

# A MOUNTAIN PASS TO THE JACOBIAN CONJECTURE

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**Abstract.** This paper presents a new injectivity theorem and a new open question. The main result of the paper is proved by means of the Mountain Pass Lemma and states that if all the eigenvalues of  $F'(\mathbf{x})F'(\mathbf{x})^T$  are bounded away from zero for all  $\mathbf{x} \in \mathbb{R}^n$ , where  $F : \mathbb{R}^n \rightarrow \mathbb{R}^n$  is a class  $C^1$  map, then  $F$  is injective. This was discovered in a joint attempt by the authors to prove a stronger result conjectured by the first author: Namely, that a sufficient condition for injectivity of class  $C^1$  maps  $F$  of  $\mathbb{R}^n$  into itself is that all the eigenvalues of  $F'(\mathbf{x})$  are bounded away from zero on  $\mathbb{R}^n$ . If true, it would imply (via *Reduction-of-degree*) *injectivity of polynomial maps*  $F : \mathbb{R}^n \rightarrow \mathbb{R}^n$  *satisfying the hypothesis*,  $\det F'(\mathbf{x}) \equiv 1$ , of the celebrated Jacobian Conjecture of Ott-Heinrich Keller. The paper ends with several examples to illustrate a variety of cases and known counterexamples to some natural questions.

## References

1. M. Chamberland and G.H. Meisters. *A Mountain Pass to the Jacobian Conjecture* to appear in the Canadian Mathematical Bulletin.