June 2005 **UNL Mathematics Qualifying Exam** **Math 970/971**

Do three of the problems from section A and three questions from section B. If you work more than the required number of problems, make sure that you clearly mark which problems you want to have counted. If you have doubts about the wording of a problem or about what results may be assumed without proof, please ask for clarification. In no case should you interpret a problem in such a way that it becomes trivial.

Section A: Point Set Topology.

1. Let A and B be subsets of a topological space X. Suppose that p is a limit point of the subset $A \cup B$. Show that p is also a limit point of A or B.

2. Let X be the set of integers, let $\mathcal{C}_1 := \{A \subseteq X \mid X - A \text{ is finite }\}$, and let $\mathcal{C}_2 := \{A \subset X \mid 0 \notin A\}$. Show that the union $\mathcal{T} := \mathcal{C}_1 \cup \mathcal{C}_2$ is a topology on X, and show that this topological space is compact.

3. Let X be a connected space with connected subset Y, and suppose that $X \setminus Y$ is not connected, with $X \setminus Y = A \cup B$ a separation of $X \setminus Y$. Show that if B is open in X, then $A \cup Y$ is a connected subset of X.

4. A topological space X is called *metacompact* if for every open cover $\mathcal{C}$ of X, there is a subcover $\mathcal{C}'$ satisfying the property that for every point $p \in X$, there are only finitely many open sets in $\mathcal{C}'$ containing p.
a. Show that metacompactness is a homeomorphism invariant.
b. Let X be the integers with the topology $\mathcal{T} := \{U \subseteq X \mid 0 \in U\} \cup \{\emptyset\}$. Show that this space is not metacompact.

Section B: Homotopy and Homology.

5. Find a cell structure for the quotient space obtained by identifying two distinct points a, b in a 2-torus to a third point c in a 2-sphere, and compute a presentation for the fundamental group of this space.

6. Show, using covering spaces, that the fundamental group of the Klein bottle is not abelian.

7. Find a Δ-complex structure on the space X obtained by identifying three distinct points a, b, c in the 2-sphere to a point, and compute the simplicial homology groups of X.

8. A space X is called *unicoherent* if whenever $X = \mathcal{U} \cup \mathcal{V}$ with $\mathcal{U}, \mathcal{V}$ nonempty, open, and path-connected, then $\mathcal{U} \cap \mathcal{V}$ is path-connected. Use the Mayer-Vietoris sequence to show that, for $n \geq 2$, S^n is unicoherent.