The Twenty-Third Annual Nebraska Conference for Undergraduate Women in Mathematics

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POSTER ABSTRACTS

Posters by Undergraduate Students

Holly Abrams, Austin Peay State University
[A][1] *Measuring Gerrymandering: Improving Upon the Efficiency Gap*

Gerrymandering is the prominent issue of drawing voting district boundaries in a way that favors one political party over another. It has been an issue since 1812, and in recent history, the U.S. courts have relied heavily on a fairly simple calculation called the efficiency gap to determine if districts have been unfairly drawn. However, the simplicity of the efficiency gap indicates a clear lack of consideration for other political factors, leading mathematicians to seek more advanced metrics to identify gerrymandering. In this presentation, we will discuss an early version of a new metric that will take into consideration the amount of seats won by the winning party compared to the total votes cast for that party, but also favor more competitive districts.

Evan Alexander, Southwestern University
Zariah Whyte, Southwestern University
[B] *Isoperimetric Problems on the Real Number Line with Prescribed Density*

Variations on the isoperimetric problem typically involve trying to find a region in some ambient space that has an optimal measurement but is subject to a fixed constraint. In the classic example, mathematicians seek out an object with optimal/maximal area, subject to a fixed/constant perimeter). Oftentimes, the isoperimetric regions that optimize these quantities (such as the circle) have symmetric or otherwise "nice" geometry. There is often a deep relationship between the stated problem and the geometry of the solution.

In our project, we examined isoperimetric regions on the real number line with prescribed densities. The densities added a weight that affected how area and perimeter were measured. We were interested in finding optimal configurations of 1, 2, or 3 regions (so-called single, double, or triple bubble problems). Specifically, we examined the number line with two densities of interest: $\mu(x) = |x|$ and $\mu(x) = ||x| - 1|$. With these densities, we found isoperimetric solutions to the single and double region problem for $\mu(x) = ||x| - 1|$ and the triple region problem for $\mu(x) = |x|$.

Lillian Baker, United States Naval Academy
[A] *Exploring Links between Climate Shocks and Migration*

This project explores increased migration from Mexico into the U.S. as it relates to climate change in Mexico and Central America. Fluctuating rainfall patterns reflect climate shocks that damage the landscape and disrupt people's lives throughout the region. These key issues explain how climate shock events affect migration and may continue to influence human movement. The project utilizes a neuron model to quantify the factors that affect migration, to include climate, cultural, and historical factors. Through further analysis of precipitation and migration data, we will explore the causal relationship between climate shocks and human migration. The end goal of the project is a new mathematical model that examines the inter-relatedness of those two data sets. The model will then be tested and validated with atmospheric and migration data.

Heather Baranek, University of Wisconsin–Eau Claire
[B] *The Game of Cycles*

The study of graphs provides an answer to questions in a variety of disciplines through the process of modeling the problem onto a graph. The Game of Cycles is a newly developed impartial mathematical game, introduced by Francis Su in his book Mathematics for Human Flourishing. The game is played on simple planar graphs in which players take turns marking edges using a sink-source rule. I will outline the strategies in winning a Game of Cycles on certain families of planar graphs.

[1] [A] indicates poster session "A", [B] indicates poster session "B"

Jessica Bennett, Brown University
Penelope Fiaschetti, Boston University
[A] *Robust and Efficient Phase Retrieval from Magnitude-Only Windowed Fourier Measurements*

We propose and analyze a new generalization of an existing algorithm to reconstruct a complex vector (up to a global phase factor) from the squared magnitude of its windowed discrete Fourier transform. This is more commonly referred to as a phase retrieval problem, since this process requires the recovery of critically important phase information from magnitude-only measurements. This is a challenging yet fascinating non-linear inverse problem since there are often several possible solutions. The proposed algorithm utilizes results from discrete Fourier analysis to linearize the governing equations and obtain a highly structured Fourier based linear system. This linear system of equations can be efficiently inverted using the fast Fourier transform (FFT) algorithm. This provides relative phase information which we use to construct a special class of banded matrices, on which we perform spectral analysis to retrieve individual phase information. In addition to developing an efficient reconstruction algorithm, we provide mathematically rigorous theoretical error bounds in the case of noisy measurements, and provide numerical simulations demonstrating that this algorithm is computationally efficient and able to recover data in the presence of noise.

Ruth Butters, Kalamazoo College
[B] *Exploring Why the Conway Knot is Not Slice*

In 2018, Lisa Piccirillo wrote a proof about the famous 11-crossing Conway knot. This knot, named for mathematician John Horton Conway who passed away in 2020 due to COVID-19, had been stumping knot theorists for more than fifty years. Thanks to Piccirillo's proof, we now know more about this strange knot and its properties. This poster will explore the history of the Conway knot as well as some of the concepts used in Piccirillo's proof.

Ivy Collins, Berry College
[B] *Octahedral Knots*

Given a knot K we define $d_8(K)$, the octahedral number of K, to be the minimum number of octahedra, connected face to face, required to construct a closed loop that represents K. In this poster we give upper bounds on octahedral numbers of knots up to seven crossings, the 8_{19} knot, and for the canonical classes of C_1 and C_2 knots for $C < 5$ (in the Alexander-Briggs notation). We also give relationships between the octahedral number of a knot and the corresponding cubic lattice and stick numbers.

Allison Cruikshank, University of Nebraska-Lincoln
[A] *The Use of Mathematical Models in Analyzing the Effects of Treatment of Pancreatic Cancer*

Pancreatic cancer is one of the leading causes of death due to cancer in the United States. Analyzing the effects of radiation is extremely valuable in determining when a patient is allowed surgical resection, which is, presently, the only potentially curative treatment for pancreatic cancer. This study examines pancreatic tumor growth and shrinkage to predict tumor response and change of resectability for pancreatic cancer patients undergoing radiation therapy. This will be done using stochastic differential equations as a mathematical model. Mathematical models have increasingly been applied to various biological systems/processes to analyze the principles involved. Specifically, stochastic differential equations prove to be especially valuable in that they suppose that certain processes fluctuate due to unpredictable biological conditions. A population dynamical model is used along with fitting assumptions to create a mathematical approach to study tumor response under treatment for pancreatic patients. The model was built by analyzing the mechanisms involved in pancreatic cancer growth and altering them based on treatment. Analysis of the model took place using modeling software (for example, MATLAB) that allows for the graphing of the growth of pancreatic cancer and its response under treatment.

Elaine Danielson, University of Florida
Angela Li, The Ohio State University
[B] *Strategies and Generalizations of the Explorer-Director Game*

We analyze the Explorer-Director game, previously introduced in 2008 as the Magnus-Derek game by Nedev and Muthukrishnan. Played on a graph G, a token starts at a vertex v. Each turn, two players, Explorer and Director, call a distance and move the token to a vertex such distance away, respectively. Explorer seeks to visit as many unique vertices as possible, whereas Director seeks to visit as few as possible. Let $f(G, v)$ denote the number of unique vertices visited after optimal play. Previously, Nedev and Muthukrishnan found $f(G, v)$ for cycle graphs, and in 2008, Nedev found an $O(n)$ round strategy for the Explorer on cycles of n vertices. We find $f(G, v)$ for other families of graphs such as lattices, trees, and paths, and lower and upper bounds on $f(G, v)$ for all graphs. Moreover, we find non-adaptive $O(n)$ round strategies for the Explorer on paths of n vertices for all starting vertices.

Annabelle Eyler, Hood College
[A] *Trends in Maryland House Prices: A Study in Multivariate Regression*

The objective for this project was to investigate the cost drivers in Maryland residential building prices. In this, I report the process of gathering and cleaning this large data set of property assessment records. With a focus on statistical analysis of patterns in the data, my findings result from both a descriptive component and a modeling component using regression techniques. Through the use of R, multilevel modeling was used to compose a model for residential cost drivers.

Penelope Fiaschetti, Boston University
[A] see **Jessica Bennett**

Kenzie Fontenot, University of North Texas
[B] *Landscapes of the Cube and Tetrahedron: an Exploration of Shortest Paths on Polyhedra*

We review the established definition of nets and Alexandrov's definition of a star unfolding, leading to the classic result that the shortest path between two points on the surface of a convex polyhedra restricted to the polyhedron is contained as a straight line segment in one of the polyhedra's nets. To do any calculations concerning paths on the surface of a tetrahedron and cube we first define a coordinate system on the surface of the polyhedron. Since the shortest path on the surface of a convex polyhedron is a line segment contained in one of the polyhedron's nets, to calculate the set of points along said paths or the length of said paths we show it is sufficient to do so for each subset of the nets in which said paths can be contained. We introduce the new notion of a landscape which will be a useful tool in determining exactly which subsets of a polyhedron's nets need to be considered for such calculations. We then establish the 5 valid landscapes of the tetrahedron and 15 valid landscapes of the cube and their proofs.

Kimberly Hadaway, Williams College
[B] *An Introduction to Parking Functions*

In 1966, Alan G. Konheim and Benjamin Weiss defined "parking functions" as follows: We have a one-way, one-lane street with n parking spaces, numbered in consecutive order from 1 to n, and we have n cars in line waiting to park. Each driver has a favorite (not necessarily distinct) parking spot, which we call its preference. We order these preferences in a preference vector. As each car parks, it drives to its preferred spot. If that spot is open, the car parks there; if not, it parks in the next available spot. If a preference vector allows all cars to park, we call it a parking function. In 1974, Henry O. Pollak proved the total number of parking functions of length n, meaning there are n parking spots and n cars, to be $(n + 1)^{n-1}$. In this poster, we describe a recursive formula, expound Pollak's succinct six-sentence proof of an explicit formula, and conclude with a discussion of other parking function generalizations.

Toni Hinskton, University of Denver
[B] *The Catalan Graph: Growth Rate and Comparison to the Pascal Adic*

We examine the right hand side of the Pascal Adic and consider this its own graph, the Catalan Graph. We find properties of the Catalan Graph based on the level and location of vertices. The graph's structure is compared to the Pascal Adic to observe the differences in the growth rate of vertices. Finally, we determine the growth of the graph at some level n and prove that the complexity sequence grows like $\ln(n)n^2$.

Catherine Huang, University of California, Berkeley
Chloe Makdad, Butler University
[A] *Connections between Graph Spectral Clustering and Partial Differential Equations*

Visualizing and interpreting a dataset can be a difficult task for high dimensional data. Clustering and dimensionality reduction help us extract important features and make the data more manageable to interpret. One approach for clustering high dimensional data is graph spectral clustering, in which data clusters are derived from spectral properties of a similarity matrix. In this work, we illustrate the connections between graph spectral clustering and a discretized diffusion process on the similarity graph modeled by partial differential equations such as the heat and wave equations.

Ariba Khan, Georgia Institute of Technology
[A] *Dynamical Characterization & Analysis of the Optimization Algorithms: LB and ISTA*

Sparse recovery optimization algorithms are utilized in machine learning, imaging, inverse parameter fitting, as well as a multitude of other fields. Compressive sensing, a prominent field in mathematics this past decade, has motivated the revival of sparse recovery algorithms with l-1 norm minimization. Although small underdetermined problems are substantially well understood, large, inconsistent, nearly sparse systems have not been investigated with as much detail. In this dynamical study, two commonly used sparse recovery optimization algorithms, Linearized Bregman and Iterative Shrinkage Thresholding Algorithm are characterized in regards to their behavior with the hyperparameter lambda and varying entry sizes. These results prompted the creation of a hybrid method which benefits from favorable characteristics from both optimization algorithms such as less chatter and quick convergence. The Hybrid method is proposed, analyzed, and evaluated as outperforming and superior to both linearized Bregman and Iterative Shrinkage Thresholding Algorithm principally due to the Hybrid's versatility when processing diverse entry sizes.

Angela Li, The Ohio State University
[B] see **Elaine Danielson**

Chloe Makdad, Butler University
[A] see **Catherine Huang**

Marcella Manivel, Carleton College
Marguerite Shaya, Carleton College
[B] *Bernoulli Numbers and Class Number of Cyclotomic Fields*

The Bernoulli numbers (B_n) are a sequence of rational numbers given by the coefficients in the generating function of $\frac{te^{xt}}{e^t-1}$. A prime number p us said to be B-irregular if p divides the numerator of at least one of the nonzero Bernoulli numbers up to B_{p-3}. Loosely speaking, the class number of an algebraic number field can be thought of as a measure of the extent to which unique factorization holds in the ring of integers. We discuss the connection between Bernoulli numbers and the class number of cyclotomic fields.

Maati McKinney, Spelman College
Laura Stemmler, Carnegie Mellon University
[A] *Modeling Cooperation Between Species*

Population dynamics informs our knowledge of ecological interactions and influences our conservation laws and wildlife management. A central model in population dynamics is the Lotka-Volterra (L-V) model, which describes a predator-prey relationship. Modeling competitive relationships is a well-established sector of population dynamics, but far less research focuses on modeling relationships in which species cooperate rather than compete. To fill this gap, we developed a simplistic model akin to the L-V, which represents a mutualistic relationship where both parties benefit. We examined the yucca moths and yucca plant as an exemplary mutualistic system; the plants rely on yucca moths for pollination, while the insects depend on the plant's fruit to nourish the next generation of larvae. Using Mathematica and R-Studio, we constructed several sets of first-order differential equation and discrete-time difference equation models for the moth-plant system. We found that a simple sign-change in the L-V model could describe a cooperative relationship but was insufficient to fully convey the complexity of the system. Additionally, we recognized that it was necessary to include the deleterious effects (i.e. larvae feeding on plant seeds) of an interaction to generate a stable equilibrium. Numerical approaches were used to analyze the stability of each system. Ultimately, we aim to establish a definitive model of mutualism with applications not only in ecology, but also in realms of human interaction, namely mathematical finance.

Shannon Murphy, Montana State University
[A] *Mathematical Modeling of a Biphasic, Isothermal DNA Amplification Method*

Isothermal DNA amplification has many possible applications; nearly all of the capabilities of standard PCR, including clinical diagnostics, environmental monitoring, and forensic testing, could be accessed in a field situation. Ultrasensitive DNA Amplification Reaction (UDAR), one of the many isothermal DNA amplification methods, is distinguished from traditional methods by use of a palindromic template, which leads to a steep secondary rise in the reaction product. Through the development, implementation, and parameterization of a differential equations model, a deeper understanding of the reaction mechanisms was developed. This model will aid in the application of UDAR to diagnostics tools by giving researchers the ability to manipulate DNA and predict outputs.

Andrew Nguyen, University of Michigan
Madelynn Roche, Vanderbilt University
[B] *Surfaces with Braided Boundaries in Blow Ups of* $D^2 \times D^2$

On this poster, we will describe our research project that we began working on during an REU at the University of Virginia. Our problem is aimed at studying certain braided surfaces in a blowup of the 4-ball by adapting graphical methods developed by Seiichi Kamada in his paper, "Graphic descriptions of monodromy representations" and in his book *Braid and Knot Theory in Dimension Four.* We devised a new method with which we could analyze the problem using cross sections of the four-dimensional surface and their monodromy representations. We conjecture that using this method we can show non-uniqueness of spanning surfaces for a particular braid, and further that these there may be infinitely many of such surfaces.

Deniz Ozbay, Lafayette College
[B] *Coprime Labeling of Ladder Graphs*

A preprint by Ghorbani and Kamali provides an algorithm for the prime labeling of the ladder $P_n \times P_2$. In this poster, we will describe our work to generalize these results to a coprime labeling of $P_n \times P_2$, where the set of labels consists of the $2n$ positive integers starting at k.

Thi Phuong Anh Pham, University of Texas at Dallas
Riley Stephens, University of Texas at Dallas
[B] *On minimality of two-bridge knots of the form* $\mathfrak{b}(p, 7)$

A two-bridge knot, or rational knot, has the form $\mathfrak{b}(p, q)$ such that $\frac{p}{q} = k + \frac{1}{a+\frac{1}{b}}$. Another notation for $\mathfrak{b}(p, q)$ is then $[k, a, b]$. This type of knot gives k-long, a-long, and b-long twists. While the minimality of rational knots for any q up to 5 is known, we want to study the minimality of knots with $q = 7$ of the form $\mathfrak{b}(7k \pm 3, 7)$ (because the cases where $p = 7k \pm 1$ or $p = 7k \pm 2$ have been studied).

A knot is called minimal if its knot group admits epimorphisms onto the knot groups of only the trivial knots and itself. It's sufficient to study the irreducibility of the non-abelian $SL_2(\mathbb{C})$ character variety of the knot group to study its minimality. We first study the presentation of the knot group. Next, we find its representation. Then, we examine the non-abelian character variety of the group, which amounts to studying the irreducibility of the Riley polynomial.

Madelynn Roche, Vanderbilt University
[B] see **Andrew Nguyen**

Kaleigh Rudge, Colorado School of Mines
[A] *Simulating Nonlinear Waves on Vortex Rings in Ideal Fluids*

Vortex filaments are a fundamental structure in fluid dynamics for transferring energy. Understanding their motion and the relationship between the transfer of energy and the helical patterns of the vortex lines are crucial in understanding the decay of free quantum turbulence. Building these geometric vortex lines can be performed by using Jacobi elliptic functions to piece together nonlinear waves on a closed curve in space. We can use the velocity the vortex induces to simulate changes in the curve geometry over time, specifically linked to the dynamics of the tangent and binormal directions along the curve. This then allows for an exploration of the curvatures and torsions of these helical structures on the curve as they travel through time. Once an understanding of one structure is gained, we can further modify our equations to localize ideal curvature and torsion along the curve. From here we can use this mathematical model to approximate and predict the behavior of vortex structures in superfluids, and ultimately understand energy transfer patterns within them.

Delaney Scheiern, Colgate University
[A] *Genetic Algorithms for Cluster Analysis: An Application to Environmental Factors and Healthcare*

This poster explores Genetic Algorithms for Cluster Analysis, aided by a discussion of more simplistic clustering techniques such as k-Means and Gaussian Mixture Model clustering. Genetic algorithms use Charles Darwin's Theory of Evolution to discover solutions to optimization problems. Generation after generation, the algorithm explores a large proportion of the solution space and solutions move closer to global optima. In the case of cluster analysis, optimal solutions occur when the observations within each cluster are most similar and the observations between different clusters are least similar.

The genetic algorithm implemented in this project focuses on a density-based initialization of potential solutions, followed by distance-based optimization. The k-Means, Gaussian Mixture Model, and Genetic Algorithm methods are demonstrated in an application of environmental conditions in the United States for clusters created from health care coverage statistics.

Marguerite Shaya, Carleton College
[B] see **Marcella Manivel**

Laura Stemmler, Carnegie Mellon University
[A] see **Maati McKinney**

Riley Stephens, University of Texas at Dallas
[B] see **Thi Phuong Anh Pham**

Yuki Takahashi, Grinnell College
Zhichun Zhang, Swarthmore College
[B] *Frobenius templates in certain* 2×2 *matrix rings*

The classical Frobenius problem, also known as a "Chicken Nugget" problem, is to find the largest integer that cannot be written as a linear combination of a given set of positive, co-prime integers using non-negative integer coefficients. Prior research has been done to generalize this classical Frobenius problem from a topic in number theory to a topic in ring theory; the Frobenius problem has been generalized from the ring of integers to the ring of Gaussian integers as well as to the rings $\mathbb{Z}[\sqrt{m}]$, where m is a square-free positive integer.

In this poster, we will introduce a new generalization of the classical Frobenius problem to the commutative ring of 2×2 upper triangular matrices with constant diagonal. We will present answers to two research questions: for which lists of matrices is the Frobenius set non-empty, and for each list such that the Frobenius set is non-empty, what are the matrices in the Frobenius set? For the lists of two matrices, we will show the construction of every matrix in the Frobenius set. Moreover, for the lists of more than two matrices, we will explain the conditions under which the construction can be extended. Finally, we will introduce a new species of Frobenius problems for future studies.

Myka Terry, Morgan State University
[A] *Rainbow Cells: Tracking Cell Proliferation using Motzkin Paths*

A *proliferated* cell is a cell that descended from a stem cell, does not have a specific task but can be differentiated to create a specialized cell type. When a *proliferated* cell divides into two cells, the resulting cells are either both proliferated, both differentiated, or one of each. Many biologists aim to determine the rate at which a proliferated cell will divide into each scenario. It has been shown that this cell division can be modeled as a random walk that starts at the origin, never goes below the x-axis, and ends at the point $(n, 0)$. A random walk is essentially the path used to get to a specific point, there is an infinite amount of possibilities. More specifically, we model this division using Motzkin paths. A Motzkin path is a random walk that starts at the origin and ends on the x-axis using three different steps; an up step $(1, 1)$, a down step $(1, -1)$, or a level step $(1, 0)$. This research is focused on finding new ways to visualize the Motzkin paths to determine relative information about the depth of the relationship between the various types of proliferated cell division. We have created various representations for a Motzkin path and describe any characterizations about the cell divisions these visualizations were able to determine. This can in the future be used to predict the state of a cell in a specific phase of its life.

Gabriella Torres Nothaft, University of Minnesota–Twin Cities
[A] *Resilience of Ocean Circulation to Changes in Temperature and Salinity in a Classical Box Model*

Ocean circulation is essential when considering heat absorption and transportation around the globe. Box models have been used to conceptualize the effects of temperature and salinity on the circulation. In a classic 1961 paper, Stommel introduced a two box model to illustrate that ocean circulation could have multiple stable steady states depending on the parameters of temperature and salinity. Changes in these parameters can eliminate one of the steady states, resulting in the system equilibrating at a new steady state. However, when the parameters are returned to their previous values, the system remains in the new state and does not return to its previous state. Whether or not the system switches states depends on the magnitude and duration of the parameter change. In this work we quantify the boundary between the values of magnitude and duration causing the system to switch states and those values where the system returns to its previous state.

Christina Tran, Harvard University and Institute for Pure and Applied Mathematics
[A] *A Reinforcement Learning Approach to Packet Routing on a Dynamic Network*

This presentation explores various packet routing methods on dynamic networks. Satellite networks, which can be modeled as discrete dynamic networks, are used for tasks such as GPS and communications. To perform these efficiently and reliably, suitable algorithms must be developed to decide how a packet of information should travel from satellite to satellite to reach its destination. Traditional algorithms direct packets on the calculated shortest path, but these cannot adapt to changes in satellite connections as they leave and re-enter service range. Furthermore, in a high-traffic network, these algorithms cause congestion in a few center satellites while under-using periphery satellites, slowing delivery time of these packets. This presentation will explain how reinforcement learning can be used to teach a satellite network how to route packets in a way that achieves faster delivery and less congestion. We will explore how Q-learning and Deep Q-Learning techniques are applied to training nodes in a network to make routing decisions. Then, we will explain the results of our research that show how Q-learning is able to improve upon the traditional shortest path algorithm, and how Deep Q-Learning is able to expand the capabilities of Q-Learning to larger networks at the cost of longer training time. Finally, we will cover some challenges involving Q-learning and Deep Q-Learning as well as recommendations for usage of these various packet routing algorithms.

Zariah Whyte, Southwestern University
[B] see **Evan Alexander**

AliAnn Xu, University of Georgia
[B] *Gröbner Basis and the Ideal Membership Problem*

In this presentation, we study Gröbner basis from Algebraic Geometry. A Gröbner Basis is a set of multivariable polynomials that helps us solve certain algorithmic problems. For instance, given a polynomial ideal, how do we determine if an arbitrary polynomial is a member of the ideal? We show that Gröbner Bases are central to the solution of the ideal membership problem.

Zhichun Zhang, Swarthmore College
[B] see **Yuki Takahashi**

Kenna Zimmerman, Arizona State University
[A] *Utilizing Statistics and Machine Learning to Detect Events and Summarize Basketball Game Footage*

The NBA (National Basketball Association) yields billions of dollars each year and serves as a past-time and hobby for millions of Americans. However, many of us simply don't have the time to watch several 2-hour games every week, especially when only a fraction of the game is actually exciting footage. The goal of my research is to apply statistics and machine learning to create an application that takes all the "fluff" out of these games and distills them into only the most exciting and relevant events. My proposed model takes in a video file containing a full-length basketball game and analyzes the visual and auditory features of every nth frame. Data is recorded on the normed difference between the pixels of the current and previous frame, sound amplitude, time left in the game (using an optical character recognition model), and camera angle (using a machine-learning model). After all of this data is collected, each frame's data is analyzed and the most "exciting" clips are chosen, then played in sequential order.

Laura Zinnel, University of Wisconsin–La Crosse
[B] *Rainbow Solutions to $x + y = z$ in $[m] \times [n]$*

Consider the equation $x + y = z$ and the set of integers $[n] = \{1, 2, 3, \ldots, n\}$. A *solution* to the equation is a set of three integers that satisfy the equation. For example, $\{2, 5, 7\}$ is a solution in [8], but $\{3, 8, 11\}$ is not a solution in [10] since 11 is not in [10]. Now, we are going to color each integer in [10] and let $r = red$, $b = blue$, $g = green$, and $y = yellow$.

1	2	3	4	5	6	7	8	9	10
r	b	r	b	g	g	y	r	y	b

Once a set of integers has been colored, we can describe a rainbow solution. A *rainbow solution* is a solution where each element in the solution is a different color. Thus $\{3, 4, 7\}$ is a rainbow solution, and $\{1, 3, 4\}$ is not. The problem we will be discussing is how to use as many colors as possible in the set $[m] \times [n]$ while avoiding rainbow solutions. Solutions in $[m] \times [n]$ are sums of ordered pairs of integers. For example, $\{(2, 3), (1, 5), (3, 8)\}$ is a solution to $x + y = z$ in $[4] \times [8]$ since each of the first components are in [4] and each of the second components are in [8]. However, $\{(2, 3), (1, 5), (3, 8)\}$ is not a solution in $[4] \times [5]$ since 8 is not in [5].