
Course Materials, 2011

MATH IN THE CITY



UNIVERSITY OF NEBRASKA-LINCOLN

FALL 2011

Ananthnarayan Hariharan

GTA: Lauren Keough

Copyright 2011. Permission is given to use these materials only for educational purposes. The authors acknowledge NSF support through the award DUE-0941132.

Math in the City - Fall 2011

Class Journal

December 1, 2011

Summary of Assignments

Assignment Description	Assigned	Due
Journal	Weekly	Every Monday throughout the semester
Homework 1 - Individual Topics	Week 1	First draft - Week 3, Final draft - Week 4
Homework 2 - Estimation/Math Models	Week 4	Week 5
Group Memo 1 - Estimation/Project Questions	Week 4	Week 5
Group Memo 2 - Sage Programming	Week 6	Week 7
Homework 3 - Linear Programming	Week 7	Week 9
Group Memo 3 - Project Proposal	Week 7	Week 8
Group Memo 4 - Project Title and Introduction	Week 10	Week 11
Group Memo 5 - Methodology	Week 12	Week 13
Rough Draft	Week 13	Week 14

Tools Used

- Sage - An open source mathematics software. Students used Sage, in particular, to do linear programming problems.
- L_AT_EX - A front end for T_EX. Students completed all assignments and class journals to prepare them for writing their final reports in L_AT_EX.
- Dropbox - A web based file hosting service. Instructors used dropbox to share homework assignments and data. Students used dropbox to share data with the class and rough drafts with their group among other things. Saved a lot of emails!
- Blackboard - Used the SafeAssign feature to collect student journals. This way only the instructors could read the student journals.
- Doodle - A free service that allowed us to conduct a poll of student availability to schedule final presentations.

Week by Week Summary

Week 1

- Students created Sage accounts and installed Lyx.
- Groups were formed based on the topic (Budgeting, Investing & Investing ++).
- Students were introduced to the class journal which is to be done in Lyx. Journals are due every Monday by class time. Journals are submitted as PDF's on Blackboard.
- Assignment 1 was assigned. Each student is to become an expert in an assigned topic and write a paper on it. The topics are:
 - Treasury
 - Notes & Bills, Corporate Papers
 - Sales Tax
 - Property Tax
 - Police and Fireman Pension
 - City Revenue
 - City Expenses
 - FFCB, FHLB, FMac, FMae
 - CD's, Bonds, & Money Market Accounts
 - Income Tax
 - Stocks and Bonds
- Talked about making intelligent guesses. Estimated the number of plumbers in the U.S.
- Guest speaker - Melinda Jones, the City Treasurer gave a presentation on the city investments and policies.

Week 2

- Used a Doodle survey to find times for groups to present at the conference.
- Guest speaker - Sherry Wolf, from the budgeting department gave a presentation on the city's budget.
- Students were introduced to Dropbox where the data is stored and where students can share files with their group members.
- Discussed the exponential growth model.

Week 3 (9/5-9/9)

- Assignment 1 is turned in. Instructors edited the assignment and the next draft is due the next week. Notes on the first assignment to the students:
 - Assignment should be readable for people that have no idea about your topic.
 - Graphs and tables are helpful
 - Should be organized into sections: Overview, Specifics of applications to the city (multiple sections), Conclusion, References
 - Should be grammatically correct.
 - The resources should be credible and relevant.
- Covered more estimation examples.

Week 4 (9/12-9/16)

- Collected final drafts of the first homework. The homework is put in the dropbox for the students to share.
- Second homework is assigned. This homework is on guesstimates/the natural growth model/writing about a mathematical model. We tried to justify the importance of each problem to the goals of the class. Students have 9 days to complete this homework assignment.
- Began to discuss linear programming (a tool for students to use with their projects).

Week 5 (9/19-9/23)

- Second homework assignment is collected.
- Continued to discuss linear programming in depth.
- Students turn in the first group memo that estimates the lifetime expenditure of the average United States citizen (began to work together as a group). Also brainstorm possible questions they would like to answer throughout the semester.

Week 6 (9/26-9/30)

- Students met in the computer lab to work with Sage twice this week.
 - First day a short introduction to Sage. Students are working in their groups to complete a worksheet that asks a couple of beginning programming problems and one linear programming problem.

Week 7 (10/3-10/7)

- Students turn in the second group memo which is the solutions to the problems from Sage that were assigned last week.
- Assigned a homework assignment on Linear Programming. A couple of the problems involve using Sage to verify answers.
- Students begin working on their projects in greater depth. Finalizing questions that they would like to answer.

Week 8 (10/10-10/14)

- Students put together the third group memo - a project proposal that includes:
 - Main question.
 - A fall back option.
 - Strategies for both options with details.
- Students do a (surprise!) presentation on their progress so far.

Week 9 (10/16-10/21)

- HW3 on Linear Programming is due this week.
- Students are encouraged to collect all the data they need by the weekend.

Week 10 (10/24-10/28)

- Students went on a field trip and met with collaborators to discuss their progress and ask questions.
- Guest speakers came to give students advice on writing their report.
- One group each available day presents their progress over the past week.

Week 11 (10/31-11/4)

- Students turn in a memo with their title and introduction.
- One group per day presents their progress over the past week.

Week 12 (11/7-11/11)

- Students meet outside of class with the instructor to discuss the mathematics in their projects.
- Students present to the class the mathematics in their projects.
- Journal submissions have started to drop off, but journals are still being collected and read by the instructors.
- Groups are meeting very frequently outside of class to work on their projects at this point.

Week 13 (11/14-11/18)

- Groups are still working on getting results for their projects. There is some frustration, but they are pushing through.
- Students turn in a memo that should serve as a rough draft for the math in their project.
- An outline (with lengths) was given for their reports as follows:
 - Introduction (4-6 pages)
 - Math (4-6 pages)
 - Data/Assumptions/Analysis
 - Big Picture Conclusions
 - Further Work
 - References
 - Appendix (Sage code, etc.)
- A first draft of the report is expected by the following week so the instructors can edit it.

Week 14 (11/21-11/25) (Thanksgiving)

- First draft is collected and returned to them with comments. A lot of comments.
- These are some general comments sent to all students:
 - Make sure you use the articles (a, an, the), and use them correctly.
 - When using a formula, make sure all the terms are explained the first time, even if they seem fairly common like x^T (transpose) or partial derivatives. If you are using text in a formula, make sure they

are not in italics (there is a "text" style that can be used in a formula; use `\text` and use `\spacebar` to insert a space). – Except for the "future work" section, use "have done" instead of "will do". The report is primarily to show what you have already done. – Unless you can justify, avoid using superlatives like "best". Also, if you cannot justify something, use phrases like "reasonable to assume/suppose" and "supports" instead of "we know" and "confirms". – a good introduction offers the reader the context of the mathematical problem as well as the motivation. – the mathematical background should be presented in a self-contained manner – explain clearly how you obtained your results, how you pursued the mathematical analysis, and explain the assumptions incorporated into the model. – conclusions should be based on the model and recommendations based on that. – The names of the group members in the report should be ordered alphabetically by last name.

Math in the City

Homework 1 - Property Tax

The projects this semester will revolve around budgeting and investing at the city level. You are to research and write about property tax. Your essay (of about 3-4 pages) should contain information such as, but is not limited to:

1. An explanation of how property tax works. How is property tax for an individual property determined? In particular, what does “17 cents per \$100 in valuation” mean?
2. How much is the property tax in Lincoln currently? Give a couple of examples of what the property tax would be on different property values.
3. How has property tax changed in the past 5 years? How does the property tax rate in Lincoln compare to the property tax rate of other cities in Nebraska? What about other states?
4. Make an educated estimate of the income the city of Lincoln receives from property tax. What data did you use to draw this conclusion? How would you expect property tax revenue to change throughout the year? What actual data can you find on the revenue Lincoln receives from property tax?
5. In the United States, what programs does property tax usually support? What programs does the property tax in Lincoln support?

Math in the City Homework 1 - Sales Tax

The projects this semester will revolve around budgeting and investing at the city level. You are to research and write about sales tax. Your essay (of about 3-4 pages) should contain information such as, but is not limited to:

1. An explanation of how sales tax works. What items are eligible for sales tax in Lincoln? Are there any items that are not?
2. What is the sales tax rate for the city of Lincoln? How does the "haymarket tax" work? Is there also a county sales tax imposed on the city of Lincoln? What about a state sales tax? Give a couple of examples of sales tax on various items (being specific about what tax goes where).
3. How has sales tax changed in the past 5 years? How does the sales tax rate in Lincoln compare to the sales tax rate of other cities in Nebraska? What about other states?
4. Make an educated estimate of the income the city of Lincoln receives from sales tax. What data did you use to draw this conclusion? What actual data can you find for income from sales tax?
5. In the United States, what programs does sales tax usually support? What programs does the sales tax in Lincoln support?

Math in the City

Homework 1 - Income Tax

The projects this semester will revolve around budgeting and investing at the city level. You are to research and write about income tax. Your essay (of about 3-4 pages) should contain information such as, but is not limited to:

1. An explanation of how income tax works.
2. What is the income tax rate for the city of Lincoln? Give a couple of examples of what the income tax would be for different incomes.
3. How has income tax changed in the past 5 years? How does the income tax rate in Lincoln compare to the income tax rate of other cities in Nebraska? What about other states?
4. Make an educated estimate of the revenue the city of Lincoln receives from income tax. What data did you use to draw this conclusion? What actual data can you find on the revenue from income tax?
5. In the United States, what programs does income tax usually support? What programs does the income tax in Lincoln support?

Math in the City

Homework 1 - City Revenue

The projects this semester will revolve around budgeting and investing at the city level. You are to research and write about the revenue the city receives. Your essay (of about 3-4 pages) should contain information such as, but is not limited to:

1. Name the various avenues through which the city receives money. Give a brief description of each. How much did the city receive from each over the past year?
2. Of the various revenue sources, which have increased the most over the past five years? What do you think is the cause of these changes?
3. Has any source decreased (or made an insignificant increase) over the past five years? What do you think the cause of this is?
4. What do the various sources of revenue fund?

Math in the City

Homework 1 - City Expenses

The projects this semester will revolve around budgeting and investing at the city level. You are to research and write about the expenses the city has. Your essay (of about 3-4 pages) should contain information such as, but is not limited to:

1. Name the various expenses the city has to pay over a given year. Give a brief explanation of each. Last year, how much money did each expense cost?
2. Of the various expenses, which have increased the most over the past five years?
3. Has any expense decreased over the past five years? How do you think this affected the city?
4. What are the various expenses funded by?
5. What debts does the city of Lincoln currently have? When are they expected to be paid off by? Explain the concept of a debt schedule.

Math in the City

Homework 1 - FFCB, FHLB, Fannie Mae, Freddie Mac

The projects this semester will revolve around budgeting and investing at the city level. You are to research and write about some of the investment opportunities the city has. Your essay (of about 3-4 pages) should contain information such as, but is not limited to:

1. Explain what the Federal Farm Credit Banks Funding Corporation is. What are the Federal Home Loan Banks? What kind of interest earning programs do they offer to the city?
2. Explain what Fannie Mae does. What options does the city have as an investor in Fannie Mae?
3. Explain the differences and similarities between these investment opportunities.
4. How much is the city allowed to invest in each of these? Why is there a limit? How much does the city invest in these?
5. Include specific examples of each type of investment (i.e., find typical numbers and figure how much the city earns and when it earns).
6. What types of investment vehicles is the city not allowed to invest in?

Math in the City

Homework 1 - Certificates of Deposit, Bonds, and Money Market Accounts

The projects this semester will revolve around budgeting and investing at the city level. You are to research and write about some of the investment opportunities the city has. Your essay (of about 3-4 pages) should contain information such as, but is not limited to:

1. Explain how a Certificate of Deposit works.
2. How does a bond work?
3. What are the differences between CD's and bonds?
4. How much is the city allowed to invest in CD's? How much in bonds?
5. What is a money market account? What are the benefits and risks associated with such an account? Typically, when does the city use money market accounts?
6. Include specific examples of each type of investment (i.e., find typical numbers and figure how much the city earns and when it earns).
7. What types of investment vehicles is the city not allowed to invest in?

Math in the City

Homework 1 - Treasury Bills and Notes, and Corporate Papers

The projects this semester will revolve around budgeting and investing at the city level. You are to research and write about some of the investment opportunities the city has. Your essay (of about 3-4 pages) should contain information such as, but is not limited to:

1. Explain how treasury bills work. What are the benefits and risks associated with treasury bills? How much is the city allowed to invest in treasury bills? Does the city currently use treasury bills?
2. How do treasury notes work? What are the differences between treasury notes and treasury bonds? How much is the city allowed to invest in treasury notes? Does the city currently use treasury notes?
3. What are corporate papers? How do they differ from treasury bills and notes? How much is the city allowed to invest in corporate papers? Does the city currently use corporate papers? Why do you think this is? What are the benefits and risks of investing in corporate papers?
4. Give specific examples of a typical treasury bill, a typical treasury note, and corporate papers (e.g., give a dollar amount, length of investment, rate, how much it earns, and when it earns).
5. What types of investment vehicles is the city not allowed to invest in? Why do you think it is limited? Who regulates these rules?

Math in the City

Homework 1 - Police and Firemen Pension

The projects this semester will revolve around budgeting and investing at the city level. You are to research and write about some of the investment opportunities the city has. Your essay (of about 3-4 pages) should contain information such as, but is not limited to:

1. What are the different plans for Police and Firemen Pension? What are their benefits? What is the eligibility for each plan?
2. Who is responsible for managing and investing these funds? Who constitute the investment committee?
3. What types of investment vehicles is the city allowed/not allowed to invest the Police and Firemen Pension funds in?
4. Compare the advantages and disadvantages of the Police and Firemen Pension with 401(K).

Math in the City

Homework 1 - Stocks and Mutual Funds

The projects this semester will revolve around budgeting and investing at the city level. You are to research and write about some of the investment opportunities the city has. Your essay (of about 3-4 pages) should contain information such as, but is not limited to:

1. What exactly are stocks? Who determines which stocks are better to invest in than others?
2. Why are they considered riskier as compared to other types of investments?
3. What are Mutual Funds?
4. How do Mutual Funds differ from Stocks?
5. Is the city permitted to invest in Stocks and/or Mutual Funds? If yes, give specific examples of these types of investments in which the city has invested. If no, why not? Are there certain branches of the city government (e.g., the Police and Firemen Pension Fund) who can invest in stocks and mutual funds? Give examples.

Homework 2

Due Friday, September 23

You may collaborate ONLY on problems 1 and 3b); if you decide to collaborate, you need to write your own solutions and acknowledge the collaboration in your work.

Your solutions should use proper English and complete statements. Make sure your notation and assumptions are stated clearly at the beginning of each problem. **Please read your solutions entirely before you turn them in.**

- Guesstimates.** (20 points) Being able to make reasonable estimates for data you can not find will be an important skill in this class. To practice, provide an order of magnitude in the following problems. Clearly write your **assumptions** and **arguments**.
 - How many dentists are in Las Vegas?
 - How many pairs of shoes can be made from a cow? (consider a spherical cow and a spherical shoe.)
- Natural Growth Models** (30 points) One model that may be important to your project is the natural growth model.
 - Use the natural growth model to predict the US population if the growth rate is assumed to be the same as the growth rate of a) 1980; b) 1990; c) 2000.
 - Assume an investment grows according to the natural growth model at a rate of 20% a year. How long will it take to double its value? Explain why this time does not depend on the amount of money initially invested in the account. Next assume that at the beginning of every year, starting with the second year, \$100 is taken out of the account. Under the new circumstances how long will it take for the investment to double its value? Does your answer depend now on the initial investment? Why or why not?
- Mathematical Modeling.** (30 points) There are many mathematical models that have been developed to fit various situations. Here, to gain more experience with another model, write an essay about a mathematical model (one you may have encountered in a Calculus, Differential Equations, Linear Programming, Probability, or other mathematics course). In addition, communicating quantitative information is an important aspect of this class. Your write up should contain the following:
 - general description of the model, where and how it is used.
 - mathematical set up; all mathematical quantities, notation, range of parameters are specified.
 - the equations describing the state/phenomena.
 - mathematical conclusions for the model (if it can be solved, include the solution); results, possible graphs.
 - physical interpretation of the results.
 - a concrete example (with numbers) that shows the model being applied.

Math 435: Math in the City
October 5th, 2011.

Homework 3

Due Wednesday 19th Oct

This is an individual assignment. You may only consult the instructor or the TA with any questions that you may have. In order to receive full credit, clearly present and motivate all the steps in your solutions.

1. (20 points) Consider the problem:

$$\text{maximize: } f = x_1 - x_2$$

$$\text{subject to: } \begin{cases} -x_1 + 2x_2 \leq 3 \\ x_1 + x_2 \geq 1 \\ x_2 - x_1 \leq 1 \\ x_1 - 3x_2 \leq 1 \\ x_1, x_2 \geq 0. \end{cases}$$

Show the feasible region and sketch the level sets in order to maximize f .

2. (20 points) A hospital patient is required to have at least 90 units of drug I and 120 units of drug II. The drugs are both contained in two substances S_1 and S_2 . Suppose a gram of S_1 contains 6 units of drug I and 4 units of drug II, and a gram of S_2 contains 3 units of drug I and 3 units of drug II. But in addition, each gram of S_1 contains 2 units of a mildly toxic drug and each gram of S_2 contains 1 unit of this other undesirable drug. How much of each substance should be given to the patient to achieve the medication requirements with minimal dosage of the toxin? How much of the toxin does the patient receive with this optimal mixture? Verify your answers using Sage.
3. (20 points) Solve the problem below. Show that the feasible region is unbounded.

$$\text{maximize: } f = -2x_1 + x_2 + x_3$$

$$\text{subject to: } \begin{cases} -x_1 + x_2 + x_3 \leq 2 \\ x_1 - x_2 + x_3 \leq 2 \\ x_1, x_2 \geq 0. \end{cases}$$

4. (40 points) *Linear best fit to data.*

Consider a set of n data points $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ that are the observed outcomes of an experiment. If we think that y_i is linearly related to x_i , we would like to find the *linear best fit*; that is, the constants m and b such that $y = mx + b$ is the equation of the line that best fits that data.

- (a) How do we measure the “fit” of the data? One way is to use the ℓ_1 -norm, which is the sum of the absolute values of the differences between the observed value y_i and the predicted value $mx_i + b$:

$$\sum_{i=1}^n |(mx_i + b) - y_i|.$$

We wish to minimize this function over all values of $m, b \in \mathbb{R}$. Formulate an equivalent linear program that solves this minimization problem. (*Hint.* Introduce a new variable to eliminate each absolute value.)

- (b) Use Sage to find the linear best fit under the ℓ_1 -norm for the data: (5.38, 8.47), (0.53, -0.17), (8.8, 13.05), (7.77, 14.14), (0.55, 2.63), (6.14, 7.43), (10.0, 14.5), (2.4, 4.92), (9.03, 10.41), (6.84, 9.85), (1.23, 3.84), (0.17, 3.61), (3.68, 3.69), (8.27, 12.15), (3.02, 7.34), (4.85, 3.63), (6.26, 5.39), (9.94, 16.37), (3.03, 3.78), (5.41, 8.41). Plot the points and the line on the same graph in Sage.

Sage Worksheet

Math in the City: Fall 2011

September 28, 2011

1. Write a Sage function that takes a matrix and computes its determinant. If the matrix is invertible, find its inverse.
2. Write a Sage function that takes a sequence of n numbers and finds the i th smallest integer.
3. Consider the following flow diagram where I is a source with infinite capacity, O is a sink and the arrows are the pipelines with the numbers representing their capacities. Use Sage to determine the maximum amount of flow through the diagram assuming that the inflow is equal to the outflow at each intermediary joint.