

Department of Mathematics

Graduate Advisory Committee

April 2021

Analysis Qualifying Exam Syllabus

The analysis portion of the Ph.D. Qualifying Examination is a three hour written exam which covers the fundamentals of Real Analysis and its standard applications. The main topics are listed below, together with a few references. This list is intended only as a guide.

Metric space topology: countable and uncountable sets, including Cantor's diagonal arguments; topology of Euclidian space and metric spaces, norms and inner products; parallelogram law; compactness in terms of subsequences and open covers; the Heine-Borel theorem; connected sets;

Sequences and series: convergence of sequences in metric spaces; Cauchy sequences and completeness; lim sups and lim infs; comparison, ratio, root, integral tests for convergence of series; absolute and conditional convergence; rearrangements of series; alternating series test; summation by parts;

Continuity of functions: continuity and uniform continuity for functions of one or several variables; images of connected sets; intermediate value theorem; images of compact sets; extreme value theorem; monotonic functions;

Calculus of functions of one variable: definition of derivative; mean value theorem; intermediate value property for derivatives; Taylor's theorem; Riemann integrals; fundamental theorem of calculus; improper integrals and Cauchy principal values; functions of bounded variation; Riemann-Stieljes integrals;

Sequences and series of functions: uniform convergence; conditions for term-by-term continuity, integration, and differentiation; Weierstrass's M-test; power series and the radius of convergence; Stone-Weierstrass theorem; equicontinuity and the Arzela-Ascoli theorem.

References.

- T. Apostol, *Mathematical Analysis*, 2nd Ed., Addison-Wesley, 1974.
- K. Davidson & A. Donsig, *Real Analysis and Applications*, Prentice-Hall, 2002.
- W. Rudin, *Principles of Mathematical Analysis*, 3rd Ed., McGraw-Hill, 1976.
- R. Strichartz, *The Way of Analysis*, Revised Ed., Jones & Bartlett, 2000.
- W. Wade, *An Introduction to Analysis*, 3rd Ed., Pearson Prentice Hall, 2004.