The Twenty-Third Annual Nebraska Conference for Undergraduate Women in Mathematics

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TALK ABSTRACTS

PLENARY TALKS

Dr. Cecilia Aragon
Director, Human Centered Data Science Lab, and Professor in Human Centered Design & Engineering
University of Washington, Seattle
From Flying Geometry to Random Forests in the Stars: A Math Story

What do the following two careers have in common: (1) data science professor, and (2) airshow pilot?

Spoiler alert: math

It turns out it was a good thing Cecilia Aragon was a math major as an undergrad, because math ended up being useful in these two very different careers she pursued during her life. Math was also her personal superpower: she used it to overcome agonizing fears and accomplish life goals.

In this talk she tells her math story, and how math helped her go from painfully shy daughter of immigrants to: working with astronomers and Nobel Prize winners to solve some of the greatest mysteries of the universe; teaching astronauts to fly; and working with NASA to design software for Mars missions. She'll present examples from her work in aviation, astrophysics, and data science using images, video, and even equations.

As the first Latina full professor in the College of Engineering at the University of Washington, author of over 130 articles and 3 books, and winner of the 2008 Presidential Early Career Award for Scientists and Engineers, she'll also talk about overcoming stereotypes and low expectations as one of the few women — or Latinas — in both of her careers. For further detail, her recent memoir, FLYING FREE: My Victory over Fear to Become the First Latina Pilot on the US Aerobatic Team, describes how she used math to overcome fear and imposter syndrome to get to where she is today.

Dr. Ruth Haas
President, Association for Women in Mathematics, and Professor and Graduate Chair of Mathematics
University of Hawaii, Mānoa
Reconfiguration in Graph Coloring (and other contexts)

In mathematics, as in life, there are often multiple solutions to a question. Reconfiguration studies whether it is possible to move from one solution to another following a given set of rules. Is it possible? How long will it take? In this talk, we will consider reconfiguration of graph coloring as well as reconfiguring a mathematical life.

Here's a mathematical description of the graph theory part of the talk: A proper coloring of a graph is an assignment of a color to each vertex of the graph so that neighboring vertices have different colors. Suppose we change the color of just one vertex in a graph coloring. Can we get from one coloring to another by a sequence of vertex changes so that each step along the way is a proper coloring? The answer is yes, if we are allowed an unlimited number of colors. But, what is the fewest colors we can have for this to work? How many steps might it take? We will look at this, related questions and variations.

Dr. Talitha Washington
Inaugural Director of Atlanta University Center Data Science Initiative, and Professor of Mathematics
Clark Atlanta University
Be the 'Square Root of Possible'

Beginning to navigate the mathematical community can feel abstract and vague. As a first-year undergraduate student, Talitha Washington withdrew from a mathematical proof course because of the rigorous demand. Now, she is a full professor of mathematics who leads a major initiative to develop data science that is grounded in ethics and addresses topics in Black America. She will share how she rose above obstacles to become a mathematician because "the square root of impossible is possible" in us all.

Talks by Undergraduate Students

Frances Aponte-Caraballo, University of Puerto Rico at Cayey
Assessment of mitigation strategies for Sars-CoV-2 using a mathematical epidemiological model

The first reported case from COVID-19 in Puerto Rico occurred on March 13, 2020, after the Costa Luminosa Cruises arrived. As the epidemic progresses, the governor implemented various executive orders to safeguard the citizens' lives. SMICRC (Sistema Municipal de Investigación de Casos y Rastreo de Contactos) was established by a group of young epidemiologists and statisticians to contact trace and cut transmission chains by identifying infected cases and possible contacts. All these measures are considered non-pharmaceutical interventions, and they directly affect the course of the epidemic. We aim to analyze the effectiveness of the actions taken to mitigate the virus based on data behavior. An epidemiological mathematical model was developed to understand the epidemic evolution. The data was retrieved directly from the Health Department to determine new cases and deaths per day. As preliminary results, after applying releasing measures in the executive orders, an increase in new cases is evident. In six months, April to September 2020, the month with more confirmed new cases is August and more confirmed deaths is September. As data and the decisions taken re-shape the epidemic curve, we are not close to being over on this battle against the COVID-19 epidemic in Puerto Rico.

Taylor Boatwright, Francis Marion University
Solving the Navier Stokes Equation

The Navier Stokes equations are used to model fluid flow. Examples include fluid-structure interactions in the heart, climate and weather modeling, and flow simulations in computer gaming and entertainment. The Navier Stokes equations date back to the 1800's, and research and development of numerical approximation algorithms and finite element methods continue to be an active area. Using C++ and the academic software library dealii, we implement a least-squares finite element algorithm to approximate solutions to the Navier Stokes equations. We review the Navier Stokes equations, we derive the least-squares method algorithm, and we discuss its implementation.

Tori Braun, Ripon College
Jenna Gorham, The University of Arizona
Classifying toric 3-fold codes of dimensions 4 and 5

A toric code, introduced by Hansen to extend the Reed-Solomon code as a k-dimensional subspace of $(\mathbb{F}_q^*)^m$, is determined by a toric variety or its associated integral convex polytope P, where $k = |P \cap \mathbb{Z}^m|$. Previous authors have classified toric surface codes with dimension up to $k = 7$. We classify all 4-dimensional toric codes of polytopes in $\mathbb{R}^3$ using White's description of polytopes with 4 lattice points, and we present progress toward the same for dimension 5 codes using work by Blanco and Santos. In particular, for $k = 4$ we first prove formulae for the minimum distances of codes coming from empty tetrahedra of the form

$$T(s,t) = \text{conv}\{(0,0,0),(1,0,0),(0,0,1),(s,t,1)\},$$

where $\gcd(s,t) = 1$, which occurs as a subpolytope for each subsequent case. We then uncover which disagreements in parameters s and t two polytopes may have while still yielding codes which are generated by matrices identical up to scaling and reordering their columns. Likewise, for $k = 5$ we prove bounds on the minimum distances of each class and indicate where uncertainty in a total classification remains.

Darcy Brunk, California State University, Stanislaus
Image Segmentation via Hypergraphs using k-Means Clustering

Image processing techniques have continued to advance to keep up with complex data sets generated by experimental facilities. The Advanced Light Source at Lawrence Berkeley National Laboratory houses a micro-tomography instrument which uses X-rays to visualize cross-sections of the object and recreate it as a virtual 3D model. Medical imaging utilizes this technique due to its ability to see inside an object without destroying it. Segmentation is an image processing technique used to create a simplified representation of the image so that it is more meaningful and easier to analyze. This research develops a hypergraph-based image segmentation algorithm that can be parallelized to handle large volumes of data. This algorithm is built off the Parallel Markov Random Field (PMRF) graph-based image segmentation algorithm. We build a hypergraph model of the image using the k-means clustering algorithm. The hyperedges are then augmented to respect the graph partitions used in the PMRF model, which allows for parallel computations. When optimizing the associated energy function, we incorporate a higher-order term which calculates the energy from the hypergraph model. When applied to micro-CT images from the Berkeley Sandstone Data Set, this algorithm shows improved accuracy compared to previously tested algorithms.

Emily Cairncross, Oberlin College
Emily Lopez, University of California, Santa Barbara
Ashley Zhuang, Harvard University
Throttling for standard zero forcing on directed graphs

Zero forcing is a coloring process performed on a graph with initially blue and white vertices. The *color change rule* is applied repeatedly: a blue vertex u can force a white vertex w to become blue if w is the only white neighbor of u. This coloring procedure has connections to the inverse eigenvalue problem and has applications to PMU placement in electrical engineering and quantum control in physics. *Throttling* is a process that gives the optimal balance of resources and time when zero forcing. In particular, the *throttling number* of a graph minimizes the sum of the number of vertices initially blue and the number of time steps needed to color every vertex blue. The process of zero forcing can also be extended to directed graphs in which a white vertex w can be forced if it is the only white out-neighbor of a blue vertex u. In this talk, we demonstrate how this extension motivates throttling for directed graphs and present some recent results.

Jiuqi Chen, Dartmouth College
Periodic Infinite Friezes of Type $\Lambda_{p_1,\ldots,p_n}$ and Dissections on Annuli

Frieze patterns are arrays of shifted rows of numbers that satisfy the unimodular property and are related to polygon dissections, quivers, and cluster algebra. Finite frieze patterns of positive integers are first defined by Coxeter and have been extended to infinite cases by Baur, Parsons, and Tschabold. Finite friezes are generalized to algebraic integers by Holm and Jørgensen, which we called friezes of type $\Lambda_{p_1,\ldots,p_n}$. We extended these Holm-Jørgensen friezes to their infinite cases and discussed their geometric interpretations and the corresponding realization criterion. We proved that for all friezes that are realizable, their entries are in the form of the sum of path weights. This property allows us to prove that all friezes that have normal dissected annulus interpretations are positive and to find combinatorial interpretations of other statistics of frieze patterns such as the growth coefficients.

Annika Christiansen, University of Minnesota–Twin Cities
Nhung Pham, University of Minnesota–Twin Cities
Symmetries of linear systems on graphs

In the divisor theory of graphs, a finite, connected graph is viewed as a discrete analog of a Riemann surface. A divisor D on a graph is an assignment of integers to each vertex of the graph. An important statistic in this setting is the complete linear system of D, which is the collection of effective (e.g. non-negative) divisors linearly equivalent to D via the discrete Laplacian operator. Recently, S. Brauner, F. Glebe, and D. Perkinson characterized all complete linear systems on a finite graph G. We extend their results by computing the subset of divisors in any complete linear system that are fixed by a symmetry of G. When G is the cycle graph, we present a bijection between certain effective divisors and a subposet of Young's lattice defined by R. Suter; using the combinatorics of these divisors, we give an alternate proof of a result by H. Thomas and N. Williams that Suter's poset exhibits the cyclic sieving phenomenon (CSP).

Zoe Daunt, Northeastern University
Elliptic Curves and the Probability of Prime Torsion

Elliptic Curves are the research focus of several disciplines. But what are they? And why are they so special? It turns out that depending on how you define an elliptic curve, they are particularly interesting to those who study them in the context of cryptography. A commonly considered question in public-key cryptography is the discrete logarithm problem. As with many cryptography problems, one is particularly interested in groups for which there is no efficient way to solve it. One example is the groups of rational points on elliptic curves over finite fields. In this talk, I will discuss the probability that an elliptic curve defined over a finite field has a rational point of prime order. In order to do so, we will review some key concepts and background material to gain a fundamental understanding of elliptic curves. Then, I will lead you through the derivation of combinatorial formulas to describe this probability.

Lam Doan, La Salle University
Graph Coloring for the Assignment of Seats to Fans

Graph coloring, in which labels are assigned to elements of a graph in such a way that specified conditions are met, has many variations. We consider an application of graph coloring to the assignment of seats to sports fans. Each seat is represented by a vertex, and pairs of vertices associated with adjacent seats are connected by edges. Each team has a unique color. The vertices of the graph are colored, subject to certain constraints, to indicate the placement of fans in seats. For various families of graphs, we determine which collections of fans can be seated under the stated compatibility conditions, and we provide optimal assignments in the feasible cases. Fans supporting more than one team are also considered; this results in multi-colored vertices when seating arrangements are determined.

Vonnie Dobbs, Marquette University
Lino Yoshikawa, University of Hawaii at Hilo
Enumerating "Good" Permutations

Two permutations π and σ are said to be *order isomorphic* if they are equivalent after pattern reduction, in other words, if they are in the same relative order. We call a permutation "good" if its first ℓ entries are order isomorphic to the last ℓ entries. Given a k, we wish to enumerate all good permutations on $[k]$. This will enable us to enumerate those pairs of permutations that overlap consecutively and which are order isomorphic. We do this for whenever $\ell \leq k/2$ and via experimentation we conjecture that whenever $\ell > k/2$ the number of good permutations is polynomial in $k - \ell$. We also make a connection of enumerating good permutations to the problem of explicitly determining the expected number of distinct permutation patterns contained in a random permutation.

Samantha Dukes, Furman University
Accordion Solitaire

Accordion Solitaire is a complex single player card game in which the player aims to consolidate a standard 52-card deck into one pile. To begin, the player randomly deals all 52 cards face up in a straight line. Throughout the game, cards that meet certain conditions shift leftward to cover other cards, thus compressing the deck like an accordion. The game concludes when there are no more available moves. We use mathematical concepts from areas such as graph theory, algebra, and statistics to develop winning strategies and algorithms for this game.

Adriana Duncan, Tulane University
Simran Khunger, Carnegie Mellon University
Generalizations of Alder's Conjecture via a Conjecture of Kang and Park

Integer partitions have long been of interest to number theorists, perhaps most notably Ramanujan. Here, we focus on partitions with gap conditions and partitions with parts coming from fixed residue classes.

Let $\Delta_d^{(a,b)}(n) = q_d^{(a)}(n) - Q_d^{(b)}(n)$ where $q_d^{(a)}(n)$ counts the number of partitions of n into parts with difference at least d and size at least a, and $Q_d^{(b)}$ counts the number of partitions into parts $\equiv \pm b \pmod{d+3}$. In 1956, Alder conjectured that $\Delta_d^{(1,1)}(n) \geq 0$ for all positive n and d. This conjecture was proved partially by Andrews in 1971, by Yee in 2008, and was fully resolved by Alfes, Jameson and Lemke Oliver in 2011. Alder's conjecture generalizes several well known partition identities including Euler's theorem.

In 2020, Kang and Park constructed an extension of Alder's conjecture which relates to the second Rogers-Ramanujan identity by considering $\Delta_d^{(a,b,-)}(n) = q_d^{(a)}(n) - Q_d^{(b,-)}(n)$ where $Q_d^{(b,-)}(n)$ counts the number of partitions into parts $\equiv \pm b \pmod{d+3}$ excluding the $d+3-b$ part. Kang and Park conjectured that $\Delta_d^{(2,2,-)}(n) \geq 0$ for all $d \geq 1$ and $n \geq 0$ and proved this for $d = 2^r - 2$ and n even.

We prove Kang and Park's conjecture for all but finitely many d. Toward proving the remaining cases, we adapt work of Alfes, Jameson and Lemke Oliver to generate asymptotics. Finally, we present a generalized conjecture for higher $a = b$ and prove it for infinite classes of n and d.

Montana Ferita, Westminster College (PA)
Not Your Normal Fibonacci Sequence

The Fibonacci sequence is perhaps the most well-known sequence in the field of mathematics. In the Fibonacci sequence each element in the series is the sum of the previous two numbers. We will denote the first two numbers in the series as a and b, where a is less than or equal to b. Given a number n we seek to find the smallest positive value b such that n appears in the Fibonacci series starting with a and b. One can determine a and b given n by performing an exhaustive search for various combinations of a and b. We investigate quicker methods than this brute force approach for n values with certain properties.

Madison Ford, Wayne State University
Sarah Pritchard, Georgia Institute of Technology
Exploring Solvability of the String Link Concordance Group using Milnor's Invariants

Suppose you have n knotted-up pieces of string tangled together. Is there a mathematical way to describe just how tangled they are? The Milnor's invariants of a string link provides us with information about how "linked" components are. In this talk, we'll discuss two ways to calculate the Milnor's invariants of a string link: a group-theoretic method and a method that involves generating surfaces bounded the link. Then, we'll discuss some results concerning the solvability of the string link group; by calculating the Milnor's invariants of string link commutators, we can learn more about how things commute in the string link group.

Chloe Frechette, Northern Arizona University
On Deleting Directed Graphs

Sylver Slayer is a 2-player game played on directed graphs, inspired by Conway's game Sylver Coinage, with the goal being to eliminate the digraph. We will present new results on families of digraphs that can be eliminated by any initial choice of vertex.

Julie Fucarino, IPAM/Wellesley College
Analyzing, Predicting, and Mitigating Defect Formation in Metal Additive Manufacturing

Recent advances in metal additive manufacturing have made this technique a promising direction in industry for printing metal parts, such as for biomedical devices and automobile machinery. Despite its ability to efficiently print highly specific parts, this process is prone to errors during printing, causing pores to form in the metal part. These pores weaken the metal by increasing the likelihood of internal cracks forming. With current techniques implemented by HRL Laboratories, these defects are only able to be detected after printing, which impedes any efforts to correct these errors in real-time. In order to provide a real-time quality control framework, we developed a machine learning model that inputs the laser printing parameters and predicts whether a defect will form at a given location, which is informed by the X-ray images of the metal scanned after printing. This model provides an in-situ prediction for the formation of defects in real-time so that corrections can immediately be made. In addition to building this predictive model, we provide a semi-automated process for preprocessing and registering the X-ray images with the in-situ laser data.

Arlena Liryce Gavino, California State University, Stanislaus
Informing the Need of Critical Thinking in Mathematics

Based on the National Assessment of Educational Progress, it seems that the public education system in California is failing to provide students with essential problem solving and critical thinking skills in mathematics. Many studies have proven that students do not rationally think about mathematics word problems. Though, it comes to question that this issue may come from outdated and unrelatable terminology, or context of mathematics problems. Using a survey that contains the original and modernized version of the "How Old is the Sheppard?" problem (which states "There are 125 sheep and 5 dogs in a flock. How old is the shepherd?") in schools in California's Central Valley, this research discovered that word choice does not play a role in students' performance on nonsensical mathematics problems; students significantly increase their critical thinking skills from middle school to high school; and performance on traditional mathematics problems has no correlation to their performance on nonsensical problems. We speculate possible distractions in their testing environment and psychological tendencies that may have influenced our data, such as talking in the classroom and how math is commonly taught; and recognize that students' critical thinking has improved from about 35 years ago (when the original study was done), but it still needs to be improved. This information may help teachers, textbook authors, and others invested in mathematics education to create better material for students.

Jenna Gorham, The University of Arizona
see **Tori Braun**

Sara Grey, Austin Peay State University
The Gender Gap of Doubt

In an effort to measure the effectiveness of their calculus courses, APSU mathematics faculty administered the "Calculus Challenge" assessment designed with an atypical, penalized scoring method, which unintentionally produced an achievement gap between men and women. The penalized scoring structure discouraged guessing by deducting 1 point for incorrect answers, while awarding 0 for unanswered questions. Analyzing two years of historical data revealed that, on average, women answered significantly fewer questions and scored far lower than men on this particular test. These findings motivated the hypothesis that penalized scoring adversely affects the prevalence to which women experience self-doubt, their willingness to guess, and overall score. To test this, we enlisted 287 participants to complete an anonymous survey and mathematics assessment utilizing the Calculus Challenge's penalized scoring structure. Preliminary analysis of the collected data supports the hypothesis and suggests a need to develop a more equitable assessment method.

Kristen Hallas, The University of Texas Rio Grande Valley
Joan Mattle, Ithaca College
Deanna Perez, California State University, Fullerton
Forms of Golden-type Recursive Polynomials

In this talk we will present some properties of Fibonacci-type recursive polynomials. After introducing the classical Fibonacci-like polynomials and the so-called Golden polynomials, we introduce recursive polynomial sequences defined by

$$G_{n+1}(x) = x^k G_n(x) + x^l G_{n-1}(x),$$

for k and l positive integers with $G_0 = -1,\ G_1 = x - 1$. We discuss Binet forms, Pascal-like triangle representations and matrix representations for G_n. We derive interesting sequences and identities. Lastly, we present analytic and numerical results on the nature of the real roots of G_n. Our work extends known results for Fibonacci-like polynomials.

Kristen Hartley, Georgia College and State Univeristy
Comparing Virtual and Concrete Manipulatives

Defined as "an object that can be handled by an individual in a sensory manner during which conscious and unconscious mathematical thinking will be fostered," (2010, p. 14) manipulatives are used in mathematic classrooms all across the United States and other countries. Stein and Bovalino state that manipulatives provide a concrete way to link abstract information to already established knowledge thus giving new concepts a deeper meaning (2001). The purpose of this study is to compare concrete and virtual manipulatives to see if one fosters a deeper conceptual understanding of the FOIL Method. Students in a Middle Grades Cohort at Georgia College were given both a pre and post assessment to assess their level of understanding of the FOIL Method after a lesson using either virtual or concrete manipulatives. They then were taught using the other type of manipulative to assess whether students prefer virtual or concrete manipulatives.

Emily Howe, Georgia College and State Univeristy
Lie-derivations of three-dimensional Leibniz algebras

The concept of derivations play an important role in understanding algebraic structures such as Leibniz algebras which are a generalization of Lie algebras, in which the bracket operation is not necessarily skew symmetric. A new notion of derivations, namely Lie-derivations, was recently introduced as a generalization of the notion of derivations for non-Lie Leibniz algebras, relying on the fact that the quotient space $\frac{\text{Lieg}}{\text{Leib(Lieg)}}$ of a Leibniz algebra Lieg by the two-sided ideal Leib(Lieg) is a Lie algebra, where $\text{Leib(Lieg)} := \langle [x, x]\ x \in \text{Lieg} \rangle$ is referred to as the Leibniz kernel of Lieg. In this talk, we will discuss how to determine the Lie algebras of Lie-derivations of non-Lie Leibniz algebras. Our calculations will focus on the case of three-dimensional non-Lie Leibniz algebras in which we completely determine the basis of the Lie algebras of Lie-derivations and classify the basis elements as inner or outer derivations.

Sidra Jawaid, University of Central Oklahoma
Influential nodes detection on complex networks via network topology

Identifying influential nodes in complex network is an important graph mining problem that has wide applications in many areas such as market advertising and rumor controlling. Nodes in a network can have varying roles in information diffusion. In this research project, we use the diffusion Frechet function (DFF), a function that leverages network topology and is robust to noise in data, to identify the most influential nodes in networks. We apply our method to various real-world network datasets to identify influential nodes. Its performance is then compared to that of the classical graph-theoretic centrality measures using the Susceptible-Infected-Recovered (SIR) simulation model. Our experimental results suggest that our method is promising in influential nodes detection and more effective than the classical centrality measures.

Lisa Johnston, Kalamazoo College
Coxeter groups and the Bruhat Order

We examine Coxeter groups, specifically the symmetric group, through a combinatorial lens. Furthermore, we present properties of intervals in posets on type A Coxeter groups. In Type A (the symmetric group), it is known how and when a principal order ideal of some permutation w is isomorphic to a generic interval. In particular, if w is decomposable, then w does not force a factor. We investigate whether this same property also holds for type B Coxeter groups (the group of signed permutations).

Riley Juenemann, Tulane University
Statistical Dashboard for Categorizing Particle Movement in Cells

At the intersection of nanoscience and biology lies the question of precisely how particles move within cells. In contrast to in vitro particle tracking experiments, wherein there are great controls on particle and environmental homogeneity, live cell tracking features tremendous diversity in particle movement. Within this research area, the use of mathematics has allowed for a better description of movement categorizations and quantitative methods to differentiate between them. In this work, we have developed a first-pass statistical dashboard to categorize disparate types of particle trajectories. The tools we developed for the categorization process include the correlation between consecutive increments and effective diffusivity from a maximum likelihood estimation. The standard deviation for the major and minor axis and the creation of a parameterized path to represent a fictional moving anchor employed principal components analysis. This anchor estimation allowed the computation of the average distance the particle deviated from the anchor. Based on these data measures, K-means clustering was utilized to distinguish between free diffusion, stuck diffusion, directed transport, tracker error, subdiffusion, and skating diffusion. This automated categorization process proved to be successful on data simulated using stochastic differential equations and provided interesting results on the live cell data.

Miriam Kharbat, North Carolina State University
Using Modeling to Characterize Patients with Pulmonary Hypertension

Pulmonary hypertension is a deadly disease characterized by high blood pressure above 25 mmHg in the pulmonary arteries. For this study, we use mathematical modeling to simulate and analyze right heart catheterization data from five patients diagnosed with pulmonary hypertension. To assess hemodynamics, we develop a compartment model analogous to an electric circuit in order to predict the flow, pressure, and volume of the cardiovascular system. This closed-loop cardiovascular system estimates model parameters that minimize the difference between model predictions and right-heart catheterization data from 5 patients. We use sensitivity analysis, covariance analysis, and parameter subset selection in order to determine a set of identifiable parameters that are fitted specific to each data from each of the 5 patients. Results show that hypertensive patients have increased pulmonary vascular resistance and decreased compliance compared to normotensive control patients. By simulating pulmonary hypertension treatments, we can achieve normotensive pressure values in both the right ventricle and pulmonary artery, while simultaneously reducing the pressure in the right atrium and systemic arteries. Since pulmonary hypertension is a chronic and progressive disease, recognizing the distinguishing features of a hypertensive network can facilitate diagnosis and improve treatment strategies.

Simran Khunger, Carnegie Mellon University
see **Adriana Duncan**

Gillian King, Bowdoin College
Digitizing Maine's Voting History with a Statistical Analysis of Error Rate

The purpose of my project, completed remotely via Bowdoin Summer Research Fellowships, was to digitize Maine voting data and analyze the error rate using three statistical tests. This research was done under the umbrella of my advisor's project: to create a single online resource for researchers that stores election data throughout Maine's history. Voting records were digitized through an Optical Character Recognition Software (OCR), ABBYY Finereader, and then converted to files that could be made public.

In the second part of the project, I plotted election results from different columns against one another, as the proportion of "yes" and "no" votes per town remained largely proportional (a column represents vote totals per question, and a row represents vote totals per town). I then used RStudio to create a line of best fit using a least squares regression which allowed me to identify original outliers in this patterns. I also calculated the p-values of each row of data, and flagged vote totals that fell out of the traditionally "acceptable range" for this test. The second test involved summing columns of voting results by town and comparing the result to the vote total noted on the original voting record, while the third used random-selection code on RStudio to check randomized entries.

My results suggest that the first two tests were very effective when used in tandem, as there were few errors identified in the third across all county sheets.

Elana Kozak, United States Naval Academy

Analysis of Monte Carlo Tree Search Methods in a 2D Lattice Search Detection Problem

Monte Carlo Tree Search (MCTS) is an artificial intelligence game player method that uses many simulations to learn about a decision space and choose the best move based on a balance of exploration and exploitation. We apply this theory to a simple game of search and detection, using one stationary target and one moving searcher in a 2D lattice with periodic boundaries. Through simulated data and theoretical proofs we show that the MCTS performs optimally when the target is from a known delta distribution and it performs as a nearly self-avoiding random walk when the target distribution is uniform.

Cassandra Lem, University of California, Santa Barbara
Ivan Matos, North Carolina State University

Numerical Range of Composition Operators on the Hardy Space of the Unit Ball

The numerical range of a bounded operator T on a Hilbert space is defined as

$$W(T) = \{\langle Tx, x\rangle : \|x\| = 1\}.$$

We explore the numerical range of a composition operator C_A on the Hardy space, $H^2(\mathbb{B}_n)$, of the open unit ball $\mathbb{B}_n$ of $\mathbb{C}^n$, induced by an $n \times n$ matrix A that is a self-map of $\mathbb{B}_n$. The composition operator C_A is defined by

$$C_A f = f \circ A \quad \text{for all} \quad f \in H^2(\mathbb{B}_n).$$

We show that spaces of homogeneous polynomials of degree k, denoted $H_k(\mathbb{B}_n)$, are invariant under C_A. We find a matrix representation of C_A restricted to $H_k(\mathbb{B}_n)$. We then characterize the numerical range of C_A induced by a variety of matrices, including forward shift, backward shift, circular shift, anti-diagonal, and arbitrary permutation matrices. For example, we show that $W(A) \subseteq W(C_A)$ in general, and $W(A) = W(C_A)$ when A is the circular shift matrix. To achieve this, we compute the invariant subspaces of $C_A|_{H_k(\mathbb{B}_n)}$ and decompose the matrix representation of $C_A|_{H_k(\mathbb{B}_n)}$ as a direct sum of matrices. We then use combinatorial and computational methods and well-known numerical range results to characterize the numerical ranges of the composition operator.

Anqi Li, Massachusetts Institute of Technology
Local Properties of Difference Sets

Erdős and Shelah asked what we can learn about a large and complicated object X from properties that are satisfied by each small piece of X. We study the following variant of this problem, first studied by Erdős and Sós. Given a set of real numbers A, we consider the *difference set* $A - A = \{|a-b| : a, b \in A\}$. While a random set A is expected to have $|A - A| = \Theta(|A|^2)$, arithmetic progressions satisfy $|A - A| = \Theta(|A|)$. Let $g(n, k, \ell)$ denote the minimum size of $|A - A|$, taken over all sets A of n numbers that satisfy the following local property: every subset $A' \subset A$ of k numbers satisfies $|A' - A'| \geq \ell$. Intuitively, every k numbers from A span many differences. We derive several new bounds for $g(n, k, \ell)$. We now state two of our results.

Erdős and others were interested in *linear thresholds* of local properties problems: the smallest ℓ for which the size of the global property is superlinear. We establish the linear threshold of the differences problem. We prove that for every k, we have $g(n, k, k-1) = n - 1$ and $g(n, k, k) \gg n$. The following is the simplest of a family of bounds that we derive: When k is a power of two, we have

$$g\left(n, k, \frac{k^{\log_2(3)} + 1}{2}\right) = \Omega\left(n^{1+\frac{1}{k-1}}\right).$$

Lu Li, Macalester College
An Empirical Study on Minimal Generators of Topological Features from Persistent Homology

This work provides an empirical study of the computational cost and effectiveness of several common optimization procedures applied to homological generators in dimension one, including algorithms to minimize L_0 and L_1 norms, bounding area, and "volume." We conduct these optimizations via standard linear programming methods, applying general-purpose solvers to optimize over column bases of simplicial boundary matrices. Our key findings are (i) the computational cost of optimizing a basis of generators exceeds the cost of computing such a basis, for each of the tested general-purpose solvers, (ii) optimization is generally effective in reducing the size of generators, though the extent of the reduction varies according to the distribution of the underlying data, and (iii) the space of L1 optimal solutions properly contains that of L0 optimal solutions in the majority of Vietoris-Rips complex studied, though none of these complexes had totally-unimodular boundary matrices in degree one, a sufficient condition found in other studies.

Emily Lopez, University of California, Santa Barbara
see **Emily Cairncross**

Amanda Mallott, Gonzaga University
Total Roman Domination of Kneser Graphs

In graph theory, Total Roman Domination (TRD) colors every vertex in a graph with a 0, 1, or 2 by specific rules that originate from Roman military strategy. The TRD-number $\gamma_{tR}(G)$, or "weight" of a graph, is the smallest possible sum of all the vertices' numbers that follow TRD rules. In this talk, we discuss strategies for finding the TRD-number of generalized Kneser graphs, a type of graph of which the Petersen graph is a well-known example, and we will demonstrate some explicit bounds. We will begin with some basic definitions of graphs, domination, and the "rules" for Total Roman Domination.

Ivan Matos, North Carolina State University
see **Cassandra Lem**

Joan Mattle, Ithaca College
see **Kristen Hallas**

Sydney Meier, University of Nebraska at Omaha
A Discrete Event Simulation Model to Overcome Human Trafficking in the U.S. and the State of Nebraska

Human trafficking is a complex issue that affects society and the global economy, which involves the commercial exchange and exploitation of people through forced labor, domestic servitude, and sex trade, and is considered the third most profitable organized crime in the world. Nebraska and the U.S. are not excluded from this societal issue, and Omaha is one of the main hubs for human trafficking in the state. By analyzing the flow of monetary gains/resources, information, and trafficked people from the perspective of traffickers, police force, and advocacy organizations, this research project develops a discrete event simulation model to represent this complex system. Different scenarios are proposed, and the main goal is to block the monetary flow and minimize the number of trafficked humans within the trafficking ring over the years. To achieve this goal, these scenarios study how an increase in government funds and private organizations can maximize information and resources within the police force and advocacy organizations. The solution of this model can assist governmental organizations to create public policies to diminish the number of trafficked individuals in the U.S. and the state of Nebraska over the years.

Uyanga Naranbaatar, Concordia College, Moorhead
Eliana Rutherford, Concordia College, Moorhead
A Games of Best Choice: The Hiring Problem

We explore the strategies to solve the classical version of the scenario called the Secretary Problem, in which an administrator is looking to hire the best candidate from a pool of applicants. After interviewing each candidate individually, the administrator must immediately make a decision to hire or reject the current applicant. The Secretary Problem asks us to consider the probability that the administrator hires the best candidate given the above constraints. Our project focused on first understanding the classical scenario and the known optimal 37% solution and proof. We then explored concepts of incentive compatibility and top t choices. Incentive compatibility was used to look at the optimal solutions to stay and go probabilities, as well as to compare strategies to each other. Whereas the classical Secretary Problem looks at the probability of choosing the best or top one candidate, we looked at how strategies changed when top two, three, or more candidates were considered desirable. Python was used to look at larger probability trees in order to avoid human error and provide a faster computation device for the probabilities.

Anushka Narayanan, Cornell University
Effects of Mars' Obliquity on Ice Cover: A Budyko Model Approach

Studies suggest that the evolution of Mars' rotational axis is chaotic and may have varied by as much as 60 degrees over its history. This large and unpredictable range in the orientation of the planet makes Mars' climate history uncertain. A planet's climate largely depends on the characteristics that describe the planet's specific orbit. Specifically, in the case of Mars, its obliquity, which refers to the angle of the tilt of its rotational axis, has a prominent role in the extent of ice cover on the Martian surface. Ice cover is particularly interesting due to its potential implications from geological and biological perspectives. To observe how Mars' changing obliquity potentially impacted ice cover dynamics over time, we use an obliquity-dependent energy balance model examining the incoming and outgoing energy. In this talk, we will discuss our simulations of Mars climate and the different resulting ice regimes. We find that as obliquity changes, there are large oscillations in polar ice cover. We see certain regions of parameter space where the model indicates stable oscillations between partial ice cover and ice-free states. The model also shows no stable partial ice cover on the Martian surface with the poles being ice-free.

Megan Osborne, University of Scranton
An Unstructured Mesh Approach to Nonlinear Noise Reduction

In any type of data acquisition, the event of gathering undesirable noise along with desirable data is inevitable. To denoise signals originating from smooth, chaotic attractors, the Air Force Research Laboratory (AFRL) adapted the time-delay embedding theory of Takens' Theorem (1981) and the causation-detecting method of Convergent Cross Mapping (CCM) to develop a grid-based denoising technique. Given a clean signal from such a dynamical system, AFRL's technique attempts to denoise a corrupted signal observed from the same system. To improve this grid-based method, we implement an unstructured mesh based on triangulations and Voronoi diagrams that better distributes data over mesh cells and improves the accuracy of the reconstructed signal. Our method achieves statistical convergence with known test data and reduces synthetic noise on experimental signals from Hall Effect Thrusters (HETs) with greater success than the grid-based strategy.

Deanna Perez, California State University, Fullerton
see **Kristen Hallas**

Nhung Pham, University of Minnesota–Twin Cities
see **Annika Christiansen**

Sarah Pritchard, Georgia Institute of Technology
see **Madison Ford**

Miranda Reed, Western Washington University
Economic Drivers in Modeling Pandemics

Kermack and McKendrick's SIR model is a standard basis for epidemiological models. The model uses parameters, such as social distancing, vaccines, and treatments for a disease, to account for the aspects of our life that can affect the spread of a disease. Using this model, we find that social distancing is the most important factor in limiting the spread of a disease outbreak preventing an epidemic. Given the importance of social distancing, we expand the standard SIR epidemic model so it divides the population into three groups to account for the differences in possible disease exposure given an individual's occupation and economic status. Next, we worked to create a model of the economy that could be coupled to our expanded SIRD model. The levels of social distancing and economic activity for various groups of the population vary based on socioeconomic status. As such, our model will show that an individual's financial situation is an important predictor of how their health and future financial situation will be impacted by an epidemic. This helps us investigate whether public health policies are reasonable and effective in curbing an epidemic and mitigating its impact on the population as a whole.

Camille Renaud, Marist College
Recurrent Neural Network Models for Predicting Ordinary Differential Equation Dynamics

Recurrent neural networks (RNN) are a class of graph-theoretic machine learning models loosely inspired by neurobiological processes. RNNs, which "learn" via vector inputs of variable length, have remarkable capabilities in applications modeling sequential data ranging from machine translation to speech and image recognition. In this talk, we apply RNN techniques to the numerical solution of certain systems of ordinary differential equations, and address whether claims of model stability made in published and unpublished work in the field can be rigorously defended.

Eliana Rutherford, Concordia College, Moorhead
see **Uyanga Naranbaatar**

Shraddha Shankar, Denison University
Random Walk on the English Language Dictionary

The English language dictionary, consisting of approximately 171,000 words, is a common tool for linguists, writers, and Scrabble players everywhere. We take the English language dictionary and study it from a mathematical perspective by phrasing it as a random walk problem. A random walk on the dictionary would consist of a random walk on a graph where nodes represent words that are connected if they share some property. For example, we could define two words to be connected if they share any common letters. In order to create and study such graphs, we begin by analyzing random walks on a much smaller subset of words. We analyze such properties of the random walk as time to stationarity, hitting time, commute time and cover time. These properties provide understanding toward answering questions such as what percentage of words our random walk covers and where we would have to start to cover a maximum number of words. We talk about our progress toward extending these smaller problems to a random walk on the entire English language dictionary.

Rose Silver, Northeastern University
Chains and Antichains in the Bipartite Cambrian and Tamari Lattice

Dillworth's Theorem states that the maximal size of an antichain is equal to the minimal number of chains needed to cover the partially ordered set. We study the Greene-Kleitman partition of c-Cambrian lattices of type A. We partially compute the Greene-Kleitman partition of the Bipartite Cambrian Lattice. We also study the Greene-Kleitman partition for the Tamari Lattice, a popular poset given by triangulations of polygons and the Catalan numbers. We give a lower bound for the size of a largest antichain for the Tamari Lattice.

Gabrielle Stokely, Lawrence University
Critical groups of directed cones over cycle graphs

For any graph, we can determine a group known as the critical or Jacobian group, which takes the form of a Cartesian product of cyclic groups. The cone of a graph is the graph formed when a vertex is added and connected by an undirected edge to each existing vertex. We look at a specific class of directed graphs and the affect on the Jacobian group when we take the nth iterated cones of such graphs.

Miriam Syvertsen, Lawrence University
Jacobian Groups of Bipartite Digraphs

With a graph we can associate an algebraic structure called the Jacobian group, which can be expressed as a Cartesian product of cyclic groups. Patterns exist among the Jacobian groups of graphs within a given type. Some of these patterns are readily discovered and proven, while some require more investigation. We present one new such pattern regarding the Jacobian groups of complete bipartite digraphs on n, n vertices that were modified by removing a single directed edge.

Kelemua Tesfaye, Seattle University
Knotris Gameplay and Probability

Knotris is a new game we have developed based on mosaic knot theory. In this talk, I will discuss probabilities related to gameplay and how they give us insight into game strategies. I will also describe how coding has helped us to determine both strategy and which game states are more difficult to play within. This analysis has informed how scoring works in the game.

Yue Wang, University of Rochester
Relativistic Brownian Motion

The displacements of a particle from classical Brownian motion forms a Gaussian distribution. However, the distribution of variables drawn from a distribution with infinite variance as opposed to a finite variance in classical Brownian motion is not Gaussian. We show that the sum of a large number of log-normal variables tends to a Levy sub-ordinator distribution. This distribution is surprisingly common in real-life stochastic systems such as the stock market and relativistic Brownian motion.

Devon Waskiewicz, Furman University
Finding Minimum Dominating Sets on Three-Dimensional Grid Graphs

A dominating set in a graph is a set of vertices with the property that every vertex in the graph is either in the set or is adjacent to a vertex in the set. A minimum dominating set of a graph is one whose cardinality is as small as possible, and the cardinality of such a set is called the domination number of the graph. In general, finding the domination number of large graphs can be computationally difficult. In this project, we considered a specific family of graphs: three-dimensional grid graphs. Our method was to create integer programming models that would attempt to optimize placement of dominating vertices in the grid. The results led to the identification of patterns, and the patterns in some cases led to proofs of exact numbers. In this talk, I will discuss our models, demonstrate some interesting patterns, establish some exact results, and conjecture several others.

Ashley Weeks, Furman University
Using statistical models to analyze trap color and location for effective capture of Velvet Longhorn

Velvet Longhorned Beetles, an invasive species native to Asia, have spread to North America and are causing damage to orchards, forests, and other plant life. In order to mitigate their destructive habits, we worked with researchers to evaluate different traps used to capture the beetles. Specifically, we compared the trap color (black, purple, green) and trap location (ground versus raised). To assess which traps were most effective, negative binomial mixed effects models were fit to these data, which indicated that black traps and high traps proved to be the most effective. The United States Department of Agriculture funded this research, and the results of this study will be submitted for publication early next year.

Ally Wilson, College of Saint Benedict
Minimal Base Sizes of Symmetric Groups

This research explores how symmetric groups interact with different sets including: 2-sets, 3-sets, and 4-sets. More specifically, it seeks to find the minimal base size of symmetric groups acting on 2-sets, 3-sets, and 4-sets. To achieve this goal, the project focuses on the number of elements necessary to form a base for each set. Using this idea, lower bound formulas were created. Looking at the arrangements of elements within the set allowed for the creation of upper bound formulas. Manipulating these formulas led to the development of the greatest lower bounds and least upper bounds. Through the use of these formulas minimal base sizes for 2-sets, 3-sets, and 4-sets were found.

Yue Wu, Carnegie Mellon University
A Quantitative Analysis on Bitcoin Perpetual Inverse Futures Contract

The perpetual inverse futures contract is a recent and popular cryptocurrency derivative on crypto exchanges. Though the contract is perpetual, exchanges implement a liquidation mechanism that terminates positions that no longer fulfill maintenance requirements. Therefore, understanding liquidation will help investors optimize the use of leverage for maximizing profitability and minimizing risks; it will also benefit researchers in understanding the design of crypto derivatives. In this study, I used both theoretical and empirical approaches to quantify how factors such as the leverage of a position and volatility of Bitcoin/USD exchange rate increase the risks of liquidation. To do so, I used regression, stochastic calculus, and simulation methods to estimate the expected time and probability of liquidation.

Andrea Wynn, Rose-Hulman Institute of Technology
Spectral Touching Points in 2 Dimensional Materials

In recent years, graphene has gained significant popularity as a building material and energy storage medium with a wide variety of applications. One of graphene's most unique properties is its conductivity, which is enhanced by mass-less fermions that enable loss-less electron transfer across a graphene sheet. Previous researchers (Novoselov, 2011) have conjectured a potential relation between a special type of spectral touching point, Dirac conical points, and the unique properties of graphene, although this has yet to be formally proven. Further research from T. Weyand (2014) and R. Martin (2017) found that variations of graphene also have these touching points. I expand upon this idea by searching for other materials possessing spectral touching points, which may indicate the presence of properties similar to those of graphene. I verify the existence of these touching points by modeling a material as a 2 dimensional infinite periodic graph, the spectrum of which can be found using Floquet-Bloch theory. I find the fundamental domain of this graph, find the corresponding magnetic flux Schrodinger operator, and then take the union over all possible values of magnetic flux. This allows me to graph the infinite spectrum and search for touching points, which indicate repeated eigenvalues. I then verify touching points by calculating the eigenvalues and eigenvectors at probable locations to show that there exist linearly independent eigenvectors for the same eigenvalue. In my presentation, I will explain how, using the methods described above, I found examples of these materials and proved the existence of various types of touching points within them under different symmetry conditions.

Lino Yoshikawa, University of Hawaii at Hilo
see **Vonnie Dobbs**

Ashley Zhuang, Harvard University
see **Emily Cairncross**