# Math in the City
# The Nuts and Bolts of a Hands-on Learning Experience

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# Early History of Math in the City

Spring 2005:

- taught 3 sections of Differential Equations: students (even the good ones) have trouble putting word problems into equations
- Department is looking for ideas to increase interaction with the local businesses; propose idea of creating a course that would have students work with data provided by businesses; no volunteers for instructors

Fall 2005:

- need ideas for educational component for grant proposal; volunteer myself for running the course, the Department commits to run the course even if the proposal does not get funded
- ITLE (Initiative for Teaching and Learning Excellence) internal grant proposal funded
- prepare the first pilot offering of Math in the City

# Early History of Math in the City (cont.)

Spring 2006

- first pilot offering the course runs
- components of the course: project and “standard” lectures, homework, quizzes, final exam
- great performance by the students: students do very well despite previous less positive experience in other math courses, excellent results on the projects, very good final presentations
- first employment success: student gets job after completing Math in the City
- instructor gets hooked on the course

# Course Timeline

**Before classes start:**

- talk to people in the Department about offering the course (best time to offer it to increase enrollment)
- think about suitable projects and collaborators
- start recruiting students: put posters up, send emails to students and student advisors, announce course to classes
- contact and meet with collaborators
- see sample of the data and sketch a road map for the project
- prepare a set of lecture notes that contain the necessary material for the project
- continue to recruit students

# Project Timeline

**After classes start:**

Project Phase I (weeks 1–5)

- collect data provided by local businesses or reseach centers at the beginning of the semester; students understand data and extract data necessary to populate models
- students gather additional data from other sources
- students research and write about a topic related to the project; each student becomes an expert on one topic relevant to the course
- discussion of mathematical modeling (introductory concepts that allow students to think outside the box) and background material that will be necessary for the work on the project (presented at the level that will be needed)

## Project Phase II (weeks 6–12)

- the mathematical content has already been covered, students are ready to use the tools on the data provided
- the work in groups begins; whole class discussions are limited only to topics that concern all groups
- mathematical models created: set up of variables, equations, find restrictions for the variables and the parameters; discuss existence of solutions
- models implemented as computer programs
- Repeat process:

  model solved $\Rightarrow$ results interpreted $\Rightarrow$ refine model

  until satisfactory result achieved
- preliminary results are obtained; students may give poster presentations
- students start writing their results

## Project Conclusion (weeks 13–16)

- students reconsider the assumptions, create more realistic model
- during this final period there is a lot of brainstorming going on: students fully understand the problem and try different ways to come up with an optimal solution; they need guidance to channel their ideas, otherwise they start feeling overwhelmed by the scope of the problem
- compile results, give conclusions, and offer recommendations
- write final report as an iterative process of students sending drafts and instructors providing comments.
- prepare slides, give mock presentations in front of the classmates; give final hour-long presentation for each group in front of a large audience

# Ingredients for Projects: Course Material

- adjusted each semester to better fit the projects
- different topics, but the material is self contained
- sets of 8–10 self-contained lectures in Mathematical Modeling, Statistics, Differential Equations, Linear Algebra, Linear and Integer Programming, Traffic Models, Routing Methods, and Math Programming
- no textbook, but students are encouraged to read the references given to them, relevant to the topic (information on the web, information provided by the collaborator, handouts and textbooks for mathematical content)
- Computer programming: Excel, SPSS, Maple, Sage

## **Spring 2006** Tell-tale of Heart Attacks

- in collaboration with Jane Meza from Univ of Nebraska Medical Center
- used data collected by UCLA which found the factors that predict heart attacks for a group of patients
- statistical variables (age, smoking, history of heart attacks, heart irregularities) have undergone a basic analysis and logistic linear regression for **all** patients
- same results obtained as in the UCLA study for the **entire** group, but with different, simpler methods
- students' statistical analysis found that men and women have different variables that predict their risk of future heart events; the heart predictors for each gender are different from heart predictors from the entire group (UCLA study found the heart predictors for entire group)

**Spring 2006** Lake McConaughy, is your water gonna stay?

- with Shuhai Zheng from Dept of Natural Resources
- 7-year drought in Nebraska; the lake is very important to agriculture (irrigation), entertainment, wildlife, hydroelectric power
- data recorded water levels over the last 50 years, evaporation rates, precipitation
- students set up a differential equations model for the water levels and performed numerical computations based on Euler's method
- small error for water volumes $< 5 - 10\%$ (very good) for September - June

# Lake McConaughy, Is Your Water Going to Stay?

## A Numerical Approximation of the Water Levels of Lake McConaughy


**Melissa Ackerman, Eric Robbins, & Caleb Sweetser**

University of Nebraska – Lincoln Mathematics Department (Undergraduate)

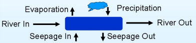

## Background

- Water level low due to 5 year drought
- Goals:
  - Model evolution of water levels over past 50 years
  - Predict future water levels
- Help in decision making processes for hydroelectric power, irrigation, and recreation purposes

## Model

- Based on the mass balance equation
  - Change in volume =

    Volume In – Volume Out

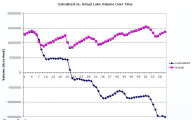



## Equation

$$\frac{dV}{dt} = R_{in} - R_{out} + P - E + S_{in} - S_{out}$$

- $\frac{dV}{dt}$ is the rate of change in volume
- $R_{in}$ is the amount, in acre-feet, of water being let into the lake
- $R_{out}$ is the amount, in acre-feet, of water being released from the lake
- P is the amount, in feet, of precipitation the lake receives
- E is the amount, in feet, of water that is evaporated from the lake
- $S_{in}$ is the amount, in feet, of water that seeps into the lake from the ground
- $S_{out}$ is the amount, in feet of water that seeps out of the lake into the ground

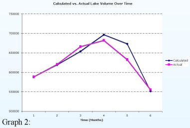
## Analysis of Results




Graph 1:

Calculated vs. Actual Lake Volume
January 1990 – December 1994

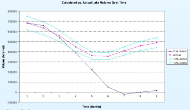
- Graph shows that the numerical method matches the slope of the actual data




Graph 2:

Calculated vs. Actual Lake Volume January – June 2004

- Within 10% above and below the actual data




Graph 3:

Calculated vs. Actual Lake Volume
April – December 2004

- August affects the graph considerably

## Conclusions

- Seepage contributes significantly, especially over summer months
- Precipitation factor is not a good estimator for seepage
- Model Error
  - Seepage error
  - Data collection error of 5-10%
  - Numerical method produces error
- A model starting in January produces a good estimate for six months
- Summer months historically seem to contain a factor which increases the error of the next step by ~ 20%
  - Model cannot recover from loss of water from this period

## Future Research

- Find estimator for seepage
- Determine a possible cause for the May/August phenomenon
  - Find a counterbalance for the effect
- Use an improved method of calculations


## Acknowledgements

- UNL Math Dept. "Math in the City" course taught by Dr. Petronela Radu
- Dept. of Natural Resources: Contact Dr. Shuhai Zheng
- Central Nebraska Public Power & Irrigation District: Contact Jeremie Kerkman
- UNL High Plains Regional Climate Center

**Fall 2006:** Traffic in Lincoln

- in collaboration with Mark Lutjeharms from Schemmer Associates
- students performed an analysis of traffic in downtown - how traffic is affected if a one-way street or a two-way street is blocked
- found a nonlinear relationship between speed and traffic density that maximizes the capacity of a road
- they solved a nonlinear conservation law to determine traffic at all times on a road at all times (use the continuum model for traffic flow)

**Fall 2006:** Was there a bubble in the Lincoln real estate market?

- in collaboration with Michelle Slack from Lincoln County Assessor
- data on residential property sales during 1995-2005
- nationwide increases in home-sale prices (double digit percent increase/year in some metropolitan areas)
- established linear regression equation between the price of a house and its features(s.f., lot, bedrooms, amenities); showed that over the past decade only the high-priced real-estate followed the national trend

**Fall 2008** Benefits and costs of sustainable design

- in collaboration with Jim Dyck from The Architectural Partnership
- students worked on 3 projects at two LEED certified buildings: The Earth House at the Prairie Hill Learning Center and the Nature Center at Pioneers Park
- students used linear programming to analyze savings vs. costs for green features: fluorescent lightning, strawbale walls, sink aerators, low flow toilets, etc. They also analyzed the green benefits of sustainable design: quantifiable and unquantifiable.
- colorgreen students found for different investment levels what sustainable design features one should choose to recuperate the investment, or to increase the maximum green benefits; financial factors (e.g. inflation, the utility function of money) were incorporated into their models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# PRAIRIE HILL Learning Center ‘Earth House’

# Future projects

- analyze financial investments considering different financial factors: performance, risk, fees (ordinary differential equations, linear programming, mathematical finance)
- the travel salesman problem for deliveries to a grocery store (routing, travel salesman problem, graph theory)
- model electrical circuits (ordinary differential equations))
- biology models (ordinary differential equations and partial differential equations)

# Assessment of student work

- most of the work is done in groups, how should individuals be assessed?
- the grade is a linear measure to assess nonlinear performance; what is more important: mathematical analysis or interpretation/communication of results?
- most of the credit is assigned to work on the project, but the grade for the final report provides only a portion of the grade; other components: participation during the semester (weekly updates and student journals), individual presentations, homework assignments

# Challenges

**From the instructor's point of view:**

- finding a good project: (i) what type of math problems would be suitable for undergraduate students? (ii) what businesses to approach? what data is available?
- different backgrounds for the students
- scheduling time for lectures and meetings; possible approaches: work with each group for 15-20 minutes every lecture, or meet with every group once a week for a full hour.
- management of group work; possible solution: weekly updates provided by individual students on a rotating basis

# Challenges (contd)

- instructor needs to be able to adjust (at any time) the course of the project in case the data is insufficient or incompatible with the mathematical tools available; handle the case when no optimal solution can be found
- motivate students and keep them focused when a roadblock appears, or when they feel overwhelmed (especially during initial and final stages)
- assess individual work
- keep and increase enrollment

# Challenges (contd)

**From the student's point of view:**

- tackling an open-ended problem where the approach is not clear (even to the instructors!)
- learning non-mathematical background necessary for the project
- need to effectively communicate with peers, instructors, and collaborators, both quantitatively and qualitatively
- teamwork is necessary; students depend on each other to complete their work, so they have to trust their teammates with the work that is being done (and graded)
- conflicts in their schedules (they have to meet to do the work)
- need to research mathematical and non-mathematical issues without a textbook

# Development of Math in the City

Development has been supported by

- University ITLE grant (2006-2007, PI Radu)
- NSF – CCLI award (2010–2013, PI Radu, Co-PI Hartke) to ensure sustainability and dissemination of the course to other institutions

Sustainability

- provide course structure and documentation: instructor handbook, student handbook
- create a portfolio of 4-6 projects and provide ideas for possible collaborators
- develop a network of businesses that provide projects on a continuous basis
- modules for mathematical content
- involve other instructors in the course

# Development of Math in the City

Dissemination

– occuring through the workshop and presentations at conferences and other institutions

**Workshop**

- 2-day “initiation” workshops
- 8–10 nonlocal instructors who have an interest in starting a Math in the City course at their institution
- includes the students’ final presentations
- discussion of structure of course, how to find projects, course management, etc.
- website and mailing list for future discussion; develop a network for Math in the City instructors

# Benefits to the Students

Educational:

- students understand how to translate a complex real-life situation into a mathematical model
- develop better communication skills in writing and for oral presentations
  - slide presentation in front of undergraduate, graduate students and faculty during the last week of classes
  - poster presentations at the Nebraska Research Expo (organized in collaboration with UNL Epscor)
  - poster presentations at the UNL Undergraduate Research Fair
- learn mathematical software: SPSS, Maple, Sage
- exposure to workplaces outside academia (they can show off and improve their "employable skills")

# Benefits to the Students

Personal:

- learn how to deal with setbacks, meeting deadlines for their project, how to work in groups, learn to take initiative
- increased self confidence and sense of achievement
- connect with possible employers in the city; find job or internship opportunities
- the course provides a non-standard experience that students will remember and tell to others

# Future of Math in the City

- Math in the City offered at UNL on a yearly basis; expect an increase in enrollment, but support offered by GTA
- Math in the City disseminated to other institutions; create forums and networks to enhance the course offerings and provide support to new instructors
- create networks of business collaborators and portfolios of projects
- mathematical communities will become more aware of the need for courses that show how mathematics is used outside academia
- create versions of Math in the City to be offered to high-school students
- more tomorrow!