

Teaching Mathematical Epidemiology at a Variety of Levels Using Multiple Representation Theory

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Part 1: Theory

1. Multiple representation theory
2. Some modeling activities
3. Narrow and broad views

Multiple representation theory

Multiple representation theory was introduced by Diaz-Eaton et al (PRIMUS, 2019). It is a variant of the “Rule of Four.”

- ▶ Models have multiple representations:
 - Verbal
 - Visual
 - Symbolic
 - Numerical
 - Experiential

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- ▶ **Modeling: making connections between representations.**

Some modeling activities

- ▶ Choosing questions
- ▶ Choosing outcomes to report
- ▶ Choosing assumptions
- ▶ Translating assumptions to mathematics
- ▶ Determining parameter values
- ▶ Doing graphical analysis
- ▶ Doing symbolic analysis
- ▶ Running simulations
- ▶ Presenting results
- ▶ Communicating results

Narrow and broad views

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- ▶ **Broad view:** A map from the space of parameters to the space of outcomes.
- ▶ **The technical work is in the narrow view, but most of the modeling is in the broad view.**
 - Example: Plot a graph of the final susceptible population as a function of the basic reproductive number.

Part 2: Practice

1. A classroom activity
2. Questions
3. Model development
4. Simulation
5. Parameterization
6. Comparisons
7. Parameter studies
8. Reporting results

A classroom activity

- ▶ Students have status cards: green for susceptible, yellow for asymptomatic infectious, red for symptomatic infectious, blue for recovered.
- ▶ Students are paired randomly in each time step. If green meets yellow or red, transmission occurs with probability $5/6$.
- ▶ Transitions from yellow to red and red to blue take exactly one day.
- ▶ Students record, graph, and discuss results.
- ▶ Variants can include isolation of symptomatics and vaccination.

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- ▶ What outputs do we want to measure?
 - Graphs of class populations
 - Maximum size of class I
 - Time for peak of infections
 - Final percentage susceptible
 - Hospitalizations
 - Deaths

Model development - assumptions

We focus on the SEIR epidemic model as an example.

- ▶ Four classes:
 - S: Susceptible (can be infected)
 - E: Exposed/Latent (infected, but cannot transmit)
 - I: Infectious (can transmit)
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- Susceptible (S) to latent (E) by transmission at rate βSI .
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- ▶ Two scenario parameters:
 - Add a contact factor δ that accounts for masks.
 - Add an initial immunity fraction V for follow-up scenarios.

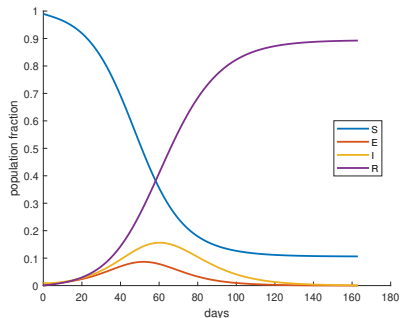
Model development - equations

- Susceptible (S) to latent (E) by transmission at rate $\delta\beta SI$, where δ is a “contact factor” that represents policy and behavior.
- Latent (E) to infectious (I) by incubation at rate ηE .
- Infectious (I) to removed (R) by recovery/death at rate γI .

$$\begin{array}{ll}
 S' = \delta\beta SI, & S(0) = 1 - E_0 - I_0 - V; \\
 E' = \delta\beta SI - \eta E, & E(0) = E_0 \quad ; \\
 I' = \eta E - \gamma I, & I(0) = I_0 \quad ; \\
 R' = \gamma I, & R(0) = V \quad .
 \end{array}$$

Simulation

- ▶ I have simulation programs in Matlab, R, and Excel for SIR, SEIR, and SEAIHRD (Covid-19).
- ▶ Students enter scenario data and get a graph.



Parameterization (for Covid-19)

- ▶ Best estimates of incubation period and infectious period are $t_e = 5$ and $t_i = 10$, or $\eta = 0.2$ and $\gamma = 0.1$.
- ▶ For β , assume we know the early phase doubling time t_d .
 - Assume $E' = \lambda E$, $I' = \lambda I$, where $\lambda = \ln 2/t_d$

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 - Assume $E' = \lambda E$, $I' = \lambda I$, where $\lambda = \ln 2/t_d$
 - Substitute $E = \rho I$, $I' = \lambda I$, $S = 1$, $\delta = 1$ into E and I equations (note $\langle \rho, 1 \rangle^T$ is an eigenvector):

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$$\beta = \frac{(\lambda + \eta)(\lambda + \gamma)}{\eta}, \quad (1)$$

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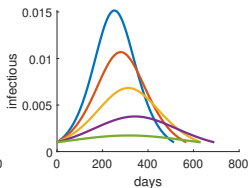
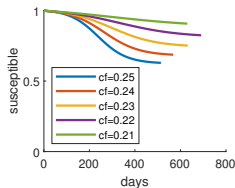
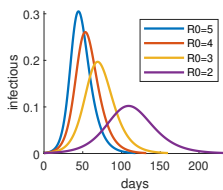
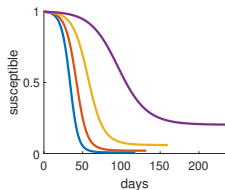
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- With $t_d \in [3.5, 4.0]$, we get

$$\mathcal{R}_0 = \beta t_i = \frac{(\lambda + \eta)(\lambda + \gamma)}{\eta\gamma} \in [5.1, 5.9].^* \quad (2)$$

Comparisons

- ▶ Vary infectivity β with no interventions or immunity. Examine time series for S and I . (top)
- ▶ Vary contact factor (with $R_0 = 5.9$ for Covid-19). (bottom)



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 - Use Excel with a template.
 - Students bring some prior experience.
 - Limited programming capacity.
 - Provide Matlab programs (usable with Octave).
 - Some modifications needed.
 - If properly designed, the modifications are minimal!

Making programming easy – SEIR_simplot.m

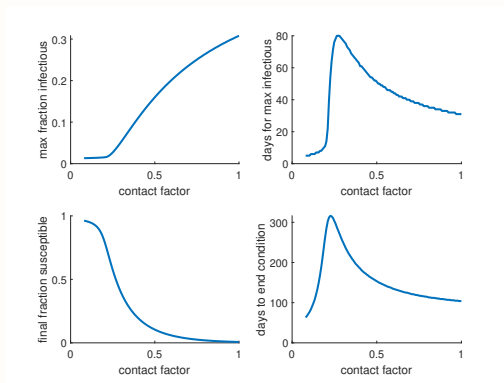
```
% DEFAULT SCENARIO DATA
beta = 0.5;
etc
V = 0;

% INDEPENDENT VARIABLE DATA
xvals = [0.5,0.4,0.3,0.2];

% COMPUTATION
for n=1:N
    beta = xvals(n);
    [S,E,I,R] = seir_sim(beta,eta,gamma,E0,I0,V)
    etc
end
```

Parameter studies

- ▶ How does the extent of mask use affect key outcomes: maximum I , final S , times for these events?
 - Use `SEIR_paramstudy.m`.



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 - Verbal *descriptions*, like “The function increases to a maximum and then decreases to 0”, are not enough.
 - Insist on *explanations* and/or *interpretations*, like “The infectious population reaches a maximum because transmission slows down as the number of susceptibles decreases.”

Resources

- ▶ Thanks for “coming”!
- ▶ See <https://www.math.unl.edu/SIR-modeling> for
 - Details on the classroom activity, including directions for an online implementation;
 - Materials for using spreadsheets to teach epidemic modeling;
 - Links to some useful resources.
- ▶ See <https://www.math.unl.edu/covid-module> for similar spreadsheet materials.
- ▶ These pages will be updated soon to include Matlab materials, SEIR versions, and notes and videos from my epidemiology class just finished.
- ▶ **Shoot me an email to receive updates on my materials.**