Do three questions from Section A and three questions from Section B. You may work on as many questions as you wish, but indicate which ones you want graded. When in doubt about the wording of a problem or what results may be assumed without proof, ask for clarification. Do not interpret a problem in such a way that it becomes trivial.

Section A: Do THREE problems from this section.

(1) Given any topological space Z and subset $D \subseteq Z$, let $Cl_Z(D)$ denote the closure of D in Z. Show that if X and Y are topological spaces and $A \subseteq X$, $B \subseteq Y$, then $Cl_{X \times Y}(A \times B) = Cl_X(A) \times Cl_Y(B)$.

(2) Let X be a connected space and $A, B \subseteq X$ be closed subsets of X with $X = A \cup B$ and $A \cap B$ a connected subset of X. Show that both A and B are connected.

(3) Let X be the set of real numbers, let $\mathcal{T}_E$ be the Euclidean topology on X, and let $\mathcal{T}_0$ be the excluded point topology (that is, $\mathcal{T}_0 = \{U \subseteq X \mid 0 \notin U\} \cup \{X\}$). For each of the following topological spaces, determine whether or not the space is compact.

(3a) The set X with the topology $\mathcal{T}_E \cap \mathcal{T}_0$.

(3b) The set X with the topology generated by the subbasis $\mathcal{T}_E \cup \mathcal{T}_0$.

(4) Suppose that the space X has the fixed point property (that is, for any continuous function $f : X \to X$ there is a point $p \in X$ with $f(p) = p$). Suppose also that $A \subseteq X$ is a subspace admitting a retraction $r : X \to A$. Show that A also has the fixed point property.

Section B: Do THREE problems from this section.

(5) Let $X = S^1 \times S^1$, also thought of as the standard quotient of the unit square $[0, 1] \times [0, 1]$, and let $A = \{(x, x) \ : \ x \in S^1\}$ be the diagonal of X. Show that A is a retract of X, but not a deformation retract of X.

(6) A group G is called *residually finite* if for every $g \in G$ with $g \neq 1$, there is a finite group H and a (surjective) homomorphism $\varphi : G \to H$ with $\varphi(g) \neq 1$. Let G be a residually finite group and let X be the presentation complex for a presentation of G, with vertex x_0. Show that for any loop $\gamma : I \to X$ at x_0 with $1 \neq [\gamma] \in \pi_1(X, x_0)$, there is a finite-sheeted covering space $p : \widetilde{X} \to X$ and basepoint $\widetilde{x_0} \in p^{-1}(\{x_0\})$ such that γ does <u>not</u> lift to a loop at $\widetilde{x_0}$.

(7) Let $p : \widetilde{X} \to X$ and $q : \widetilde{Y} \to Y$ be covering spaces of path-connected, locally path-connected spaces X and Y with $\widetilde{X}$ and $\widetilde{Y}$ locally path-connected and simply-connected. Show that if X and Y are homeomorphic, then $\widetilde{X}$ and $\widetilde{Y}$ are homeomorphic.

(8) Construct a Δ-complex structure, and use it to compute the simplicial homology groups, for the connected sum of two projective planes.