Math 830-831, Qualifying Exam, June, 2003

1. Apply the LaSalle Invariance theorem to the system

$$\begin{aligned} x' &= -x + 6y^5, \\ y' &= -x. \end{aligned}$$

2. Use Putzer's algorithm to find e^{At}, where

$$A = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}.$$

   Then use the variation of constants formula to solve the IVP

$$x' = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix} x + \begin{bmatrix} e^{2t} \\ 2e^{2t} \end{bmatrix}, \quad x(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

3. Using an appropriate Green's function, solve the BVP

$$x'' + x = t, \quad x(0) = 0, \quad x(\frac{\pi}{2}) = 0.$$

4. Show that the system

$$\begin{aligned} x' &= x + y - x(x^2 + 2y^2), \\ y' &= -x + y - y(x^2 + 2y^2) \end{aligned}$$

   has a limit cycle.

5. Assume μ_0 is a Floquet multiplier for the Floquet system $x' = A(t)x$, where ω is the smallest positive period of the matrix function $A(t)$.

   (a) Show that there is a nontrivial solution $x_0(t)$ satisfying

$$x_0(t + \omega) = \mu_0 x_0(t), \quad t \in \mathbb{R}.$$

   (b) Show that there is a nontrivial solution of the form

$$x(t) = p(t)e^{\alpha t},$$

   where p is a continuously differentiable function on $\mathbb{R}$ which is periodic with period ω and α is a constant.

   (c) Show that if $\mu_0 = -1$ there is a nontrivial solution of the Floquet system which is periodic with period 2ω.