Masters and Qualifying Exam, **January, 2004.**

1. Use Putzer's algorithm and the variation of constants formula to solve the IVP

$$u(t+1) = \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix} u(t) + \begin{bmatrix} 3^{-t} \\ 0 \end{bmatrix}, \quad u(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

2. For each of the following systems determine if it has a limit cycle or not:

   (a)
   $$x' = x + y - x^3$$
   $$y' = -x + y - y^3$$

   (b)
   $$x' = y$$
   $$y' = -x - y + x^2 + 2y^2.$$

3. Show that if $\Phi(t)$ is a fundamental matrix for the vector difference equation $u(t+1) = A(t)u(t)$, then $\Psi(t)$ is a fundamental matrix of this vector difference equation if and only if $\Psi(t) = \Phi(t)C$ for some nonsingular constant matrix C.

4. State and prove the Putzer algorithm for finding e^{At}.

5. (a) Given that
   $$x(t) = \begin{bmatrix} e^{2t}\sin(2\pi t) \\ e^{2t}\cos(2\pi t) \end{bmatrix}$$
   is a solution of a Floquet system with $\omega = \frac{1}{2}$. Use this to find a Floquet multiplier of the Floquet system.

   (b) Without actually finding the Floquet multipliers of the Floquet system
   $$x' = \begin{bmatrix} 2 & \sin(3t) \\ \cos(3t) & 1 + \sin(3t) \end{bmatrix} x,$$
   find the product of the Floquet multipliers.

6. (a) Show that if $h(x, y)$ is a Hamiltonian function for a two dimensional Hamiltonian system, then $h(x, y)$ is constant along solutions of the Hamiltonian system.

   (b) Apply the LaSalle Invariance Theorem to the system
   $$x' = -x + y$$
   $$y' = -y - x^3.$$