will help us hire the best candidate in variations of the secretary problem. These include permutation pattern avoidance to avoid less-desirable scenarios, consecutive permutation pattern avoidance, and optimal strike set.

Izah Tahir, Georgia Institute of Technology
see **Viridiana Neri**

Yuki Takahashi, Grinnell College
Lifting Methods in Mass Partition Problems

Ham sandwich theorem guarantees that we can cut a three-ingredient sandwich once so that each ingredient is split exactly in half; more precisely, it proves the existence of a hyperplane that simultaneously splits d given measures in $\mathbb{R}^d$. Variations of this theorem that split more than d measures in $\mathbb{R}^d$ are proved by lifting $\mathbb{R}^d$ to a higher-dimensional space. By extending this lifting argument to polyhedral surfaces, we proved the existence of equipartitions of $d+1$ measures in $\mathbb{R}^d$ by parallel hyperplanes and of $d+2$ measures in $\mathbb{R}^d$ by concentric spheres.

Chi Tran, Concordia College, Moorhead
see **Kayla Selvig**

Pamela Vargas, Smith College
see **Parneet Gill**

Ella Wilson, Kenyon College
Using Circle Packings to Approximate Harmonic Measure Distribution Functions

Harmonic Measure Distribution Functions, or h-functions, encode information about the geometry of domains in the plane. Specifically, given a domain and a basepoint within it, for a fixed value r, the value $h(r)$ is the probability that a particle under infinite random motion first exists the domain within distance r of the basepoint. There are many domains such as the disk and the inside and outside of a wedge for which we can do direct computations of the h-function. However, for domains with more intricate boundaries exact computation is often difficult or impossible, so we need methods for discrete approximation. The mathematical process we used to discretize the domain is known as circle packing. I present two methods, one using geometric and one using probabilistic techniques, to create discrete analogues of the h-function. Both techniques utilize circle packings and have connections to open problems in the field of h-functions.

Suixin (Cindy) Zhang, Colby College
Bounding the L-invariant of Spun Knots

A trisection, introduced by Gay and Kirby in 2012, is a simple decomposition that can be used to construct a 2-dimensional diagram of a closed, compact, connected, oriented 4-manifold. In 2015, Meier and Zupan introduced the notion of bridge trisection and showed that every knotted surface in 4-sphere admits such a simple decomposition. The L-invariant measures the complexity of bridge trisections using the pants complex of the surface. In this talk, I will first give the definition of a trisection and show the example of the trisection of an unknotted sphere. After discussing tangles and spun knots, I'll describe bridge trisections. I will define the L-invariant and conclude with some of our results on the estimates for the L-invariant for spun knots: the simplest of all knotted spheres.