Department of Mathematics

Graduate Advisory Committee
May 2020

Differential Equations Qualifier Exam Syllabus

ODEs Topics:

(1) Mathematical modeling. Dimensional analysis, scaling, physical derivations. (~1 week)
(2) Contraction Mapping Theorem, Implicit Function Theorem, and Existence and Uniqueness Theorem for ODEs. (~1 week)
(3) Linear Systems of ODEs, phase-plane method for 1 and 2 dimensional systems. (~2 week)
(4) Lyapunov Theorem of Stability for nonlinear systems. Hartman-Grobman Theorem for hyperbolic fixed points. (~2 week)
(5) Variation of Parameters Formula, Stable Manifold Theorem, Center-Manifold Theorem (statement only). (~2 week)
(6) Elementary Bifurcations for ODEs: saddle node bifurcation, Hopf bifurcation. (~2 week)

References:

[1] (Main textbook) Carmen Chicone. Ordinary Differential Equations with Applications, 2nd edition, 2011. (The 1999 edition is available online at UNL library.)
[2] Jack K. Hale. Ordinary Differential Equations, 2009.
[3] J. David Logan. Applied Mathematics, 4th edition, 2013. (The 1997 edition is available online at UNL library.)

PDEs Topics:

(1) Introduction, Classification (~1 week)
(2) Transport equation and wave equation, Characteristics, Quasilinear equations, Energy methods, d'Alembert's solution (~4 weeks)
(3) Heat equation, Fundamental solution (~1 week)
(4) Fourier series, Series solution of PDEs (~3 weeks)
(5) Maximum principles for Laplace equation, Mean value formula (~1 week)

Reference:

[1] (Main textbook) E. C. Zachmanoglou, Dale W. Thoe. Introduction to Partial Differential Equations with Applications, Dover Publications, 1987.
[2] D. Borthwick. Introduction to Partial Differential Equations, Springer, 2016. (available online at UNL library.)

Numerical Topics

(1) Numerical methods for ODEs and numerical integration. Includes: Euler method, Stability and consistency for Euler, Runge-Kutta method, Dahlquist Equivalence Theorem, Numerical quadrature (~3 weeks)
(2) Finite element method. Includes: Weak derivatives and weak solutions, Finite element approximation, Finite element solution of the Poisson equation, Error estimates, Finite element solution of the heat equation (~5 weeks)
(3) Finite difference method for convection. Includes: Semi-discrete schemes, Upwinding schemes, CFL condition, Solvers for the wave equation (~2 weeks)

Reference:

[1] (Main textbook) Arieh Iserles. A first course in the numerical analysis of differential equations, Cambridge University Press, 2009.
[2] Michael T. Heath. Scientific Computing: An Introductory Survey, Revised 2nd Edition, SIAM, 2018.
[3] C. Grossmann, H.-G. Roos, M. Stynes. Numerical Treatment of Partial Differential Equations. Springer, 2007. (available online at UNL library.)