**Qualifying Exam 830-831** **June, 1998**

- a Show that if $(\lambda_0, x_0)$ is an eigenpair for the matrix A, then $x(n) = \lambda_0^n x_0$ is a solution of the difference equation x(n+1)=Ax(n).

  b Consider the Floquet system $x'(t) = A(t)x(t)$ where $\omega$ is the smallest positive period of A(t). Show that if $\mu = -1$ is a Floquet multiplier, then there is a nontrivial solution which is periodic with period $2\omega$.

- a Write the differential equation $x'' = x - 2x^3$ as an equivalent two dimensional system.

  b Draw the phase diagram for the system you got in a.

- Use the variation of constants fomula to solve the initial value problem

$$x' = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} x + \begin{bmatrix} e^{2t} \\ 0 \end{bmatrix},$$

$$x(0) = \begin{bmatrix} 3 \\ 2 \end{bmatrix}.$$

- Determine if each of the following has a limit cycle or not.

  a

$$\begin{aligned} x' &= xe^{2y} + y^3 - x^2y, \\ y' &= y^3 + x^2y + xy^2. \end{aligned}$$

  b

$$\begin{aligned} x' &= x^2 + 2y^2, \\ y' &= x - 2. \end{aligned}$$

  c

$$\begin{aligned} x' &= -12xy + x^3, \\ y' &= 4y. \end{aligned}$$

- Find the stable subspace for the system

$$u(n+1) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{3} & \frac{1}{9} & -3 \end{bmatrix} u(t).$$

- a Factor the difference equation

$$u(n+2) - (n+3)u(n+1) + 2nu(n) = 0.$$

  Show that the factors you got do not commute.

  b Solve the above difference equation.

- Find a strict Liapunov function for the system

$$x(n+1) = \begin{bmatrix} x_2(n) - x_2(n)[x_1^2(n) + x_2^2(n)] \\ x_1(n) - x_1(n)[x_1^2(n) + x_2^2(n)] \end{bmatrix},$$

  in some ball about the origin. What conclusion can you draw from this?

- Use Putzer's algorithm to find $e^{At}$, where

$$A = \begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix}.$$

  Use your answer to solve the initial value problem

$$x' = \begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix},$$

$$x(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$