Math 830-831, Qualifying Exam, January, 2015

Work all of the following problems:

1. Show that for any $F, G \in C^1(\mathbb{R})$ that
$$u(x,t) = F(x+t) + G(x-t) \tag{1}$$
is a solution of the one dimensional wave equation
$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} = 0.$$
Using equation (1) derive that
$$u(x,t) = \frac{1}{2}[\phi(x+t) + \psi(x-t)] + \frac{1}{2}\int_{x-t}^{x+t} \psi(s)ds$$
solves the IVP
$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} = 0$$
$$u(x,0) = \phi(x), \quad -\infty < x < \infty$$
$$\frac{\partial u}{\partial t}(x,0) = \psi(x), \quad -\infty < x < \infty,$$
where ϕ, ψ are given C^1 functions on $\mathbb{R}$.

2. Use the Putzer algorithm to help you solve the IVP
$$x' = \begin{bmatrix} -1 & 4 \\ -3 & 6 \end{bmatrix} x + \begin{bmatrix} 0 \\ e^{3t} \end{bmatrix}, \quad x(0) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

3. Let μ_0 be a number. Show that μ_0 is a Floquet multiplier of the Floquet system $x' = A(t)x$ if and only if there is a nontrivial solution of the Floquet system satisfying $x(t+\omega) = \mu_0 x(t)$ for $t \in \mathbb{R}$, where $\omega > 0$ is the prime period of $A(t)$. What can you additionally say if $\mu = -1$ is a Floquet multiplier (prove your answer)?

4. Use the Cauchy-Kovalevsky method to solve the IVP
$$u_t = u_x u_y, \quad u(0,x,y) = x + y - 2x^2$$
explicitely finding all terms of degree two and less.

5. Show that the matrix norm $\|\cdot\|_1$ for the set of all $n \times n$ matrices induced by the l_1 vector norm is given by
$$\|A\|_1 = \max_{1 \le j \le n} \sum_{i=1}^{n} |a_{ij}|.$$

6. (a) Find the integral surface of the vector field $\vec{V}(x,y,z) = < y - z, z - x, x - y >$ that contains the curve
$$C : x = t, \quad y = 2t, \quad z = 0, \quad 0 < t < \infty.$$
(b) Derive the formula for infinitely many integral surfaces of the vector field $\vec{V}(x,y,z) = < 1, 1, z >$ which contains the curve
$$C : x = t, \quad y = 1 + t, \quad z = e^t, \quad 0 < t < \infty.$$