Qualifying Exam, Math 830-831 **June, 2008.**

1. (a) Use Putzer's algorithm to find e^{At} given that $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 12 & -16 & 7 \end{bmatrix}$.

   (b) Use your answer in (a) to solve the initial value problem

$$x' = Ax + \begin{bmatrix} 0 \\ e^{2t} \\ 0 \end{bmatrix}, \quad x(0) = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}.$$

2. Apply the LaSalle Invariance Theorem to the system

$$\begin{aligned} x' &= xy^2 - y \\ y' &= x^3 - 4x^2 y. \end{aligned}$$

3. Show that the Green's function for the BVP

$$x'' = 0, \quad x(0) + x'(0) = 0, \quad x(1) + x'(1) = 0$$

   exists and then find $G(t, s)$. Use your answer to solve the BVP

$$x'' = t^3, \quad x(0) + x'(0) = 1, \quad x(1) + x'(1) = 2$$

4. (a) Show that

$$Q[x] = \int_1^{e^2} \{t(x'(t))^2 - \frac{1}{t} x^2(t)\}\, dt$$

   subject to $x(1) = 1$, $x(e^2) = 1$ has a proper global minimum at some $x_0(t)$ and then find $x_0(t)$.

   (b) Show that the problem

$$I[x] = \int_0^{2\pi} \left[e^t x^2(t) - 3t^2 x(t) x'(t) - (\cos t)(x'(t))^2\right]\, dt$$

   subject to $x(0) = 2$, $x(2\pi) = 1$, has no local extremums.

5. Given that λ_0 is an eigenvalue of the constant matrix A. Find an eigenvalue (i) for the transpose A^T, (ii) for A^n (n a positive integer) and (iii) for the inverse A^{-1} (assuming $\det A \neq 0$). Be sure to verify your answers.

6. (a) Show that if $x(t)$ is a nontrivial solution of the Floquet system $x' = A(t)x$, where $A(t)$ has minimum positive period of $A(t)$ is ω and if $\mu = -1$ is a Floquet multiplier, then there is a nontrivial solution of the Floquet system with period 2ω.

   (b) Give a formula for the matrix norm induced by the traffic norm (l_1) norm. Verify this formula.

7. State several results concerning the Sturm–Liouville problem

$$(p(t)x')' + q(t)x = \lambda r(t)x, \quad \alpha x(a) - \beta x'(a) = 0, \quad \gamma x(b) + \delta x'(b) = 0.$$

   Prove one of your stated results.