Conference Talks

Hakima Bessaih, Florida International University

Continuous Data Assimilation for Displacement in a Porous Medium

We propose the use of a continuous data assimilation algorithm for miscible flow models in a porous medium. In the absence of initial conditions for the model, observed sparse measurements are used to generate an approximation to the true solution. Under certain assumption of the sparse measurements and their incorporation into the algorithm it can be shown that the resulting approximate solution converges to the true solution at an exponential rate as time progresses. Various numerical examples are considered in order to validate the suitability of the algorithm.

Elizabeth Carlson, University of Victoria

Accurately and Efficiently Modeling Turbulent Flows: Data Assimilation
and Parameter Recovery

One of the challenges of the accurate simulation of turbulent flows is that initial data is often incomplete. Data assimilation circumvents this issue by continually incorporating the observed data into the model. A recent approach to data assimilation known as the Azouani-Olson-Titi (AOT) algorithm introduced a feedback control term to the 2D incompressible Navier-Stokes equations (NSE) in order to incorporate sparse measurements. The solution to the AOT algorithm applied to the @D NSE was proven to converge exponentially to the true solution of the 2D NSE with respect to the given initial data. In this talk, we present our tests on the robustness, improvements, and implementation of the AOT algorithm, as well as generate new ideas based off these investigations. First, we discuss the application of the AOT algorithm to the 2D NSE with an incorrect parameter and prove it still converges to the correct solution up to an error determined by the error in the parameters. This led to the development of a simple parameter recovery algorithm, whose convergence we recently proved in the setting of the Lorenz equations. The implementation of this algorithm led us to provide rigorous proofs that solutions to the corresponding sensitivity equations are in fact the Fréchet derivative of the solution to the original equations. We may also briefly touch on a nonlinear data assimilation algorithm and applications to climate models.

Hongqiu Chen, University of Memphis

Co-existence of stable and unstable solitary-wave solutions

It is well-known that solitary-wave solutions for single KdV-equation are orbitally stable. However, we recently encountered a class of systems of coupled nonlinear dispersive equations that possess both stable and unstable solitary-wave solutions. Numerical simulations show that some unstable solitary waves resolve into a train of stable solitary waves together with a dispersive tail, and some simply blow up in finite time.

Aslihan Demirkaya, University of Hartford

Kink Dynamics in a Parametric Biharmonic Phi4- Model

We consider the interaction of solitons in a biharmonic, beam model analogue of the well-studied phi4 Klein-Gordon theory. Specifically, we calculate the force between a well separated kink and antikink. Knowing their accelerations as a function of separation, we can determine their motion using a simple ODE. There is good agreement between this asymptotic analysis and numerical computation. Importantly, we find the force has an exponentially-decaying oscillatory behavior (unlike the monotonically attractive interaction in the Klein-Gordon case). Corresponding to the zeros of the force, we predict the existence of an infinite set of field theory equilibria, i.e., kink-antikink bound states. We confirm the first few of these at the PDE level, and verify their anticipated stability or instability. We also explore the implications of this interaction force in the collision between a kink and an oppositely moving antikink.

Scott Hansen, Iowa State University

Spaces of exact controllability of some PDE involving internal point masses

It is known that a wave traveling on a 1-d elastic string which encounters a point mass has a part that reflects with the same regularity and another part that transmits with 1 Sobolev degree higher regularity. As a consequence, the space of controllability with control active at one end of the string is *asymmetric* interms of Sobolev regularity with respect to the point mass. Recently this result has been investigated and partly generalized to other PDE's involving internal point masses. For example, in the case of a Schrödinger equation the EC spaces can be described precisely characterized in terms of some weighted Fourier series spaces, which can be symmetric, asymmetric or a combination, depending upon coefficients and boundaryconditions. A similar characterization also applies to an Euler-Bernoulli beam with an internal point massin some situations.

Weiwei Hu, University of Georgia

Boundary Control for Optimal Mixing via Navier-Stokes Flows

We discuss the problem of optimal mixing of an inhomogeneous distribution of a scalar field via an active control of the flow velocity, governed by the Navier-Stokes equations, in a two dimensional open bounded and connected domain. We consider the velocity field steered by a control input that acts tangentially on the boundary of the domain through the Navier slip boundary conditions. This is motivated by the observation that moving walls accelerate mixing. Our main objective is to design an optimal Navier slip boundary control that optimizes mixing at a given final time. Non-dissipative scalars, both passive and active, governed by the transport equation will be discussed. In the absence of diffusion, transport and mixing occur due to pure advection. This essentially leads to a nonlinear control problem of a semi-dissipative system. A rigorous proof of the existence of an optimal controller and the first-order necessary conditions for optimality will be presented. Numerical experiments on mixing in Stokes flows will be presented to demonstrate our design and ideas.

Ning Ju, Oklahoma State University

Uniqueness and global uniform boundedness for some classes of weak solutions
to viscous 2D Primitive Equations

My results on uniqueness of weak solutions to viscous 2D Primitive Equations (PE) will be discussed. The problem has been studied by researchers, but is still not yet completely resolved to my knowledge. One of the main difficulties of the problem is that the non-linearity in weak formulation of IVBP of 2D viscous PE seems about as strong as for 3D Navier-Stokes Equations, for which the problem of uniqueness of Leray-Hopff weak solutions has been studied for years. Related to the problem of uniqueness of weak solutions of PE, my results on improved regularity properties for weak solutions to viscous 2D PE along with longtime dynamical behavior of the weak solutions with partial regularities will be discussed as well.

Tetyana Malysheva, University of Wisconsin-Green Bay

Mathematical Analysis of a Landscape Evolution Model

We consider a nonlinear advection-diffusion equation motivated by a landscape evolution model arising in geomorphology applications. The underlying landscape evolution model describes changes in Earth's surface elevation in response to fluvial and hillslope bedrock erosion and localized uplift relative to magmatic intrusions. In the first part of the talk, we present results on the global existence and uniqueness of a weak solution to the equation and its continuous dependence on data and model parameters. The proofs are based on the Galekin method with a compactness argument and energy estimates derived by means of the Gronwall-type argument. The second part of the talk is focused on estimation of model parameters from output measurements representing surface elevation. The approach is based on minimization of an error functional. The obtained results are illustrated by a numerical example for a landscape evolution model with Henry Mountains data. This is joint work with Luther W. White, Oregon Applied Mathematics Institute, and Leif Karlstrom, The University of Oregon.

Dionyssis Mantzavinos, University of Kansas

Initial-boundary value problems for dispersive PDEs

Nonlinear dispersive equations have been a topic of major interest within the broader PDE community. In particular, the initial value (Cauchy) problem for these equations has been studied extensively via a range of tools and techniques from areas such as harmonic analysis, differential geometry, and completely integrable systems. For example, the study of the IVP for the nonlinear Schrödinger and the Korteweg-de Vries equations respectively led to the discoveries of the celebrated Bourgain spaces and the inverse scattering transform method. Contrary to IVPs, where the spatial domain is unbounded, our understanding of dispersive PDEs is far less developed when these are paired with spatial domains that involve a boundary (e.g. the half-line). In such cases, in addition to the usual initial data one needs to also prescribe appropriate boundary data, formulating an initial-boundary value problem (IBVP). The slow progress in the study of IBVPs becomes evident from the fact that, even the fundamental question of well-posedness, which refers to the existence and uniqueness of solution as well as to the dependence of the solution on the data, is still largely unexplored. In this talk, we will discuss a new method developed in collaboration with Alex Himonas and Thanasis Fokas for proving local well-posedness of nonlinear dispersive PDEs in the IBVP setting. We will consider different types of boundary conditions, in one or more spatial dimensions, and for various PDEs, including also a non-dispersive example.

Leo Rebholz, Clemson University

Applications of Anderson acceleration to algorithms for Newtonian and non-Newtonian fluid simulation

After reviewing recent theoretical results for Anderson acceleration (AA), we consider its application to solving incompressible Navier-Stokes equations and regularized Bingham equations. For NS, the classical penalty method is considered, which typically will only work with very small penalty (but very small penalty causes issues with iterative solvers, making it not practical for large scale use). For regularized Bingham, we consider a Picard type iteration that has trouble converging for small regularization parameter. We show that both of these methods can be cast as a fixed point iterations that fall into the AA theory framework, which allows for improved convergence rates to be proven. Moreover, numerical results reveal that with AA, the classical penalty method is very effective even with O(1) penalty parameter and regularized Bingham Picard iteration is dramatically improved and nearly robust with respect to the regularization parameter.

Michael Renardy, Virginia Tech (retired)

Controllability of linear viscoelastic flows

The lecture will review results on controllability of viscoelastic flows obtained over the last decade. For linear viscoelastic flows with any finite number of relaxation modes, approximate controllability has been shown. The result can be extended to an infinite number of relaxation modes, provided the relaxation times satisfy a gap condition. If there is more than one relaxation mode, exact null controllability does not hold. Approximate controllability holds within a subspace of the state space, which is invariant under the evolution of the system. This invariance is destroyed by nonlinear terms, as well as by variable coefficients. This leads to challenging and large unresolved problems.

Gieri Simonett, Vanderbilt University

On the Navier-Stokes equations on surfaces

I will consider the motion of an incompressible viscous fluid on a compact surface without boundary. Local-in-time well-posedness is established in the framework of Lp-Lq-maximal regularity. It will be shown that the set of equilibria consists exactly of the Killing vector fields. Moreover, it will be shown that each equilibrium is stable and that any solution starting close to an equilibrium converges at an exponential rate to a (possibly different) equilibrium as time tends to infinity.

Edriss Titi, Texas A&M and The University of Cambridge

On Recent Advances of the 3D Euler Equations by Means of Examples

In this talk we will use a basic example of shear flow to demonstrate some of the recent advances in the three-dimensional Euler equations. Specifically, this example was introduced by DiPerna and Majda to show that weak limit of classical solutions of Euler equations may, in some cases, fail to be a weak solution of Euler equations. We use this shear flow example to show the immediate loss of smoothness and ill-posedness of solutions of the 3D Euler equations, for initial data that do not belong to $C^{1,\alpha}$. Moreover, we also show the existence of weak solutions for the 3D Euler equations with vorticity that is having a nontrivial density concentrated on non-smooth surface (vortex sheet). This is very different from what has been proven for the two-dimensional Kelvin-Helmholtz (Birkhoff-Rott) problem where a minimal regularity implies the real analyticity of the interface. Furthermore, we use this shear flow to provide explicit examples of non-regular solutions of the three-dimensional Euler equations that conserve the energy, an issue which is related to the Onsager conjecture. Eventually, we will discuss the recent remarkable work of De Lellis and Székelyhidi concerning the wild weak solutions of Euler equations and their non-uniqueness. In particular, we propose the following ruling out criterion for non-physical weak solutions of Euler equations: "In the absence of physical boundaries any weak solution of Euler equations which is not a vanishing viscosity limit of Leray-Hopf weak solutions of the Navier-Stokes equations should be ruled out''. We will use this shear flow, and other solutions of Euler equations with certain spatial symmetry, to provide nontrivial examples for the use of this ruling out criterion. If time allows we will also discuss (i) recent progress concerning the Onsager conjecture in bounded domains; (ii) the nonuniqueness of weak solutions to the 3D Navier-Stokes equations with Hyper-viscosity $(-\Delta)^{\theta}$, for $\theta < 5/4$, demonstrating the sharpness of the J.-L. Lions result.

Poster Session

Animesh Biswas, University of Nebraska-Lincoln

Nonlocal curvature with integrable kernel

The focus of this talk will be on the recently introduced topic of nonlocal curvature, defined as

$$H^J_\Omega(x) := \int_{\mathbb{R}^n} J(x-y)(\chi_{\Omega^c}(y) - \chi_\Omega(y))dy,$$

where $x \in \mathbb{R}^n$, $\Omega \subset \mathbb{R}^n$, χ is the characteristic function for a set, J is a radially symmetric, nonegative, nonincreasing convolution kernel. Several papers have studied the case of nonlocal curvature with nonintegrable singularity, a generalization of the classical curvature concept, which requires the regularity of the boundary to be above C^2. Nonlocal curvature of this form appears in many different applications, such as image processing, curvature driven motion, deformations. Our results offer some generalizations and extensions to the constant mean curvature problem, where counterparts to Alexandrov's theorem in the nonlocal framework were established independently by two separate groups: Ciraolo, Figalli, Maggi, Novaga, and respectively, Cabre, Fall, Sola-Moreles, Weth. By using the concept of nonlocal curvature for integrable kernels J, as discussed by Mazon, Rossi, Toledo, we are able to lower requirements on the smoothness of the boundary.

Nicole Buczkowski, University of Nebraska-Lincoln

Stability of solutions to nonlocal models with respect to changes in data

Stability of solutions with respect to changes in data is an important feature for solutions that model physical phenomena. As measurements of physical parameters can never be exact, small changes in measured data will hopefully generate similarly small changes in the solutions. The exact dependence can be studied with analysis and partial differential equation tools and serve engineers, computational scientists, and also be important in other theoretical studies. Nonlocal models, with their capability to handle discontinuities, record long range interactions through a kernel which gives additional flexibility. In this poster, we show several results on the continuous dependence (or stability) of the solution with respect to changes in (linear and nonlinear) forcing terms, boundary or collar data, as well as with respect to the kernel of the nonlocal operator. Numerical results are also given.

Hengrong Du, Vanderbilt University

On partial regularity of the Ericksen–Leslie system in 3-D

The Ericksen–Leslie system models the hydrodynamics of nematic liquid crystals. It is a strongly coupled PDE system between incompressible Navier–Stokes equations for the underlying fluid velocity field and gradient-flow-like equations for the director field describing the averaged alignment of liquid crystal molecules. In dimension three, we establish partial regularity of suitable weak solutions to the Ericksen–Leslie system with the Ginzburg–Landau approximation which asserts that the solution is smooth away from a closed set with parabolic Hausdorff dimension at most 15/7. This is joint work with Changyou Wang (Purdue).

Thomas Hogancamp, University of Missouri

Broadening Global Families of Anti-Plane Shear Equilibria

This poster is based on some recent global bifurcation results in nonlinear elastostatics. Specifically, we consider anti-plane shear deformations for several classes of materials. In one case, we show that a broadening phenomena occurs. Roughly speaking, this corresponds very wide and flat equilibria solutions in the global continuum. Broadening has been predicted numerically for internal solitary water waves, yet no rigorous construction of such waves exists. Our result seems to be the first to show broadening in the PDE context. Another class of materials is shown to suffer a breakdown of ellipticity in the limit. This behavior has been linked to failure mechanics for nonlinear elastic materials.

Josue Knorst, University of Kansas

Benjamin Krewson, University of Missouri

Defect Dynamics of Quenched Striped Patterns

We study transverse modulational dynamics of striped pattern formation in the wake of a directional quenching mechanism. Such mechanisms have been proposed to control pattern-forming systems and suppress defect formation in many different physical settings, such as light-sensing reaction-diffusion equations, solidification of alloys, and eutectic lamellar crystal growth. Furthermore, they are a simple model of a growth process in a patterned biological system. In the context of the complex Ginzburg-Landau and Swift-Hohenberg equations, two prototypical models of pattern formation, we show that long-wavelength and slowly varying modulations of striped patterns are governed by a one-dimensional viscous Burgers equation, with viscous and nonlinear coefficients determined by the quenched stripe selection mechanism. This reduced model allows for accurate description of defect dynamics as shock/rarefaction dynamics.

Hayley Olson, University of Nebraska-Lincoln

Nonlinear Diffusion in the Nonlocal Calculus Framework: Convergence Results

Nonlocal calculus operators can be used to model phenomena classical differential operators cannot capture -- such as super- and subdiffusion processes. These nonlocal integral operators act on an interaction horizon about the point of interest which allows the models utilizing them to have solutions with lower requirements on regularity including functions which are not differentiable and even discontinuous. Here, we investigate the introduction of nonlinearities to the nonlocal calculus framework and provide convergence results for certain classes of nonlinearities to show they behave similarly to classical operators when the interaction horizon shrinks down to the point of interest.

Abba Ramadan, University of Kansas

Standing Waves of the Schrödinger Equation with Concentrated nonlinearity

We study the concentrated NLS on $\mathbb{R}^n$, with power non-linearities, driven by the fractional Laplacian, $(-\Delta)^s, s > \frac{n}{2}$. We construct the solitary waves explicitly, in an optimal range of the parameters, so that they belong to the natural energy space H^s. Next, we provide a complete classification of their spectral stability. Finally, we show that the waves are non-degenerate and consequently orbitally stable, whenever they are spectrally stable.

Daniel Sinambela, University of Missouri - Columbia

Large-amplitude solitary waves in two-layer density stratified water

We present a large-amplitude existence theory for two-dimensional solitary waves propagating through a two layer body of water. The domain of the fluid is bounded below by an impermeable flat ocean floor and above by a free boundary at constant pressure. For any piecewise smooth upstream density distribution and laminar background current, we construct a global curve of solutions. This curve bifurcates from the background current and, following along the curve, we find waves that are arbitrarily close to having horizontal stagnation points. The small-amplitude waves are constructed using a center manifold reduction technique. The large-amplitude theory is obtained through analytical global bifurcation together with refined qualitative properties of the waves.

Majed Sofiani, University of Kansas

Global existence of the general poiseuille flow of nematic liquid crystals via the full Ericksen-Leslie model

The Poiseuille flow of the full Ericksen-Leslie model for nematic liquid crystals is a coupled system of partial differential equations consisting of two equations. One is a quasi-linear hyperbolic wave equation representing the orientation of the molecules, the crystallization of the material, and the other is an equation of parabolic type derived from Navier-Stokes equations representing the velocity of the flow, the liquidity of the material. In this work, we study the Cauchy problem of the system for a general choice, up to physical constraints, of the viscous (Leslie) stress tensor coefficients. We show the global existence of a weak solution of the Cauchy problem that is Holder continuous.

Duygu Vargun, Clemson University

An efficient nonlinear solver and convergence analysis for a viscoplastic flow model

This poster studies a finite element discretization of the regularized Bingham equations that describe viscoplastic flow. Convergence analysis is provided, as we prove optimal convergence with respect to the spatial mesh width but depending inversely on the regularization parameter ε, and also suboptimal (by one order) convergence that is independent of the regularization parameter. An efficient nonlinear solver for the discrete model is then proposed and analyzed. The solver is based on Anderson acceleration (AA) applied to a Picard iteration, and we prove accelerated convergence of the method by applying AA theory (recently developed by authors) to the iteration, after showing sufficient smoothness properties of the associated fixed point operator. Numerical tests of spatial convergence are provided, as are results of the model for 2D and 3D driven cavity simulations. For each numerical test, the proposed nonlinear solver is also tested and shown to be very effective and robust with respect to the regularization parameter as it goes to zero.

Collin Victor, University of Nebraska-Lincoln

Modifications of a Data Assimilation Method for Turbulence Modeling

Data assimilation is a technique for increasing the accuracy of simulations of solutions to partial differential equations by incorporating observable data into the solution as time evolves. In this work we consider modifications to the Azouani-Olson-Titi algorithm, which is a continuous data assimilation scheme that incorporates observations continuously in time at the PDE level. We demonstrate computationally in the case of the 2D incompressible Navier-Stokes equations that the simulated solution generated by the algorithm converges exponentially fast in time to the true solution for observations that are sparse in time or moving continuously in space. We prove these results hold analytically in the case of sparse in time observations. Additionally, we demonstrate computationally that sparse in time observations produce exponential convergence to the true solution for the 3D incompressible Navier-Stokes equations.

Anh Vo, University of Nebraska-Lincoln

Nonlocal Advection-Convection Equation

In this study, we investigate the convergence of solutions of nonlocal advection-diffusion PDE to the local counterparts in 1D. Nonlocal operators are integral operators that mimic differential operators but account for long-range interactions over a finite horizon. Nonlocality appears in many physical phenomena and has a wide range of applications. In the past, it was only shown that the solutions of the nonlocal Burgers equation converge to the local counterpart. In our research, we generalize the nonlocal advection operator. In the limit when the horizon parameter approaches zero, we prove nonlocal operators convergence pointwise to its local counterpart. Then, we apply the result to show the convergence of the solutions of the nonlocal advection-diffusion equation to the local counterpart

Ahmed Zytoon, Iowa State University