

1. Apply the LaSalle invariance theorem to the system

$$\begin{aligned}x' &= -2x - 2xy \\y' &= 2x^2 - y.\end{aligned}$$

2. a) Find a fundamental matrix for

$$x' = \begin{bmatrix} 2 & 1 \\ -9 & -4 \end{bmatrix} x.$$

- b) Use the variation of constants formula and your answer in a) to solve the initial value problem

$$x' = \begin{bmatrix} 2 & 1 \\ -9 & -4 \end{bmatrix} x + \begin{bmatrix} 2 \\ 1 \end{bmatrix};$$

$$x(0) = \begin{bmatrix} 3 \\ 2 \end{bmatrix}.$$

3. Draw the phase diagram for the system

$$\begin{aligned}x' &= -y + x(x^2 + y^2) \sin\left(\frac{\pi}{\sqrt{x^2 + y^2}}\right) \\y' &= x + y(x^2 + y^2) \sin\left(\frac{\pi}{\sqrt{x^2 + y^2}}\right).\end{aligned}$$

Find the limit cycles for this system. Is the origin asymptotically stable?

4. Find the Floquet multipliers for the Floquet system

$$x' = \begin{bmatrix} -1 + \cos t & 0 \\ \cos t & -1 \end{bmatrix} x.$$

5. State and prove the Picard-Lindeloff Theorem.
6. State and prove the Contraction Mapping Theorem.