

Department of Mathematics

Graduate Advisory Committee
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Topology Qualifying Exam Syllabus

The Topology Qualifying Examination covers the fundamentals of point-set and algebraic topology, along with standard applications. The main topics are listed below, together with a few references.

Point-set Topology

Topological spaces and continuous functions: Topology, open and closed sets, basis, sub-basis; continuous function, homeomorphism; closure, limit points; subspace topology, product topology, and quotient/identification topology.

Homeomorphism invariants: Separation properties (T_0 , T_1 , Hausdorff, regular, normal), countability properties; connectedness, path connectedness, components; compactness, metrizability. Applications.

Continuous deformations: Retraction, deformation retraction, contractible, mapping cylinder, homotopic maps, homotopy type.

Algebraic topology

Fundamental groups: Fundamental group, induced homomorphism; free group, group presentation, Tietze's theorem, amalgamated product of groups, Seifert - van Kampen Theorem; cell complex, presentation complex, Classification of surfaces.

Covering spaces: Covering map, Lifting theorems; covering space group action; universal covering, Cayley complex; Galois Correspondence Theorem, deck transformation, normal covering; applications to group theory.

Homology: Simplicial homology, singular homology, induced homomorphism, homotopy invariance; exact sequence, long exact homology sequence, Mayer-Vietoris Theorem. Applications.

References

- J.R. Munkres, Topology (Second Edition), Prentice-Hall.
- A. Hatcher, Algebraic Topology, Cambridge.
- W.S. Massey, A Basic Course in Algebraic Topology, Springer.