Department of Mathematics

Graduate Advisory Committee
April 2012

Applied Math Qualifying Exam Syllabus

The following are only the core topics to be studied for the qualifying exam. Additional material may be included.

(1) Dimensional analysis; scaling; two-dimensional ODEs, including phase plane, nonlinear systems (equilibria and nullclines) and bifurcation.

(2) Perturbation Methods: regular perturbation; singular perturbation; Poincare-Lindstedt method; boundary layer analysis; asymptotic expansions of integrals

(3) Calculus of Variations: derivatives of functionals; the Euler equation; generalizations, several functions, natural boundary conditions

(4) Orthogonal expansions and Fourier series; Sturm Liouville problems; Integral equations: Volterra integral equations; Fredholm integral equations with separable kernels; symmetric kernels; Greens functions (boundary value problems, initial value problems)

(5) Introduction to partial differential equations: linearity versus nonlinearity, superposition; derivation of PDEs: conservation laws and constitutive equations, equilibrium equations; the heat equation; initial and initial-boundary value problems in n space dimensions; solutions by eigenfunction expansions; the Laplace transform; the Fourier transform on $\mathbb{R}^n$; stability of solutions; solutions by Green's functions

(6) Wave propagation: linear and nonlinear waves, characteristics; first-order, quasilinear conservation laws in one space dimension; shock formation, jump condition; the wave equation, initial and initial-boundary value problems, D'Alembert's solution in one space dimension. If time: Additional modeling examples

References

- J. David Logan, Applied Mathematics, Wiley-Interscience.
- C.C. Lin and L.A. Segel, Mathematics Applied to Deterministic Problems in the Natural Sciences, SIAM.