The Twenty-Sixth Annual Nebraska Conference for Undergraduate Women in Mathematics

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POSTER ABSTRACTS

Posters by Undergraduate Students

Emily Almgren, Haverford College
Dynamics of a Vibro-Impact Energy Harvester Under Non-Smooth Forcing

We study the dynamics of a ball-and-capsule vibro-impact energy harvester (VI-EH) under triangle wave forcing to contrast with ideal models based on harmonic forcing. We obtain a comprehensive bifurcation structure of our model via simulations, a semi-analytical approach based on nonlinear maps, and linear stability analysis of the system under both the triangle wave forcing and its smooth Fourier approximations. Across a range of relevant parameters, we observe and characterize general shifts of periodic solutions and the bifurcations to smaller capsule lengths. Further analysis of these bifurcation structures reveals novel phenomena not seen under harmonic forcing. Energy-harvesting analysis shows that low-order Fourier approximations provide an accurate estimate of the energy harvested under the non-smooth triangle wave. We find that the VI-EH is more efficient under the harmonic forcing than the triangle wave forcing in the same regime of motion, while the energy harvester generally remains in the more efficient regime of alternating periodic motion for a larger range of capsule lengths under the triangle wave forcing. This bridges the gap between previous work and experimental conditions.

Kathryn Altman, Southwestern University
Alleen Koenig, Southwestern University
Modeling the Optimal Growth of Gravitropic Respondent Solanum Pimpinellifolium Root Systems

The roots of wild tomato plants can be modeled as a Euclidean graph made up of the line segments comprising the main root and lateral roots. These biological networks attempt to minimize the total amount of material used (wiring cost) and minimize the time needed to transport resources to the base of the main root (conduction delay). These conditions compete against each other, therefore minimizing both simultaneously is generally impossible. In order to account for this trade off, we attempt to optimize the network under the theory of Pareto optimality. This talk explores the attempts of both analytical and algorithmic methods of generating a model for the growth of these roots under the effects of gravity. The addition of the gravity variable expands a previous model explored. We will discuss the creation and application of an algorithm to compute the Pareto front of root structures that achieve an optimal balance between wiring cost and conduction delay within a given set of gravitropic conditions. We will explore how the algorithmically modeled roots compare to actual scans of tomato roots. This talk will also discuss the progress made to find an analytical solution to this model and the challenges that face an analytically produced model.

Sami Aurin, Georgia Institute of Technology
The Curve Graph of the 5-Punctured Sphere

Hyperbolic metric spaces, as introduced by Gromov, have proven to be useful in geometric group theory. The curve graph $\mathcal{C}(S)$ of a surface S is the graph where vertices are curves and the edges represent disjointedness. It was proven by Hensel, Przytycki, and Webb in 2015 that $\mathcal{C}(S)$ is at worst 17-hyperbolic if S is connected, and we show that $\mathcal{C}(S)$ is at best 2-hyperbolic if S is the 5-punctured sphere. This research was done with Darrion Thornburgh at the Georgia Tech Math REU.

Ezra Aylaian, University of Maryland
Knot Corner-Mosaics

A knot mosaic is a grid of pictorial tiles representing a knot or link. At NCUWM 2023, Eric Rawdon, Elizabeth Paterson, and Sayde Jude introduced what we call knot corner-mosaics, a new set of tiles that also represent a knot or link. Aaron Heap, et al. independently introduced these tiles in a June 2023 preprint. We define the (corner) tile number, the minimum number of tiles needed to represent a knot as a (corner) mosaic. Our first result is a correspondence between mosaics and a certain kind of corner-mosaics, which has exciting implications for the tile number. Our second result is a classification of corner-mosaics with small tile numbers.

Isidora Bailly-Hall, Grinnell College
Virtual Resolutions of Points in Products of Projective Space

Projective space arises from the idea of perspective, where when we look into the distance it appears that parallel lines meet at infinity. The coordinate ring connects the geometry of projective space to algebraic structures. In this presentation we will define projective space and the coordinate ring before discussing free resolutions. Free resolutions are algebraic objects that encode many geometric properties. This correspondence lies at the heart of classical projective algebraic geometry. We will next show the shortcomings of free resolutions in products of projective space to motivate the introduction of virtual resolutions. Virtual resolutions were recently introduced by Berkesch, Erman, and Smith, and produce a similar correspondence for products of projective space. We will describe two methods for producing nice virtual resolutions for a finite sets of points in a product of two projective spaces.

Leah Bandy, Colorado School of Mines
Lattice-Boltzmann Modeling of Interstitial Fluid Flow over Ellipsoidal Osteocytes in Three Dimensions

Osteocytes play a pivotal role in bone growth and decay via mechanotransduction forces from surrounding interstitial fluid, but the precise mechanism remains unknown. Due to their location within mineralized matrices, in situ study is challenging, necessitating mathematical models to inform experiments. Our model explores the impact of osteocyte shape on interstitial flow-induced forces on the osteocyte surface. It features a single ellipsoidal osteocyte within an ellipsoidal lacuna with two cylindrical canaliculi along its major axis. We employ Lattice-Boltzmann equations (D3Q19) for fluid flow, utilizing bounce-back and Zou-He boundary conditions. Fluid stresses are extrapolated onto the osteocyte surface using Gaussian-weighted averages. Three trials with prolate spheroidal osteocytes of varying major axis lengths were performed, revealing that peak forces occur near the canaliculi, increasing with major axis length. Future work involves adding more canaliculi, roughening lacuna-canalicular surfaces, and simulating diverse osteocyte geometries. Long-term objectives encompass incorporating osteocyte processes and establishing a cell network. Refinements to this model may advance our understanding of osteocyte mechanosensation in bone physiology.

Lakrisha Berry, University of Central Oklahoma
From Tacos to Functions; Understanding Functions and Relations with Tacos Abstract

Intro to proof writing courses are commonly a student's first look at logic and more abstract ideas. These concepts are extremely important but can be difficult to grasp. While some texts use Venn diagrams for concepts with sets, there are few visualizations used outside of these and those that are tend to be abstract themselves. Our goal was to create effective visuals using graphic design techniques, along with the psychology of learning, to create images that will increase understanding of topics in intro to proof-writing classes. By using tacos and taco bars as fun overarching metaphors it's easier to see the relationship between these more abstract concepts. This poster will focus on the images developed for functions and relations. The images use color coordination and typography rules to make the concepts visually stimulating and easy to comprehend. A more cartoon-like aesthetic makes the images less threatening for students. While these initial visualizations were drawn with Intro to Proof Writing courses in mind, I believe these visuals can help anyone who is interested in understanding fundamental concepts pertaining to functions.

Evelyn Brylow, Northeastern University
Jordan Forms of Nilpotent Matrices that Commute

Matrices do not necessarily commute, and finding characteristics of matrices that do commute is an interesting topic. In this project, I will be exploring the polynomial ring $K[x, y]/I$ where K is an infinite field, and I is the ideal generated by the relations between two nilpotent matrices A and B. For some A, we can look at what characteristics are invariant between matrices B that commute with A.

Paola Campos, California State University, Stanislaus
Maximum Stars in Pendant Ladder Graphs

For a graph G, a family of independent r-sets is an r-star centered at v in $V(G)$ if v is in every r-set of the family. Since the intersection of any two sets in an r-star contains the center, an r-star is an example of a pairwise intersecting subfamily of the independent r-sets of G. Building upon the Erdös-Ko-Rado (EKR) theorem for sets, a graph is r-EKR if the size of every pairwise intersecting subfamily of independent r-sets is at most the size of an r-star. Holroyd, Talbot, and Spencer conjecture that G is r-EKR if r is at most the minimum size of a maximal independent set of G. Talbot suggests if a counterexample to their conjecture exists, it may be found in the class of well-covered graphs. We consider a subclass of well-covered graphs, namely pendant graphs denoted G^*, obtained by appending one pendant edge to each vertex in any base graph G. In our first result, we prove that the k-star centered at a pendant vertex in G^* is of maximum size if, for all r at most k, the r-star in the base graph G centered at the corresponding base vertex is of minimum size. In our second result, we determine the vertex in the pendant ladder graph which yields the k-star of maximum size by applying our first result in proving the r-star of the corresponding base vertex in the ladder graph has the minimum size for all r at most k.

Anastasia Chambers, Saint Mary's College
Kathleen Hurley, Saint Mary's College
The Shape of Libras: An Application of Topological Data Analysis

During our research we explored topological data analysis (TDA) specifically persistent homology and what sort of information can be learned through this new data analysis method. Topology is a field of math that, in brief, is the study of shapes and homology is a field of topology that works to classify shapes based on different characteristics called homology groups. These ideas have recently begun to be applied to data analysis with success using data from a number of fields. After gaining background knowledge on topology and homology we moved on to applying the ideas of persistent homology to a data set on Grammatical Facial Expressions in Brazilian Sign Language.

Aksheytha Chelikavada, Saint Louis University
Limit Theorems for Fixed Point Biased Permutations Avoiding a Pattern of Length Three

The study of the number of fixed points occurring in a uniformly random permutation has a long history going back to Montmort in the early 1700s. In this poster, we prove limit theorems to help understand the asymptotic behavior of pattern avoiding permutations biased by their number of fixed points. In particular, one case we study features a phase transition where the limiting distribution of fixed points changes abruptly from negative binomial to Rayleigh to normal depending on the bias parameter.

Cassi Chen, Institute for Pure & Applied Mathematics
Gauss-Markov Modeling of GPS Ephemeris and Clock Error

The Global Positioning System (GPS) is a Global Navigation Satellite System (GNSS) that consists of a constellation of satellites that broadcast signals to users so that they may compute estimates of their positioning, navigation, and timing (PNT). Additionally, the GPS ground control segment monitors the real-time position, velocity, and clocks of the satellites themselves. These estimates of GPS satellite ephemeris and clock obtained from the control segment play a crucial role in determining the accuracy of GPS user PNT solutions. In our work, we develop a Gauss-Markov model of the broadcast ephemeris and clock error. This statistical error model is generated from and validated against real errors of broadcast GPS ephemeris and clock information and can be used for uncertainty quantification of users' PNT. We initially employ an expectation maximization algorithm to refine our model parameters. However, recognizing the challenges of capturing the nonlinear relationships in the data, we also implement a Gauss-Markov model that utilizes the observed periodicity of the data and state augmentation. Moreover, our study extends its focus beyond Earth's orbit by investigating the position accuracy of GPS satellites relative to the proposed lunar space station, Gateway.

Ling Chen, Occidental College
Analogues of Alder-Type Partition Inequalities for Fixed Perimeter Partitions

In a 2016 paper, Straub proved an analogue to Euler's celebrated partition identity for partitions with a fixed perimeter. Later, Fu and Tang provided both a refinement and generalization of Straub's analogue to d-distinct partitions. They also prove a related result to the first Rogers-Ramanujan identity by defining two new functions, $h_d(n)$ and $f_d(n)$ for a fixed perimeter n, that resemble the pre-exisiting q_d and Q_d functions. Motivated by generalizations of Alder's ex-conjecture, we further generalize the work done by Fu and Tang by introducing a new parameter a, similar to the work of Kang and Park. We observe the prevalence of binomial coefficients in our study of fixed perimeter partitions and use this to develop a more direct analogue to Q_d. Using combinatorial techniques, we find Alder-type partition inequalities in a fixed perimeter setting, specifically a reverse Alder-type inequality.

Siran Chen, Carnegie Mellon University
Bounding Lifts of Markoff Triple mod p

In 2016, Bourgain, Gamburd, and Sarnak proved that Strong Approximation holds for the Markoff surface in most cases. That is, the modulo p solutions to the equation $x^2+y^2+z^2 = 3xyz$ are covered by the integer points for most primes p. In this poster, we show how the algorithm given in the paper of Bourgain, Gamburd, and Sarnak can be used to obtain upper bounds on lifts of Markoff triples modulo p. We provide numerical evidence that these bounds can be improved on average and with high probability, and present an implementation of the BGS algorithm. This is joint work with Elisa Bellah, Elena Fuchs and Lynnelle Ye.

Libbie Clevette, Doane University
Detecting Energy Consumption Anomalies Through Micro-moment Features

Anomaly detection within the data science field has been a flourishing field of study, especially in recent years. As the world moves into a more technologically advanced society, energy consumption is also advancing. The objective of this research, done during a summer research experience for undergraduates (REU) was to identify a more accurate and efficient method of identifying these anomalies, while extracting micro-moment features. Previous work has been conducted consisting of different machine learning algorithms, both supervised and unsupervised algorithms, to identify these anomalies. While previous research has been successful, such as creating a Deep Neural Network with 99.58% accuracy, we wanted to see if we could find a more accurate algorithm. We used a dataset provided by Qatar University known as (QUD), which is a simulated dataset consisting of energy consumption data in houses. We implemented four different machine learning algorithms, two supervised and two unsupervised algorithms. Our findings indicate that the four algorithms we implemented did not perform better than previous algorithms researched, as accuracy and F1 scores of algorithms were much lower than previous algorithms.

Karen Cordova, Wellesley College
A Complete Match: Deconstructing a Scheduling Problem Using Graph Theory

Suppose a professor is teaching a class with an even number of students. To avoid panic when tasked with finding a lab partner, the professor creates a schedule so that students have an assigned partner every week. In a class with an even number of students, it would be possible for the professor to designate partners so that each student is paired with everybody else in the course exactly once. However, after the end of the add/drop period, an odd number of students remain in the class and they are now assigned to groups of two and a group of three. The goal remains the same; the professor wants to ensure that every student has been partnered with everybody else in the class exactly once. We prove that this is possible only when the class has three or nine students and otherwise impossible.

Kassidy Crockett, University of Central Oklahoma
Modeling Bioaccumulation Environmental Contaminants in a Consumer-Resource System

Bioaccumulation is the process where a single organism absorbs a substance at a faster rate than they excrete it, resulting in a higher concentration of the substance. Biomagnification is the growth in the concentration of the substance as it transfers from lower to higher trophic levels. In tandem, bioaccumulation and biomagnification have led to significant environmental impacts due to the growing amount of organic pollutants. In recent years, there have been major discoveries in the rates of bioaccumulation related to pollutant exposure. This project uses differential equations models to track the concentrations of environmental contaminants (e.g., PFAS and Hg) in a simple food web. With numerical solutions, the dynamics and factors that drive biomagnification are illustrated in a consumer-resource system.

Danielle DaSilva, Elon University
Comparing Approaches to Mathematically Modeling Obesity Trends in the United States

The prevalence of obesity has drastically increased over the past several decades. This has caused strain within the healthcare system as obesity puts individuals at an increased risk for a variety of diseases and conditions. This project aims to compare various mathematical approaches to modeling obesity trends within the United States. The ideal approach would create a model that mimics the overall obesity trends in the U.S., is able to be parameterized to fit the obesity trends in different regions of the U.S., and could take into account how obesity varies due to societal factors. Factors such as socioeconomic status, income, race, ethnicity, geographic location, and personal connections have all been shown to contribute to one's likelihood of experiencing obesity. This project first looked at the overall trends of obesity, and how they have changed over time utilizing linear regression models. Then, since the rise in obesity levels has been theorized to mimic the spread of infectious diseases, an SIR-type model was created. Finally, agent-based modeling strategies were employed to create a probabilistic model of obesity trends. The benefits and disadvantages of each of these models will be compared. Developing these models will enable the investigation of various intervention strategies to reduce obesity levels within the U.S.

Armelle Duston, Colorado School of Mines
Don't PASA Your Time Writing Down Sensitivity Equations!: Automated Sensitivity Analysis for Dynamical Systems

Systems of nonlinear differential equations are a powerful modeling tool for a variety of physical and biological phenomena. Understanding how changes in the parameters of a system affect the solution can provide meaningful insight into the dynamics of the system. The technique of local sensitivity analysis tells us how sensitive the solution is to changes in the model's parameters at the current parameter values. For example, we can use this technique on systems which describe the spread of a disease to analyze how sensitive the final solution is to things like infection or recovery rates. The process of finding the sensitivity equations for a system is computationally expensive, so the purpose of this research was to write software in Python which automates the process. The software, called Python Automated Sensitivity Analysis (PASA), is user-friendly and produces visual output and/or a spreadsheet containing all of the output data. In this presentation, it is demonstrated how PASA can be used in an mathematical biology context with a very simple and a more complex example, for which would be nearly impossible to find and solve the sensitivity equations otherwise.

Sierra Edelstein, University of Florida
The Mathematics of Crochet and Knitting

Mathematicians from around the world have developed methods of transforming complex and abstract mathematical concepts into figures that can be realized through methods of crochet and closely related yarn-techniques, like knitting. These models can be found in a variety of disciplines within math, including, but not limited to, hyperbolic geometry, abstract algebra, and topology. I highlight various applications of crochet to represent certain mathematical concepts in a geometric manner. In particular, I focus on developing and understanding models of Hyperbolic Planes developed by Daina Taimina in her book "Crocheting Adventures with Hyperbolic Planes;" representations of a specific type of fractal, the space-filling fractal, with crochet and techniques of knitting torus knots and links developed by Kyle Calderhead and Sarah-Marie Belcastro, respectively, in the book "Figuring Fibers;" and a representation of the group of symmetries of a regular hexagon developed by Andrea Heald. These examples give a concrete view of the power of crochet in understanding and exploring mathematical concepts, with potential to be utilized in a classroom setting to give students an intuitive, hands-on understanding of new mathematical ideas that may seem abstract. The purpose of this presentation is to inspire mathematical creativity in the development and utilization of models to bring abstract mathematical concepts to a broader audience through the exploration of professional projects.

Zoe Erpelding, Gonzaga University
Diana Hoppe, Gonzaga University
A Combinatorial Study of the Game Triominoes

Triominoes are a variant of dominoes using equilateral triangular tiles. The corners of each tile are labeled by integers from the set $\{0, 1, 2, 3, 4, 5\}$ following certain rules. Each allowed tile appears exactly once in the game set. In this presentation, we discuss why these rules and game size are important, as well as present several combinatorial properties of these triominoes, including a counting of generalized triominoes and the implications of playing the game in the more general version.

Nayda Farnsworth, Colgate University
Marisa Zarcone, Colgate University
Reconstructing Monomial Orders

Ordering systems are invaluable in our day-to-day lives. Unsurprisingly, ordering systems also play a central role in our comprehension of mathematics. From basic counting systems to our topic of discussion, monomial orders (the ranking systems for monomials, algebraic expressions consisting of one term), ordering systems are prevalent in every area of math. Monomial orders themselves also play a pivotal role in mathematics, especially in the computation of a Gröbner Basis, a central idea in the fields of algebraic geometry and combinatorial algebra. Throughout this past summer, we explored the question of reconstructibility of monomial orders. Every monomial order (in two or more variables) has a set of smaller, or 'induced,' associated orders, each ranking the monomials missing exactly one of the variables for which the actual order is defined. We ask, which monomial orders can be reconstructed from their induced orders? In other words, given a collection of orders in $n-1$ variables, is there a unique monomial order τ in n variables such that the induced orders of τ correspond to the collection? Furthermore, we explore the question: Given a collection of induced orders, is it possible to find a monomial order for which they are induced orders? If so, how? What are the necessary and/or sufficient conditions that would allow us to do so? Throughout this presentation, we will discuss our approach to these challenges and the results of our research findings.

Amber Fisher, University of Montevallo
Congruent Number Problem

We will be exploring the congruent number problem, which is determining whether a given natural number n is congruent or not. A natural number n, is considered a congruent number if you can find a right triangle with area n that has all rational sides. In other words, we are trying to satisfy the following equations: $a^2 + b^2 = c^2$ and $ab/2 = n$, where $a, b, c \in \mathbb{Q}$. It can be proved that there is a one-to-one correspondence between the following sets: $\{(a, b, c) : a^2 + b^2 = c^2, ab/2 = n\}$ and $\{(x, y) : y^2 = x^3 - n^2x, x, y > 0\}$. This connection, along with Tunnell's Theorem and the Birch-Swinnerton-Dyer Conjecture, allows us to begin determining whether a given number is congruent.

Galileo Fries, Colorado College
The Geometry of Small Chemical Reaction Networks

Chemical reaction network theory is used in many areas of science, for example in systems biology, to study cellular signaling pathways, and in epidemiology, to study the effect of human interaction on the spread of disease. Various mathematical tools can be used to analyze chemical reaction networks, including graph theory, dynamical systems, and combinatorics. Our research focuses on understanding a reaction network's equilibrium points through the lens of algebraic geometry; under the assumption of mass-action kinetics, a network can be translated into a system of polynomial equations whose solution set is an algebraic variety. Of particular interest are the components of this variety which nontrivially intersect the positive orthant, as they record all biologically relevant steady states. Currently, there is no algorithm to calculate a network's positive steady-state variety in general. We provide a systematic classification of positive steady-state varieties produced by 2-species, 2-reaction networks, grounded in combinatorial and algebraic characteristics of the networks, with the goal of providing a foundation for the future analysis of larger networks.

Mengchan Gang, Mount Holyoke College
Integers, Permuted Digits, and Averages

This presentation starts with the following example: Begin with the positive integer 629 and write down the permutation of its digits, i.e. all ways to write a 3-digit number using the digits 6, 2, and 9. In this case, they will be 629, 692, 269, 296, 926, and 962. The average of these six permutations is $3774/6 = 629$ which is one of the permutations. Along with this example, I will present additional cases of numbers that also have this property. This presentation is the result of an investigation of why some integers have this property and others do not.

Janelyn Geronimo, University of Denver
Designing an Introductory Course on Computational Sociology

Computational sociology is a field that combines computer science and sociology through creating computer simulations. These simulations are based on theories and frameworks from sociology and are modeled through code. Through learning about computational sociology, students majoring in computer science or in sociology will be able to see connections between the social sciences, math, and computer science. Models in computational sociology are agent-based, meaning that they focus on how interactions between agents create larger patterns within the model. The agents are given simple rules, and the resulting patterns that arise from the interactions between agents is called emergent phenomena.

Makenna Greenwalt, University of Oregon
Composition of Maps for Curves Defined Over $\mathbb{Q}$

Given a Riemann surface C, Belyi's Theorem states that C is defined over $\overline{\mathbb{Q}}$ if and only if there exists a map $\beta : C \to \mathbb{P}^1$ such that all critical values of β are in the set $0, 1, \infty$. Such a map is called a Belyi map, and if it is "clean," the inverse image $\beta^{-1}[(0, 1)]$ can be realized as a graph on the original surface C. By composing arbitrary Belyi maps with constructed maps from $\mathbb{P}^1$ to $\mathbb{P}^1$, we prove that there always exists a Belyi map from a surface C defined over $\overline{\mathbb{Q}}$ that can be realized as a face-bipartite triangulation of C.

Annika Griffith, Concordia College
Data Analytics of Anxiety and Memory in Zebrafish

The focus of this project is to learn data analysis skills and techniques by analyzing datasets from undergraduate neuroscience research we conducted. The data is analyzed to evaluate whether zebrafish (Danio rerio) respond to environmental enrichment through modulation of behavior. In this study, we exposed adult leopard zebrafish (Danio rerio) to Vivaldi music and hypothesized that compared to controls, the enriched fish would demonstrate less anxiety, stronger memory, and less erratic swimming behavior. A video tracking system was used to record data from various behavior tests of individual fish, and relevant data was analyzed in RStudio. We will share results from the study, comparing pre- and post-enrichment memory and anxiety-like behavior, and discuss generating data visualizations.

Caroline Guiler, United States Air Force Academy
Utilizing a Machine Learning Approach with Agriculture Data

This study leveraged the Department of Agriculture's Feed Grain Database, a comprehensive repository of data pertaining to grain production and exportation, to develop a predictive machine learning model. Employing Python, the model was trained using a range of statistical and machine learning techniques to enhance its accuracy. The primary objectives were to construct a reliable predictive model for grain production and export trends and identify critical variables influencing these trends. This research has the potential to inform decision-making processes in the grain industry, offering insights to policymakers and stakeholders and aiding in the development of effective policies and practices for ensuring a stable and prosperous future for the grain industry, both domestically and in international trade.

Cameron Hahnfeldt, College of Saint Benedict
Finding Minimal bases for Symmetric Groups Acting on k-sets

We are looking at the action of $\mathrm{Sym}(n)$ on k-sets. For example, when $k = 3$ and $n = 5$, the element $(1, 4, 2)(3, 5)$ moves the 3-set $\{1, 2, 3\}$ to $\{4, 1, 5\} = \{1, 4, 5\}$. A base of this action is a set S of k-sets such that the only element of $\mathrm{Sym}(n)$ that fixes every k-set in S is the identity. Our goal is to find the size of a minimal base for every pair (n, k).

Emma Hayes, Carnegie Mellon University
Surrogate Modeling of PDEs and Applications to Inverse Problems

Neural networks have become a powerful tool to provide numerical solutions for scientific problems with increased computational efficiency. This efficiency can be advantageous for numerically challenging problems where time to solution is important or when evaluation of many similar analysis scenarios is required. In this work, we considered solutions of the 2D acoustic wave equation where pressure and velocity vary over space and time under the influence of a driving force at a source point. We trained neural networks on data generated with a single driving force at one source point, as well as on data with multiple source locations. Our ground truth data was generated on a coarse grid using a Discontinuous Galerkin method. We show that our neural networks are able to produce an accurate solution over a square domain over a fine grid roughly 300 times faster than the Discontinuous Galerkin method on a single source. Moreover, we study the performance of the network with physics-based regularization terms added to the loss function using a variety of schemes. We outline some of the potential failure modes of such methods, particularly in the presence of a forcing term. Finally, due to the computational efficiency and accuracy achieved via neural networks, we show that the inverse problem of detecting source location from the pressure signals at given receiver points can be solved in a Bayesian setting by using MCMC methods.

Diana Hoppe, Gonzaga University
see **Zoe Erpelding**

Anastasia Horne, Colorado School of Mines
Determining Visibility of Buried Objects Through Temperature Difference

The ability to detect buried objects is critical for the army. This research is apart of an ongoing study to create a machine learning algorithm that accurately predicts the existence of a buried object. Buried objects produce a thermal signature that is of a different magnitude to the surrounding soil. Using image smoothing and detection techniques to enhance the visibility of these objects, and calculating the temperature difference between the object and its background we can determine a temperature difference threshold, that when exceeded allows the object to be visible. The focus of this poster is to determine that temperature difference threshold using 2018 longwave infrared data collected from a testplot at CRREL.

Emily Huang, University of North Carolina at Chapel Hill
Fast Haar Transformation in Dependency Test

Compared with other basis choices, the non-standard Haar wavelet basis offers optimal local and sparse properties. It is naturally associated with an adaptive hierarchical tree structure, making it an ideal choice for analyzing high-dimensional data sets located on a low-dimensional manifold. However, to compute a number K of Haar wavelet expansion coefficients from N data points, a direct numerical method would require an expensive $O(K \cdot N)$ operations. In this research, we present a fast Haar Transform with asymptotically optimal complexity $O(N + K)$. The new algorithm uses a divide-and-conquer strategy and processes the data efficiently on a hierarchical tree structure. The algorithm collects information directly from the data for the leaf nodes and stores the information in a compressed form. For each parent node at coarser levels, information is only collected from its four children nodes and the collected information is further compressed. The compression and recycling strategies make the fast Haar transform algorithm approximately K times faster than the direct method. Numerical simulation results are presented to demonstrate the algorithm's efficiency, accuracy, and asymptotic complexity. The developed Fast Haar Transform package is a powerful tool in the frequency domain-based statistical analysis in high dimensions.

Kathleen Hurley, Saint Mary's College
see **Anastasia Chambers**

Hala Jahjah, University of Oklahoma-Norman
Application of Transformation Matrices in Developing a Forward Kinematic Model for the RobotGeek Snapper Arduino Arm with 5-DOF

In this presentation, a comprehensive exploration of the forward kinematic modeling of the RobotGeek Snapper Arduino Arm with 5-DOF is presented using homogeneous transformation matrices. Through design intricacies, the Denavit-Hartenberg (DH) method is utilized to determine joint angle vectors and establish coordinate frames. For joints whose frames do not comply with the DH convention, alternative tactics are applied, and these are highlighted in the presentation. Using this systematic approach, direct kinematics is elucidated, with the aim being to accurately determine the final position and orientation of the manipulator arm's end-effector in relation to joint angles and a coordinate system. The model is embodied in a MATLAB application, where the theoretical constructs are not only verified but also a dynamic tool for streamlined analysis is provided.

Nzingha Joseph, Carleton College
The Stable Tamari Order

The classical Tamari order is a poset of Catalan objects that is useful across algebraic combinatorics. Recent work in this area has been motivated by Mark Haiman's study of trivariate diagonal harmonics using the Tamari lattice. Early generalizations of the classical Tamari order include the r-Tamari order developed by Francois Bergeron, a poset of r-Dyck paths represented by their area sequences. Motivated by the construction of the r-Tamari order, we present a new generalization of the classical Tamari order, the stable Tamari order, a generalization of the classical Tamari order to all non-negative integer sequences. We draw connections between the algebraic and combinatorial properties of the stable Tamari order, showing possible links to graph theory with a preliminary result. This presentation is based on work conducted at the NYC Discrete Math REU at Baruch College (CUNY), in Summer 2023. This work was mentored by Professor Anna Pun (Baruch College) and conducted in collaboration with Anna Hugo (Davidson College).

Anya Kapitula, Hope College
Identity Development Among Pre-Health Students: Identifying Hidden Groups and Transitions Between Them Via Latent Class Analysis in Survey Data During the COVID-19 Pandemic

Sociology studies have been published regarding the development of medical school students, but there is a gap in research observing undergraduate students on pre-health professions tracks. Previous studies have noted a significant empathy decrease in medical school students during their third year, but no research has been conducted to identify development patterns of these students during their undergraduate years. This study aims to identify groups of undergraduates on pre-health professions tracks based on typologies formed from longitudinal survey responses and to identify any significant transitions between these groups over time. Given this study spans from 2019 to 2022, we can track student's experiences across the COVID-19 pandemic. Using our longitudinal models to compare the transitions between groups across different cohorts, we address whether the pandemic had a significant impact on the identity and empathy development of undergraduate pre-health students. Latent Class Analysis (LCA) was used to categorize our sample based on their response patterns to survey questions regarding religion, political views, moral foundations, empathy, and demographics. After comparing transitions across the groups discovered through LCA, we found significant differences in transitions between identity groups across the cohorts. Further research can be done to see if these variations in development have an impact on these individuals' future identities and empathy levels.

Camille Kennedy, Northwestern University
Subsquares in Random Latin Squares and Rectangles

A $k \times n$ partial Latin rectangle is C*-sparse* if the number of nonempty entries in each row and column is at most C and each symbol is used at most C times. We prove that the probability a uniformly random $k \times n$ Latin rectangle, where $k < (1/2 - \alpha)n$, contains a βn-sparse partial Latin rectangle with ℓ nonempty entries is $(\frac{1 \pm \varepsilon}{n})^\ell$ for sufficiently large n and sufficiently small β. Using this result, we prove that a uniformly random order-n Latin square asymptotically almost surely has no Latin subsquare of order greater than $C\sqrt{n \log n}$ for an absolute constant C.

Alleen Koenig, Southwestern University
see **Kathryn Altman**

Isabella Kulstad, Tulane University
Identifying the Downstream Effectors of Tle4-Mediated reprogramming

Cortical Progenitors in the ventricular zone differentiate to form the cerebral cortex, a six-layered dorsal structure involved in decision making, sensation, attention, and memory. Projection Neurons, or CPns, are a distinct class of neurons involved in communicating electrical impulses between the cerebral cortex and distal regions of the Central Nervous System (CNS). The question of how corticospinal projection neurons are involved in motor function and represents a fundamental and clinically important question in neurodevelopment. A network of transcription factors, including the transcriptional co-repressor Tle4, are central to specifying cortical projection neuron fates and identity. In this study, we investigate the role of Tle4 during embryonic development and post-natal circuit maturation. Utilizing a full stack RNA Sequencing analysis, we explore the identified transcriptional regulator Tle4 controls downstream gene patterning, causing reprogramming. Gene Ontology is later used to specify biological functions impacted by the reprogramming, and specific 4 cellular functions also impacted by reprogramming (Slit/Robo, Cadherin, Wnt Signaling, Angiogenesis). To further characterize downstream genetic patterns, we utilize a transcription factor analysis to identify motifs associated with loss of layer 6 and gain of layer 5 genes.

Ruiqi Lin, Wake Forest University
Invariants of Hopf actions on skew polynomial algebras

Let $\Bbbk$ be a field of characteristic $p > 0$, and let U be the restricted enveloping algebra of the 2-dimensional nonabelian solvable Lie algebra; U is a non-semisimple Hopf algebra of finite representation type with no-nontrivial grouplike elements. H.-X. Chen, D.-G. Wang, and J.J. Zhang have classified all 2- and 3-dimensional quadratic skew polynomial algebras T on which U acts non-trivially. We compute the associated invariant subrings T^U and explore their properties, including whether T^U is again a skew polynomial ring, as well as bounds on the degrees of a minimal set of generators of T^U.

Neely Lovvorn, University of North Alabama
Getting Your Hands Dirty in the World of Abstract Math

The purpose of my research was to create rigorous definitions, proofs, and theorems for concepts regarding permutations without using or being introduced to group theory. In group theory, concepts about permutations are easily derived without the necessity of full step proofs. They are based on intuition. The foundation given was that a permutation in S_n is a bijective function from $\{1, \ldots, n\}$ to $\{1, \ldots, n\}$. The final goal was to prove that every permutation is a cycle or a product of disjoint cycles. I worked for 6 weeks making definitions, theorems, and proofs to use to prove the final goal. My team was successfully able to come up with our own definition of a cycle to use in our proof as well as 10 total other lemmas and theorems that we proved and came up with by recognizing patterns, or realizing that if it could be said that something was true, it would help prove something else. Although, some of these lemmas and theorems were not used in proving the final goal. We proved that every permutation is a cycle or a disjoint cycle by using strong induction. We made a lemma to prove our base case: If a permutation has 2 elements, then it's a 2-cycle. Then we took our permutation and essentially "factored out" the first cycle by constructing a function to equal the first cycle and then proving that it is indeed a cycle. We then let whatever we didn't factor out be a different function and prove that it is a product of cycles. Finally, since we used strong induction, we were able to conclude our original function is a product of cycles.

Natalia Luna, Saint Mary's College
Belief Evolution Over Time in Social Networks

Evolution of beliefs of a society is a result of interactions between people in the society over generations. We analyze the long term dynamics of belief evolution by combining people's prior beliefs, social dynamic network structures, and the confusion that occurs between beliefs. The main contribution of this work is threefold. First, we explore the belief evolution using existing network models, such as scale free networks and small world networks, to create social network structures and belief confusion structures. Second, we model the belief evolution with homophily based models using different statistical distances. We compare the individual and societal belief distributions and trends obtained from different models. Third, we explore the evolution of religious affiliations in different countries; both large and small in size, located in different continents. We use a homophily based model to fit religious affiliation data to model the dynamics of religious beliefs of Australia and Canada over time.

Kayley McBride, University of Central Oklahoma
Baseline Agent Based Model of Fibrin Polymerization

According to the American Society of Hematology, roughly 100,000 people in the United States die from a blood-clot related complication annually. The formation of a blood clot is an intricate process with many steps, leading to the formation of a mesh-like structure of fibrin that holds the blood cells in place and creates a strong seal. The formation, structure, and kinetics of fibrin polymers have previously been studied using computational models that either contain a lot of detail but are unable to run for larger spatial and temporal scales, or are able to run for larger spatial and temporal scales at the expense of biological detail. We aim to create a 2-dimensional agent-based model that splits the difference between the extremes and includes thrombin-induced activation of fibrin, fibrin polymerization, and lateral aggregation. We present the ground-level version of this model and discuss the steps that are underway to make the model more physiologically relevant.

Morgan McCaskill, Francis Marion University
Conservation Element and Solution Element Method Applied to Euler Equations

We review the conservation element solution element (CESE) method with respect to scalar and vector differential equations. We present the application of the method to the Euler equations and a shock tube problem with respect to work done by Sin-Chung Chang. We present Python code and results of the algorithm's application and present future work.

Hannah Meit, Rhodes College
Computing Formulae for $\Gamma-$Orbifold Euler Characteristics of $O(2)$ Representations

Lie groups are, in essence, groups of continuous symmetries. Consider a circle; an arbitrarily small rotation will still preserve its structure. This "structure" is a somewhat abstract concept to consider until the group is assigned a representation. For our purposes, a representation is a combination of matrices that describe group actions, like rotations and reflections. Given a compact Lie group G, a finitely presented discrete group Γ, and a representation X of G, the Γ-orbifold Euler characteristic $\chi_\Gamma(G \ltimes X)$ is a topological invariant computed by determining the orbit types of X, each of their orbit spaces, the spaces of conjugacy classes of each isotropy, and the Euler characteristic of each of these components. We will discuss recent results in computing formulae for $\chi_\Gamma(G \ltimes X)$ with G as the group $O(2)$ of 2×2 orthogonal matrices acting on an arbitrary representation of $O(2)$, as well as the consequences of certain restrictions to G-invariant subsets of X and changes of Γ.

Sarah Mirrow, Elon University
Investigation of Silent Transmission: Mathematical Modeling Incorporating Asymptomatic and Presymptomatic Spread of COVID-19

Since appearing in the United States in January 2020, COVID-19 has caused deaths and medical care shortages and cost billions of dollars in vaccine research. COVID-19 is especially dangerous because it can spread through silent transmission; people with mild or no symptoms can unknowingly infect others. This study uses two SIR-based differential equations models to investigate the impact of silent transmission on the spread of COVID-19 in early 2020. Through reconfiguration of the basic SIR model into SEAIR and SEPIAR models, this poster considers both viral latency and symptom severity to examine how presymptomatic and asymptomatic populations impact the transmission of COVID-19. Both models were compared to data from the CDC measuring new daily cases in the early months of the pandemic. Visual sensitivity analyses found that changes to infected and presymptomatic parameters had the most impact on model output. Finally, the basic reproduction number, R0 was calculated for both models: 3.65 for the SEAIR and 3.098 for the SEPIAR. Comparison between the models has demonstrated the importance of accounting for a presymptomatic period when modeling COVID-19. As many preventative measures are oriented toward different symptomatic severities, such as isolation of severely symptomatic individuals or preemptive testing of individuals without symptoms, this understanding of the silent transmission of COVID-19 is crucial in determining how the virus spreads and how to prevent it.

Maja Palmroos, Denison University
Seeing is Believing: Applying Eye Tracking to COVID-19 Maps

The pilot study utilized a 10 person convenience sample and prompted the participants with two series of COVID-19 maps (univariate, bivariate) representing the same data. This exploration was conducted to determine if bivariate mapping is a potential tool for crisis data communication. Univariate maps display a single variable per map, where the legend is broken up by intervals. Bivariate maps display two variables per map, with a grid legend and can indicate correlation between the variables. The variables of interest were case fatality rate and vaccination rate. The participants were asked to respond to the questions on a monitor connected to a screen based eye tracker. The eye tracker recorded the participants eye movement to give insight on how information was viewed. The output from the eye tracker, responses, and mental cognitive load survey were used to generate preliminary conclusions through ANOVA analysis. There was no statistically significant difference in the cognitive load between the map stimuli, despite initial exposure to bivariate mapping. There were differences in fixations per second and perceived risk. The univariate series had more fixations per second, indicating participants skimmed the information on the map instead of focusing on areas of interest to respond to the question. This pilot study was conducted at the REU Computing for Health and Well-being 2023 at University of Iowa under the supervision of Dr. Juan Pablo Hourcade and Michalis Kantartjis.

Bhakti Patel, Francis Marion University
AI Consulting Project for CBEC Comparing Naive Bayes and Non Naive Networks to Mitigate Limitations in Collected Data

This research project focuses on using in-depth AI consulting for the Cell Biology Education Consortium (CBEC). The project explores how machine learning techniques can be leveraged to personalize the website experience. The project aims to explore the potential of Bayesian networks as a tool for predicting and modeling user behavior on the website. In this case, we compare two different Bayesian network models, a naive Bayes approach and a causal network, to predict a proxy for CBEC website traffic, and see which, if either, is better at predicting the target with a limited number of data points available. The target variable for these networks is the duration of a given user session on the website, with the hopes of identifying useful indicators of increased website engagement. Each model considers the same variables but structures the respective networks differently. Data was collected from Wix analytics, but required extensive data cleaning and preprocessing before analysis could be conducted. K-Fold validation was a crucial step in evaluating the performance of our Bayesian models developed using data collected from CBEC's website.

Reanna Pipes, University of Montevallo
What Makes Mutually Orthogonal Latin Squares?

A Latin Square of order n, $[LS(n)]$, is an $n \times n$ array filled with symbols $1, 2, \ldots, n$ such that in each row and column, each of the symbols 1 through n appears exactly once. But what I am focusing on is the Orthogonality of Latin Squares. Two Latin Squares L_1 and L_2 of order n are orthogonal if, when L_1 and L_2 are superimposed, the ordered pairs in the resulting array are all distinct. I more specifically want to focus on Mutually Orthogonal Latin Squares (MOLS), what makes them such, and how to find them with a given order n. A set $\{L_1, L_2, \ldots, L_n\}$ of Latin Squares is Mutually Orthogonal if every pair of Latin Squares in the set is orthogonal. Throughout this poster, I will be covering the topic of Mutually Orthogonal Latin Squares and for what values of n does there exist at least one pair of Latin Squares of that order that are orthogonal.

Bridget Rozema, Grand Valley State University
Maisie Smith, Grand Valley State University
Edge Covers of Tadpole and Kayak Paddle Modifications

A graph mathematically models pairwise relationships between discrete objects. We can visually represent a graph to consist of vertices (dots) and edges (lines) that join or connect a pair of vertices together. An edge cover of a graph is a subset of the graph's edges chosen in a way so that each vertex is an endpoint of at least one edge in this subset. In this project, we studied the number of sequences formed by counting the edge covers in a graph family. Certain graph families, such as path and cycle graphs, give rise to known number sequences such as the Fibonacci and Lucas numbers, respectively. The graph families of tadpole and kayak paddle graphs also produce sequences that satisfy the same recurrence relation as Fibonacci numbers. Thus these edge cover totals give rise to new combinatorial interpretations of known sequences or generate new sequences. We will report on our results on the edge cover sequences for modifications of tadpole and kayak paddle graphs.

Renata Russell, United States Air Force Academy
A Sylver Variation on the Game of Nim

Nim is a two-person game in which the players take turns removing objects from a collection of piles. On their turn, a player must remove at least one object from any one of the piles. The player may remove as many objects as they would like provided they are all from the same pile. A player wins when they take the last object. Nim, with its traditional rules, is completely understood in that every position can be quickly determined as a winning position or a losing position and optimal plays for a winning position can also be determined. Motivated by John Conway's game of Sylver Coinage, we examine a variation of the Game of Nim in which the piles have "links" between them. That is, removing enough objects from one pile may automatically remove objects from another pile. To what extent does this change the classification of winning and losing positions?

Jane Santamore, United States Air Force Academy
Modeling the Competition Dynamics Between Immune Response and Melanoma

Melanoma is a highly aggressive cancer that has a potential to spread rapidly. Immunotherapy is a kind of cancer treatment that is designed to enhance the immune system's ability to destroy melanoma cells and prevent their spread. In this study, we consider a mathematical model that consists of seven nonlinear differential equations to simulate the immune response against B16-melanoma induced by the combined treatment administration of activated OT1 CTLs and immunostimulatory monoclonal antibodies. The mathematical structure of the model are built on two compartments, the injection point compartment where treatment is administered and the skin compartment where melanoma cells proliferate and where the competition between cancer and the immune system occurs. We produce simulations of the model using different levels of immunotherapy. We also perform stability analyses of some cancer-free equilibria and aim to find parameter values that guarantee its stability. Finally, we perform sensitivity analysis, via a differential technique, to understand how the system varies under different parameter values.

Maisie Smith, Grand Valley State University
see **Bridget Rozema**

Mariana Tecalero, Ocean County College
Discrete Analytic Power Series and Rational Functions on Uniform Rhombic Lattices

In this poster, power series expansion of discrete analytic functions on uniform rhombic lattices are constructed. Reproducing kernel Hilbert spaces of such functions, analogous to weighted Hardy spaces of continuous analytic functions are introduced. Further, discrete analytic rational functions defined in terms of the convolution product and system realizations are studied. This work was completed in collaboration with Zubayir H. Kazi under the supervision of Dr. Dan Volok at Kansas State University during the SUMaR 2023 REU program.

Thi Thanh Thanh Tran, Coe College
Digitalizing Coe College Catalog

The catalog is an important document for any school. It lists the requirements to graduate, courses required in a major, and the prerequisites for individual classes. Unfortunately, Coe's catalog is only available as either a printed document or in PDF. It is not web-friendly or easily searchable. This project would aim to translate the existing catalog into a more web-friendly standard, one that can be cross-compiled into a standalone PDF or a full website. With Quarto's versatile platform, we will digitize program details and other academic resources to enhance the student experience, improve information accessibility, and simplify the process of updates and revisions.

Abby Ulrich, Doane University
Data Analysis of Customer Feature Usage

This project was completed for an Omaha SAAS (software as a service) company helping companies plan their work, organize schedules and to-do lists, and interact with other companies and customers. Using Tableau, different features were analyzed to find which ones help the most with customer retention. In addition, a timeline was created showing when the customer used the features to try to find trends. After determining the most important features, we created cohort customer retention models in MSExcel analyzing how long customers kept their subscription based on which feature they used first, and establishing the value of a customer over time. We were able to determine certain features that were most impactful for customer retention and also found insights about the different customers who used only specific features.

Joanna Van Liew, Seattle University
Comparing Nonlinear Schrödinger Model Predictions with Experimental Observations of Deep Water Waves

The nonlinear Schrödinger (NLS) equation and its generalizations model the evolution of envelopes of wave packets on deep water. We wrote Julia codes that numerically solve the NLS, dissipative NLS, Dysthe, viscous Dysthe, and dissipative Gramstad-Trulsen equations. We used these programs to compare the predictions of these partial differential equations with the nondimensionalized experimental wave tank data collected by Dr. Diane Henderson at Penn State University. We found that the dissipative models (dissipative NLS, viscous Dysthe, and dissipative Gramstad-Trulsen) significantly outperformed the conservative models (NLS, Dysthe). Our goal is to compare these results with predictions from the new broadband generalization of the NLS equation derived by Yan Li from the University of Bergen.

Laura Vaughan, Vanderbilt University
p-Adic Trees and Iterative Sequences

The p-adic valuation, $\nu_p(n)$, is the highest power of a prime that divides an integer n, which can be extended to determine the valuation of the terms in an integer sequence and then visually represented with a tree. A p-adic valuation tree is composed of nodes which can be terminating, when every term in the node has the same valuation, or otherwise non-terminating. We investigate how nodes split on a variety of trees, such as those formed from a product of linear polynomials with integer coefficients and from iterative mappings of linear functions. We describe the infinite branches of these polynomial trees and the valuations of their terminating nodes. For iterative mappings, we conjecture their behaviors and whether the sequences of valuations are constant, periodic, or unbounded, depending on the coefficients and initial values of the linear function.

Bella Villarreal, Grinnell College
(Almost) All Transverse-Free Curves are Trivially Transverse-Free

We say that a curve C in P^2 defined over F_q is transverse-free if every line over F_q intersects C at some point with multiplicity at least 2. Adopting Poonen's notion of density for sets of homogeneous polynomials, we compute an asymptotic equality for the density of polynomials defining transverse-free projective curves, which mostly resolves the question investigated in a paper by Asgarli and Freidin. En route, we specialize the machinery used to compute such densities, including generalizing the methodology of the aforementioned authors. This allows the problem to be rephrased purely combinatorially as a counting question about special subsets of $PG(2, q)$, and we obtain our bounds by exploiting some basic properties of blocking sets from the literature. In particular, we define a class of trivial blocking curves that correspond to trivial blocking sets and show that nearly all transverse free curves are trivial blocking curves for sufficiently large q.

Madeline Vinal, University of North Carolina at Chapel Hill
Hitting Probability Metric for the Analysis of Gene Expression Data on Directed Graphs

Metrics intended for undirected graphs, such as the shortest path distance and generalized effective resistance, are often used in the analysis of directed graphs, but they often overlook crucial underlying information encoded in the graph's directionality. Using the recently introduced hitting probability metric, this project leverages the differences between the dynamics of random walks on these two different types of graphs. We developed a Python implementation of this new type of metric, intended for strongly connected, directed graphs. We compared our metric to the generalized effective resistance metric in two simple synthetic graphs: one undirected, the other directed. While the results for the undirected graph did not significantly differ between the two metrics, the synthetic directed graph's results did: the hitting probability metric produced from this network better captured the underlying topology of the graph than generalized effective resistance. In ongoing work, the hitting probability metric will be applied to a gene network centered on the important tumor suppressor p53 to analyze different sarcoma types. Although the gene network is directed, the prior study used the shortest path metric on an undirected version of the network. This project will compare the results of this prior study with those generated by the new metric and investigate whether the hitting probability metric is more effective than the shortest path metric.

Janine Wang, Williams College
Combinatorial and Analytic Properties of Higher-Order Recursive Polynomial Sequences

We consider two families of polynomial sequences defined by linear recurrences of the form:

$$F_n(x) = xF_{n-1}(x) + F_{n-a}(x)$$

with initial conditions $F_i(x) = x^i$ for $0 \leq i \leq a-1$, and:

$$C_n(x) = xC_{n-1}(x) + C_{n-a}(x)$$

with initial conditions $C_0 = a$ and $C_i(x) = x^i$ for $1 \leq i \leq a-1$. Both of these sequences have natural combinatorial interpretations that lead to novel identities, including a closed form for C_n given by:

$$C_n = \sum_{k=0}^{\lfloor \frac{n}{a} \rfloor} \left(a\binom{n-1-(a-1)k}{k-1} + \binom{n-1-(a-1)k}{k} \right) x^{n-ak}$$

as well as several other identities relating the sequences to each other and to other combinatorial numbers, such as integer compositions. Finally, for both F_n and C_n with arbitrary $a \geq 2$, we obtain a precise description of where their roots lie in the complex plane using previous work done by Boyer and Tran. We also show that for even a, there are no nonzero real roots, while for odd a, the sequences of minimum real roots of F_n and C_n both converge to

$$\frac{-a}{\sqrt[a]{(a-1)^{(a-1)}}}.$$

Lillian Whitesell, Rhodes College
Integrating $O(n)$-invariant Functions via the Hilbert Embedding

Let G be a collection of $n \times n$ invertible complex matrices and define $\mathbb{C}[x_1, \ldots, x_n]^G$ to be the set of polynomials in n variables that are invariant under G, meaning that they don't change when elements of G are applied to the variables. We are primarily interested in the case where G is isomorphic to an orthogonal group and the representation is k copies of its defining representation. A theorem of Hilbert guarantees that $\mathbb{C}[x_1, \ldots, x_n]^G$ is finitely generated. This means that it is the subalgebra of $\mathbb{C}[x_1, \ldots, x_n]$ generated by a finite set of polynomials $f_1, \ldots, f_k$. Such a generating set is called a *Hilbert basis*. Given a Hilbert basis, the *Hilbert embedding* (a map $\boldsymbol{f}\colon V \to \mathbb{R}^k$) can be defined. This allows us to think of the orbit space as a subset of $\mathbb{R}^k$. The goal of my research is to integrate functions that are invariant with respect to a group action over the orbit space via the Hilbert embedding. By the Schwarz-Mather Theorem, smooth invariant functions on V can be expressed as smooth functions of the invariant polynomials, and we will discuss progress on the question of how to integrate such functions on the image of the Hilbert embedding. This is known as the "canonical measure" or "canonical volume form" on the orbit space that pulls back via the Hilbert embedding to the ordinary volume form.

Alexandra Whiteside, University of North Carolina at Chapel Hill
A Mathematical Model of the Intrinsic Pathway of Coagulation

Coagulation, the process of blood clotting to control bleeding, underlies mechanisms that deposit fibrin by thrombin generation and activate and adhere platelets. Coagulation includes two main pathways: the intrinsic pathway, which is factor XII initiated, and the extrinsic pathway, which is initiated by tissue factor. The dynamics of the system rely on the presence of silica surfaces for many of the intrinsic reactions and lipid surfaces for many of the extrinsic reactions, which many current models fail to account for. We developed a mathematical model of the intrinsic pathway coagulation that incorporates the various reactions between key enzymes, zymogens, and cofactors. We use the mathematical model to investigate the effects of various perturbations on coagulation, including variability in initial concentrations of key factors and introduction of inhibitors, and thus the model is able to predict the behavior of the intrinsic pathway under certain conditions. We specifically focus on analyzing the dynamics involving the silica surface, which initiates the autoactivation of factor XII. Since it is known that blocking factor XI and its activity reduces venous thromboembolism (VTE), our focus also extends to the main inhibitor of the system, C1-inhibitor (C1INH). The model provides a tool for studying the kinetics of the intrinsic pathway and may have implications for the development of new treatments for factor XI-deficient disorders.

Anna Wolff, Drake University
Efficacy of Body Doubling with ADHD

The purpose of this research is to develop a better understanding of the way individuals use body doubling and show its efficacy, specifically for individuals with ADHD. Body doubling refers to when one person performs a task with another person present, which has been anecdotally shown to be effective. We surveyed over 60 people to get a sense of their history (if any) with ADHD, the ways that they use body doubling, and how effective it is with different tasks in their lives. Participants in this survey use body doubling on a wide variety of tasks – household chores, studying for exams, and crafts. This research is important because it gives us a better insight into the efficacy of body double for individuals with ADHD as a tool for others to use in the future.

Marisa Zarcone, Colgate University
see **Nayda Farnsworth**

Xingyi Zhang, Carleton College

Optimal Constructions for DNA Self-Assembly of K-Regular Graphs

Within biology, it is of interest to construct DNA complexes of a certain shape. These complexes can be represented abstractly in the language of graph theory, where edges are understood to be strands of DNA joined at junctions, represented by vertices. Because guided construction of such structures is inefficient, design strategies for DNA self-assembly are desirable. Branched DNA molecules are referred to as tiles, each consisting of flexible unpaired cohesive ends with the ability to form bond-edges. We thus consider the minimum number of tiles and bond-edge types to construct a graph G (corresponding to a DNA target structure) without allowing the formation of smaller graphs, or non-isomorphic graphs of the same size. In this paper, we specifically investigate the case where G is k-regular. We introduce the concept of (un)swappable graphs, using the property to establish lower bounds on bond-edge and tile types in the unswappable case. We also introduce a method of generating upper bounds using a vertex-cover model. We apply both of these methods to prove new bounds on a number of regular families, including crown graphs, prism graphs, Kneser graphs, Johnson graphs, antiprism graphs and Archimedean solid graphs.