Math 830-831, Qualifying Exam, June, 2014 Work all of the following problems:

1. Assume a and f are C^1 functions on the set of real numbers. Find an implicit representation of the solution of the IVP
$$a(z)z_x + z_y = 0, \quad z(x,0) = f(x).$$
Then use implicit differentiation to find formulas for z_x and z_y. Finally, use your answers to show that for $|y|$ sufficiently small a shock developes.

2. Show that the vector field $\vec{V}(x,y,z) =< z-y, x-z, y-x >$ is not tangent to any point on the curve
$$C: \quad x = t, \quad y = 2t, \quad z = 3t, \quad t > 0.$$
Then find the integral surface of vector field $\vec{V}$ containing the curve C.

3. Use the Putzer algorithm to help you solve the IVP
$$x' = \begin{bmatrix} -1 & 6 \\ -1 & 4 \end{bmatrix} x + \begin{bmatrix} 2e^t \\ -e^t \end{bmatrix}, \quad x(0) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

4. Verify that the assumptions of the Cauchy-Kovalevsky Theorem for the IVP
$$u_t = \cos u_x, \quad u(0,x) = \frac{\pi}{4}x + \frac{\pi}{6}x^2$$
about the origin are satisfied and solve this IVP explicitely finding all terms of degree three and less.

5. Draw the phase plane diagram for the differential equation $x'' = 2x^3 - x$ and find the equation of the separatrix.

6. Work each of the following:
   (a) Classify the PDE
   $$u_{xx} - 9u_{yy} = 16x^2y^2.$$
   Then solve this PDE by first solving a corresponding canonical form.
   (b) Classify the PDE
   $$u_{xx} - 6u_{xy} + 9u_{yy} = 9xy.$$
   Then solve this PDE by first solving a corresponding canonical form.

7. Draw the phase plane diagram for the system
$$x' = -y + x(x^2+y^2)\sin\left(\frac{\pi}{\sqrt{x^2+y^2}}\right)$$
$$y' = x + y(x^2+y^2)\sin\left(\frac{\pi}{\sqrt{x^2+y^2}}\right).$$
The right hand sides of these two equations are understood to be zero when $(x,y) = (0,0)$. Is the origin unstable, stable or asymptotically stable?

8. State and prove Floquet's Theorem.