The Twenty-Sixth Annual Nebraska Conference for Undergraduate Women in Mathematics

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TALK ABSTRACTS

PLENARY TALKS

Dr. Erika Tatiana Camacho
Professor, University of Texas at San Antonio
Manuel P. Berriozábal, Ph.D. and María Antonietta Berriozábal Endowed Chair
The Intersection of Adversity, Resilience, Tenacity, and Models of Photoreceptor Metabolism: My Story, Passion, and Research

Faced by rapidly accelerating social, environmental, and medical/health challenges there is an urgent need to create a strong quantitative workforce. Addressing these challenges requires intense, aggressive, and innovative efforts at every level. As students, we need to work very hard, have tenacity, and be resilient so that we never give up, take every opportunity in our path, and ensure that we are educating ourselves as best as possible and becoming very quantitative regardless of our academic focus. As educators and members of a larger community, we need to create an environment where our students can be more quantitative and thrive. In this talk, I will provide insights into these challenges through my story (including highlights of my research). My research has focused on developing a mathematical framework to investigate, corroborate, and guide experimental work on photoreceptor degeneration and vitality. Photoreceptor cell degeneration, resulting in retinal diseases such as retinitis pigmentosa and AMD, is linked to metabolic issues. To identify the key mechanisms in metabolism that may be culprits of this degeneration, we develop and investigate mathematical models of the metabolic pathway of aerobic glycolysis in photoreceptors at the cone molecular and photoreceptor cellular interactions.

Dr. Emily Riehl
Professor of Mathematics, Johns Hopkins University
Categorifying Cardinal Arithmetic

In this talk we'll prove that $a \cdot (b + c) = a \cdot b + a \cdot c$ via a roundabout method that takes us on a tour through several deep ideas including categorification, the Yoneda lemma, universal properties, and adjunctions.

Talks by Undergraduate Students

Avalon Blaser, University of Utah
Molly Bradley, University of Pennsylvania
Kathy Xing, Amherst College
Sato-Tate Type Distributions for Matrix Points on Elliptic Curves and Some K3 Surfaces

Generalizing the problem of counting rational points on curves and surfaces over finite fields, we consider the setting of $n \times n$ matrix points with finite field entries. We obtain exact formulas for matrix point counts on elliptic curves and certain K3 surfaces for "supersingular" primes. These exact formulas, which involve partitions of integers up to n, essentially coincide with the expected value for the number of such points. Therefore, in analogy with the Sato-Tate conjecture, it is natural to study the distribution of the deviation from the expected values for all primes. We determine the limiting distributions for elliptic curves and a family of K3 surfaces. For non-CM elliptic curves with square-free conductor, our results are explicit.

Sophie Boileau, Institute for Pure & Applied Mathematics
Parallel Algebraic Multigrid for Fusion and Higher-Order PDEs

Multigrid methods play a key role in large-scale scientific simulation because they are among the fastest and most scalable approaches for solving the underlying sparse linear systems of equations that arise from a wide array of partial differential equation (PDE) discretizations. Algebraic multigrid (AMG) is a special type of multigrid method that depends only on the description of the linear system, giving it better portability and broader applicability than geometric multigrid, as it requires no explicit knowledge of the problem geometry. Even though these methods are widely used today, there are still applications where further development is needed. In this presentation, we focus on PDEs with higher-order terms (e.g., fourth order), concentrating on a PDE that arises in tokamak edge plasma simulations (a tokamak is a machine that confines a plasma using magnetic fields and is believed to be the leading plasma confinement concept for future fusion power plants). General multigrid relaxes a linear system on coarser grids and reverses this process with interpolation, but standard AMG methods struggle with the aforementioned higher-order PDEs. We investigate cyclic coarsening and interpolation heuristics, as well as new iterative approximation methods of refining the solution at each grid to improve the existing multigrid approach. To this end, we ensure that these techniques are transferable to a parallelized setting with LLNL's supercomputers.

Molly Bradley, University of Pennsylvania
see **Avalon Blaser**

Jenson Bridges, Oklahoma State University
Enumeration of Cyclic Permutations in One-Line and Cycle Notation

A permutation is said to avoid a given pattern if there is no subsequence of the permutation in the same relative order as that pattern. This notion of pattern avoidance has several applications, including applications to computer science, algebraic combinatorics, and dynamical systems. We investigate cyclic permutations that avoid a pattern sigma in its one-line notation and another pattern tau in its cycle notation. In this, we will prove a bijective mapping from previous permutations of the desired n by using recursive sequences.

Natalie Burton, Northern Arizona University
Tara Zurick, Northern Arizona University
Strategies in Sylver Coinage

Sylver Coinage is a two-player game played on the natural numbers, created by Conway and Guy in the 1970s. The two players take turns naming positive integers greater than 1 that are not the sum of nonnegative multiples of previously named integers. The player who is forced to name 1 loses. Very little is known about achieving and maintaining winning advantages. In a 2002 paper, Sicherman states without proof that if the opening two moves are an odd integer > 3 followed by 4, player 1 is in a winning position but has only one choice to preserve their winning position. We will show why this is the case and give potentially winning strategies for player 2 if player 1 does not play optimally in this scenario.

Morgan Carns, Furman University
Alyssa Pate, Furman University
Finding the Key Length of a Vigenère Cipher

For our research, we focused on the Vigenère Cipher, created by Giovan Battista Bellaso in 1553, and it is one of the primary examples of a polyalphabetic substitution cipher. For over 250 years, it resisted decipherment and has seen a wide range of applications in history and popular culture. More specifically, we looked at previously discovered methods to find the key length and compared performance accuracy based on different parameters of the cipher text. We wanted to see if any test performed overall better than other tests. In this talk, we will introduce these methods, which include the Index of Coincidence, Kasiski-Babbage, and more recently the Twist, and Twist+, and provide a quick explanation of how they work. We will also describe our methods of comparing the test accuracies, using Google Collab and Python. From this, we will discuss our final product, a neural network used to streamline the process of finding the key length of any given Vigenère Cipher. This research was conducted through the Summer Mathematics Undergraduate Research program at Furman University, in collaboration with Dr. Christian Millichap and Dr. Yeeka Yau.

Julia Carrigan, Occidental College
Natural Bijections for Contiguous Pattern Avoidance in Words

Two words p and q are avoided by the same number of length-n words, for all n, precisely when p and q have the same set of border lengths. However, known proofs of this result use generating functions and do not provide explicit bijections. We establish a natural bijection from the set of words avoiding p to the set of words avoiding q in the case that p and q have the same set of proper borders.

Teressa Chen, Coe College
Berezin Range of Composition Operators

Our research focuses on the Berezin range of composition operators. One property of composition operators is the numerical range. The numerical range of an operator is a special set of values that are computed using the operator and has the geometric properties of convexity and compactness. A more restrictive set known as the Berezin range must satisfy an additional condition. We seek to find operators where the Berezin range has the same geometric properties as the numerical range. If the question is reframed about the numerical range and the Berezin range using linear algebra, then we can investigate the properties of the operator using its associated matrix instead through its eigenvalues and eigenvectors.

Jessica Childress, Gonzaga University
A Component Upper Bound for Link Mosaic Diagrams

A link mosaic diagram is a link diagram made using tiles on a square $n \times n$ grid. A link consists of 2 or more components, where a component is defined to be a closed loop in a mosaic diagram. In this presentation, I will introduce a quick foundation of mosaic knot theory. I will then talk about methods for maximizing available arc tiles in a mosaic diagram and how that affects the number of components. Lastly, I will talk about generating an upper bound on the number of components.

Erica Choi, Columbia University
Katerina Stuopis, University of Wisconsin-Madison
Cubiquitous Lattices and Chi-Sliceness

A lattice is cubiquitous if it admits an embedding into $\mathbb{Z}^n$ in such a way that its image λ intersects a point in each unit cube with integer vertices. Greene and Ownes have proved that if an alternating nonsplit link L is χ-slice, then there exists an associated cubiquitous sublattice. We show various properties that a basis $B = \{v_1, ..., v_n\}$ that generates a cubiquitous lattice has, and prove that $B^T B$ has 1's, 2's, and 4's on the diagonal. This finding proves the Greene and Owens' conjecture that the converse of their theorem is true for torus links.

Catherine Cossaboom, University of Virginia
Patterns of Primes in Joint Sato-Tate Distributions

In 2013, James Maynard demonstrated that there are infinitely many pairs of consecutive primes that differ by 600 or less; the underlying ideas were also developed by Terence Tao independently. Here, we build on the Maynard–Tao machinery to show that there exist bounded gaps in distinguished positive density subsets of the primes, defined as follows. Let $\mathcal{E}_1$ and $\mathcal{E}_2$ be elliptic curves. For each prime p and $j = 1, 2$, let $a_j(p)$ be the normalized error for the number of solutions mod p, i.e. $a_j(p) := \frac{p+1-\#\mathcal{E}_j(\mathcal{F}_p)}{2\sqrt{p}}$. The now-proven Sato–Tate conjecture states that the normalized error terms $a_j(p)$ equidistribute with respect to a semicircular distribution on $[-1, 1]$. We show that if $\mathcal{E}_1$ and $\mathcal{E}_2$ are twist-inequivalent, then for all subintervals $I_1, I_2 \subset [-1, 1]$, there exist infinitely many bounded gaps between the primes p satisfying both $a_1(p) \in I_1$ and $a_2(p) \in I_2$. We additionally prove a common generalization of a Green–Tao style theorem, showing that in our set of primes, there exist arbitrarily long arithmetic progressions infinitely often, which meet bounded gaps conditions.

Sogol Cyrusian, University of California, Santa Barbara
Mayla Ward, Western Washington University
Generalized Delta sets of Numerical Semigroups

A Numerical Semigroup is a cofinite submonoid of $(N, +)$. These semigroups admit non-unique factorization into irreducibles; the factorization set of a single element is conventionally denoted as a set of vectors. Extensive existing literature considers the set of 1-norms of said vectors (the "length set") and the gaps between consecutive lengths (the "delta set"). We consider a more general family of length sets by considering the t-norm for arbitrary t in $(0, \infty)$ (under the usual l_p space definition), and identify properties of the associated Δ sets. In particular, for $t = 0$, the Δ_0-sets of all semigroups up to three generators, as well as maximal embedding dimension semigroups, semigroups with generators in generalized arithmetic progression, and semigroups with generators in a compound sequence, are explicitly given. For $t = \infty$, the Δ_∞-sets of semigroups with two generators are explicitly given, and the contents of other generalized families, including semigroups with generators in generalized arithmetic progression, are analyzed. The periodicity of the Δ_0 and Δ_∞ sets of individual semigroup elements is also proven, along with general results for t-lengths between 1 and ∞. We also relate semigroup trade structure, t-catenary degree, and Δ_t sets.

Djeneba Diop, Hobart and William Smith Colleges
Different Variations of Toggle

In the commercial one-player game Lights Out, a grid of lights is randomly generated with some lights on and some lights off. The player can press a light to flip its on/off state as well as the state of its neighbors. Toggle seeks to transform Lights Out into a variety of impartial two-player games. Two players take turns toggling the on/off state of lights in an attempt to leave the other player with no available legal moves. We analyze Toggle on various finite simple graphs and use impartial game theory to determine which player has a winning strategy given an initial Toggle configuration.

Kristen Ess, Lewis University
A Mathematical Model of C. difficile Prevention and Control in Healthcare Settings

Clostridioides difficile, C. difficile, has been the leading cause of infectious diarrhea and one of the most commonly obtained infections in United States hospitals, with half a million cases recorded each year. Those infected could have contracted a C. difficile infection due to interactions with a surface or person harboring the bacterial spores. Patients who are infected with C. difficile spread these endospores, which have proven to be difficult to remove from the hospital environment. In order to reduce the spread of this bacteria, infected patients are sometimes placed into isolation. Previous work has only considered the environment's interactions with patients and not evaluated the effect of hospital employees or isolation of infected individuals. The mathematical model developed uses a system of ordinary differential equations to examine the effect of different transmission routes such as healthcare workers, doctors, and low- and high-touch frequency fomites, objects likely to carry infection, on the spread of C. difficile in a hospital setting. This model is also one of the first to consider an isolation class for patients infected with C. difficile. The results from this model can be applied by those in healthcare to help control the spread of C. difficile in healthcare settings through the interventions considered.

Yuhan Fu, Denison University
Hyperspectral Eye Tissue Images Segmentation and Boundary Detection

Fluorescence microscopic imaging and autofluorescence are widely used in biomedical imaging and possess great potential in disease diagnosis, in particular, the biomedical hyperspectral imaging of eye tissues. We are developing a non-invasive algorithm-based analysis to extract spectral-spatial information from a large eye tissue imaging dataset and perform accurate tissue segmentation and boundary detection. At the preliminary stage, a Regions of Interest approach, an unsupervised clustering analysis, and a supervised deep learning method (3D-UNet), based on both Python and R, work well for tissue boundary detection. These non-invasive techniques help to avoid choroid staining and shorten the examination time. More mature techniques are expected to aid in eye examinations outside laboratories in the future, such as the diagnosis of Spaceflight-Associated Neuro-ocular Syndrome in astronauts and Amyotrophic Lateral Sclerosis.

Lily Gebhart, Occidental College
Adjustments for Kurtosis and Continuity on the Prentice Test

The test of Prentice is a non-parametric statistical test for the two-way analysis of variance using ranks. The null distribution of this test is approximated using the Chi-square distribution. However, the exact null distribution deviates from the Chi-square approximation in certain cases commonly found in applications of the test, motivating adjustments to the distribution. Here we present improved adjustments to this null distribution correcting for continuity, multivariate skewness, and multivariate kurtosis. The effects of alternative scoring methods as non-polynomial functions of rank sums are also presented as a broader application of the approximation.

Mia Goldstein, The State University of New York at New Paltz
Emily Herbert, The State University of New York at New Paltz
Explicit Categorical Constructions Used in Modeling Sentences

Historically, two separate mathematical frameworks are used to model sentences: a pregroup for grammar and a collection of vector spaces for semantic meaning. Through category theory a third framework was constructed, by Coecke, Sadrzadeh, and Clark, simultaneously modeling both the grammar and the meaning of a sentence. Each framework induces a monoidal category, which means that it exhibits certain properties akin to associativity and multiplicative identity. The third framework's construction requires showing that the product category of two monoidal categories is again monoidal. Our presentation will include all necessary category theory background needed to understand the construction, as well as its applications in sentence modeling.

Hannah Graff, Creighton University
On the Generalized Distance Matrix

Given a graph G and a function f, the generalized distance matrix $D(G, f)$ has rows and columns indexed by the vertices of G with the (u, v) entry being $\mathrm{dist}(u, v)$, where dist is the distance between vertices u and v. Two graphs G and H are said to be strongly distance cospectral if $D(G, f)$ and $D(H, f)$ have the same characteristic polynomial for all f. We give a sufficient condition for two graphs to be strongly distance cospectral in terms of simultaneous similarity of related 0-1 matrices. Moreover, we give a sufficient condition in terms of a block similarity matrix for two strongly distance cospectral graphs to remain strongly distance cospectral after gluing arbitrary graphs on to subsets of the vertices (i.e., coalescing).

DJ Henson, Drake University
Utilizing the Navier-Stokes Equations for Modelling Incompressible, Viscous, Non-Laminar Fluid Flow in Ultrasonic-Oscillatory Artificial Gravity

One of the most significant challenges in developing technology for long-term space flight is addressing the physiological feasibility of microgravity and artificially induced gravity conditions. We investigate the impact of artificial gravity conditions on cardiovascular fluid flow to predict the changes prolonged exposure to ultrasonic-oscillatory-induced gravity may have on blood flow in the short-term by applying the finite element method to solve the partially differential and notoriously unstable Navier-Stokes equations for incompressible and non-laminar flow of viscous fluid. By utilizing a modified form of Chorin's method in the FEniCS platform, we predict the behavior of blood flow through a short, cylindrical vessel encased in a rectangular mesh. The results from this calculation are then analyzed using the CVSim model to determine the cardiovascular feasibility of oscillatory gravity conditions.

Emily Herbert, The State University of New York at New Paltz
see **Mia Goldstein**

Annemily Hoganson, Carleton College
Moments of Random Multiplicative Functions over Function Fields

Determining to what extent prime numbers behave like random variables can have deep implications for understanding their distribution. One can approach questions in this area by studying multiplicative functions, which are characterized by their behavior on prime powers. It is sometimes possible to obtain stronger results studying these questions for multiplicative functions defined over function fields rather than over integers, and these results in the function field setting may provide insights into the integer setting. Paralleling the work of Harper, Nikeghbali, and Radziwiłł and Heap and Lindqvist over the integers, we study the natural moments of sums of random multiplicative functions over function fields. Using combinatorial and analytic arguments, we obtain an exact expression for the 4th moment, which has not been found in the integer setting, and an asymptotic expression for the higher natural moments.

Edie Irwin, Agnes Scott College
Ayechan Moe, Agnes Scott College
Complete Residue Systems in Second-Order Linear Recursive Sequences Mod a Prime

This work is a study of the behavior of sequences that follow a second-order linear recurrence relation. Such sequences $\{a_n\}$ are given by initial conditions $a_0 = 0$, $a_1 = 1$, and the recurrence relation $a_n = c_1 a_{n-1} + c_2 a_{n-2}$. Given a prime p, we investigate whether $\{a_n\}$ contains a complete residue system mod p, that is, whether there exists a_{n_i} such that $a_{n_i} \equiv i \pmod{p}$ for all $0 \leq i \leq p-1$. Such sequences are necessarily periodic modulo p, and using number theory and finite field theory we show that the existence of a complete residue system depends only on the period length. Explicit examples will be given. Finally, we will investigate the proportion of sequences which are periodic for a fixed prime p.

Julia Jammalo, Fairfield University
Lingran Zhang, Fairfield University
Elliptic Islands in Moon Billiards

Mathematical billiards are central models of dynamical systems in statistical mechanics in which point particles collide elastically with fixed boundaries. This project studies a class of billiard tables called moon billiards, whose boundary comprises two circular arcs, one concave and one convex. One of the primary objectives of this research project is to explore how the dynamics vary as a function of two system parameters: the radius of the larger circle and the distance between the centers of the circles. By studying a family of stable periodic orbits, we are able to identify elliptic islands in the phase space whose existence excludes the possibility of ergodic dynamics. By systematically varying the table parameters, we gain insights into the diverse behavior of the moon billiard system, uncovering regions of stability and hyperbolicity. This research was conducted at Fairfield University in the summer of 2023 with the support of National Science Foundation grant DMS 2055070.

Khaiylah Johnson Bustamante, Mercy University
Digitally Restricted Ostrowski Expansions

The Cantor Set is the x in $[0,1]$ whose base 3 expansions exclude the digit 1. The Hausdorff Dimension of the Middle Thirds Cantor Set is $\log(2)/\log(3)$. In fact, for any similar Cantor-like set with base b digital restrictions, it follows that the Hausdorff Dimension is $\log(\#\text{Allowed Digits})/\log b$. We will determine the Hausdorff Dimension for analogous cases of digital restrictions of a different type of expansion that has applications in Diophantine Approximation. The Ostrowski Expansion of a real number is evaluated with respect to the integers in the infinite continued fraction of an irrational number $\beta = [b_0; b_1, b_2, \dots]$, and takes the form $\sum c_{n+1} D_n$ where D_n is the difference between β and its nth convergent $[b_0; b_1, ..., b_n]$, with the condition that if $c_{n+1} = b_{n+1}$, then $c_n = 0$. The simplest case of $\beta = [0; b, b, b, \dots]$ with b_n excluded yields dimension $-\log(\#\text{Allowed Digits})/\log(\beta)$, which is satisfyingly similar to the previous Cantor Set case. We shall discuss other continued fractions beta with varying behaviors as well as the Hausdorff Dimension of sets with randomly determined digitally restrictions of their Ostrowski Expansions, which give rise to Fractal Percolation.

Nour Kawni, Institute for Pure & Applied Mathematics
Maria Nicos Alain Pasaylo, Institute for Pure & Applied Mathematics
Detection of Out-of-stock Items at Retail Stores using Computer Vision

Computer vision has emerged as a powerful technology that enables computers to perceive and understand visual information. Retail stores today have difficulty keeping track of the product inventory on shelves due to the current inventory management system that involves manual scanning and counting of products, which is time-consuming and error prone. Computer vision enables us to automate inventory management by using cameras to capture real-time data on product availability. Specifically, we use deep learning to detect out-of-stock (OOS) items and price tags on the shelves. We also implement an algorithm to match the OOS item to the appropriate price tag and extract its barcode and textual information to successfully identify OOS items.

Maggie Lai, Tulane University
Zero-Sum-Free Graph Labellings

A k-tuple over $\mathbb{Z}_n$ is zero-sum-free if every nonempty subset of its components has a nonzero sum. We provide results on zero-sum-free tuples with distinct entries and generalize the concept to zero-sum-free graph labelings. Given a graph Γ with vertices $v_1, v_2, \dots, v_k$, a $\mathbb{Z}_n$*-labeling* on Γ is an injective function that maps v_i to $a_i \in \mathbb{Z}_n$. A $\mathbb{Z}_n$-labeling on Γ is *zero-sum-free* if $\sum_{i=1}^{j} a_{\sigma_i} \neq 0$ for all positive integers j and all path subgraphs $v_{\sigma_1} v_{\sigma_2} \cdots v_{\sigma_j}$ in Γ. We find results for $\mathbb{Z}_n$-zero-sum-free labelings on classes of simple graphs and extend the notion of a $\mathbb{Z}_n$-zero-sum-free labeling to nonabelian groups.

Catherine Lillja, The College of New Jersey
Maya Powell, Berry College
NUTS for Bayes: Bayesian Generalized Weibull Regression with Applications to Survival Data

Data in reliability analysis, especially over the life-cycle of the product, can involve high initial failure rates (infant mortality), and eventual high failure rates due to aging and wearout, indicating a bathtub failure rate. Models that allow only monotone failure rates might not be appropriate or adequate for modeling these populations. We propose a Bayesian generalized Weibull regression method and develop Accelerated Time to Failure models using Bayesian methods. The parameter estimation procedure is carried out using Hamiltonian Monte Carlo algorithm with No-U-Turn Sampler and compares the results of generalized Weibull regression with exponentiated Weibull regression, Weibull regression, and log-normal distribution across simulated and clinical data sets. We examine the effectiveness of generalized Weibull distribution as a survival model and compare it to more studied probability distributions. In addition to monotone and bathtub hazard shapes, the additional shape parameter in the generalized Weibull distribution provides flexibility to model a broader class of monotone hazard rates. We found the generalized Weibull model to have superior results for multiple hazard function types. We also developed a Shiny app to dynamically visualize our models using real life data from lung cancer, heart attack, and German breast cancer studies. The app's interactive interface lets users input their own data sets and see Weibull regression analysis on their data.

Jordan Martino, Northeastern University
Relating Quiver Varieties and Hessenberg Varieties

In this presentation, I will be talking about my senior thesis research project focusing on quiver varieties. Nakajima quiver varieties are a family of algebraic varieties that can be explicitly constructed using data associated with a directed graph. They play an important role in geometric representation theory, enumerative geometry, and mathematical physics. The goal of this project was to learn about the geometry of Nakajima quiver varieties, from both an algebraic and symplectic point of view. I will be discussing my efforts toward finding new families of Hessenberg varieties that can be realized as quiver varieties (an open problem in the field) and my results.

Kathryn Massey, Marist College
Mathematical Modeling of Liquid Metal Dynamics

Eutectic Gallium-Indium (EGaIn) is a room-temperature liquid metal alloy that dramatically changes its surface tension and dynamics under an applied electric field. EGaIn has been used heavily in soft electronics engineering due to its high conductivity, malleability, and safety. However, the absence of mathematical modeling in the current literature makes its behavior difficult to understand and predict. Oxidation, while observable in the physical setting, cannot be well measured, calling for an alternative method of quantification. In this study, we present a one-dimensional lubrication model for the dynamics of an EGaIn droplet moving along an inclined plane. The thin oxide skin of the droplet, which controls the interfacial surface tension, is modeled as an insoluble surfactant at the surface. Our model incorporates essential physical effects and parameters including oxidation, capillary action, diffusion, gravity, and the Marangoni effects. Utilizing experimental data, we calibrate system parameters and qualitatively obtain numerical simulation results comparable to experimental observations. Our model has demonstrated success in reproducing the observed dynamics of an EGaIn droplet and provides a valuable resource for further investigation and uses of EGaIn.

Rhiannon Maynes, William Jewell College
The Committee Size Paradox in Three-Candidate Elections

As more jurisdictions across the U.S. look to use ranked choice voting (RCV) in elections, research on RCV has become more relevant. RCV is known to exhibit many interesting paradoxical behaviors and pathologies such as monotonicity paradoxes. We investigate a less well-studied anomaly that can occur in RCV elections known as a committee size paradox. This occurs when the winner of the one-seat election does not win if there are instead two seats available. To determine how often this phenomenon may occur, we examine elections with three candidates using Monte Carlo simulation under the impartial culture and impartial anonymous culture models of voter behavior. To supplement this theoretical work we also investigate how often such paradoxes occur in practice, using a large database of real-world, ranked-choice elections.

Rayme McCallan, Western Oregon University
Solitaire: The Strongest Hand Cipher

The Solitaire Encryption Algorithm was created by Bruce Schneier for use in the novel Cryptonomicon by Neal Stephenson. The purpose of this project was to analyze the algorithm and replicate it using computer programming software. In this presentation, we will demonstrate by hand how the encryption algorithm works. We will also discuss our procedure for verifying results about bias in the algorithm as well as other vulnerabilities.

Ayechan Moe, Agnes Scott College
see **Edie Irwin**

Kolton O'Neal, University of Nebraska-Lincoln
Primitive Solvable Permutation Groups of Rank 5 and 6

Let G be a finite solvable permutation group acting faithfully and primitively on a finite set Ω. Let G_0 be the stabilizer of a point α in Ω. The rank of G is defined as the number of orbits of G_0 in Ω, including the trivial orbit $\{\alpha\}$. We completely classify the cases where G has rank 5 and 6, continuing the previous works on classifying groups of rank 4 or lower.

Svetlana Pack, University of Rochester
Measuring Fractal Behavior in Time Series using Discrete Energy

Real world datasets are replete with examples of fractal dynamics. For instance, natural processes such as flooding cycles of the Nile river can be modeled using fractional Brownian Motion (fBM), a Gaussian process with self-similar structure and long range time dependence. Measuring the degree of fractal behavior in time series data is of interest in estimating the degree of time dependence in a time series and potentially in performing dimension reduction on large datasets while preserving their self-similar features. However, it is difficult to characterize self similar patterns in finite datasets, as most machinery for studying fractals has been developed in a continuous setting. In our research, we define the discrete energy of finite point sets, which we developed to serve as a discrete analog of the Hausdorff outer measure of sets. We show that the discrete energy of datasets sampled from well characterized fractals like the Cantor set has convergence properties consistent with what we expect from the Hausdorff outer measure. We then examine the discrete energy distribution of several simulated fBM time series generated with different kinds of long range time dependence (determined by the presence of positive, zero, and negative autocorrelation). Preliminary results show that discrete energy is an effective tool for detecting the presence of self similarity and distinguishing different degrees of time dependence in the simulated fBM time series.

Sophia Palcic, Kansas State University
Painted Tropical Complexes

From a polytope, one can create its secondary polytope, which encodes information about the original polytope's subdivisions and triangulations. In this talk, I will show the connection between secondary polytopes and tropical complexes. I will further describe a painting function of a tropical complex and discuss how secondary polytopes can also keep track of such decorations. The partially ordered set of painted trees is a well-studied example of this structure and is known to be the face lattice of a polytope called a multiplihedron. A corollary from my research provides an alternative construction of multiplihedra as secondary polytopes.

Maria Nicos Alain Pasaylo, Institute for Pure & Applied Mathematics
see **Nour Kawni**

Alyssa Pate, Furman University
see **Morgan Carns**

Isabel Pfaff, Oberlin College
The Sandpile Group of Subset Intersection Graphs

The sandpile group of a graph is a finite abelian group related to the graph's structure, whose order is equal to the number of spanning trees of the graph. We consider the Laplacian matrix of various subset intersection graphs, such as Kneser and Johnson graphs. Building on the work of Wilson and Bier, we construct matrices that conjugate this Laplacian matrix to a simplified upper triangular form that we can then fully diagonalize. This reveals the corresponding sandpile group.

Maya Powell, Berry College
see **Catherine Lillja**

Mallory Price, Grand Valley State University
Sarah Zaske, Grand Valley State University
Distinguishing Index of Mycielskian Graphs

The distinguishing number and distinguishing index tell us in some sense how symmetric a graph is. We define a distinguishing vertex coloring to be a coloring of the vertices of a graph G such that no non-trivial automorphism preserves the vertex coloring. The distinguishing number, $\mathrm{Dist}(G)$, is the smallest number of colors possible for which there is a distinguishing coloring. Similarly, a distinguishing edge coloring is a coloring of the edges of G such that no non-trivial automorphism preserves the edge coloring, and the distinguishing index, $\mathrm{Dist}'(G)$, is the smallest number of colors needed for a distinguishing edge coloring. The Mycielskian of a graph G, denoted $\mu(G)$, is an extension of G introduced by Mycielski in 1955. In 2022 Boutin, Cockburn, Keough, Loeb, Perry, and Rombach showed that for graphs on at least three vertices $\mathrm{Dist}(\mu(G)) \leq \mathrm{Dist}(G)$. We prove $\mathrm{Dist}'(\mu(G)) \leq \mathrm{Dist}'(G)$, finishing a conjecture of Alikhani and Soltani.

Erin Prins, Furman University
How Tracking Shapes Teachers' Instructional Decisions and Views of Students

There is a strong belief in the United States that tracking is a necessary structure to manage differences in students' mathematical ability. Inevitably, such demarcated placements affect how teachers think about and engage in their work. To explore teachers' decision making in general, and the role tracking plays specifically, we interviewed nine secondary teachers about how they teach factoring of quadratics. Results highlight three different rationales driving teachers' decision making. One group was quite responsive in their approach, presenting different techniques depending on the success and understanding of their students. A second group used the tracks to predetermine the instructional strategy, providing conceptual methods to students in honors and rote methods to lower tracked students. A third group also used tracking to determine their approach; however, their responses were characterized by deficit language, justifying their decisions by describing negative personality traits they associated with students in the lower track.

Tahda Queer, Hunter College, City University of New York
Rainbow Cliques Guaranteed: New Results on Colorful Turán Problems

Given an edge-colored graph G, we denote the number of colors to be $c(G)$, and the number of edges to be $e(G)$. An edge-colored graph is rainbow if every two edges have different colors. A proper mK_3 is a vertex disjoint union of m rainbow triangles. Rainbow problems have been studied extensively in the context of anti-Ramsey theory, and more recently, in the context of colorful Turán problems. B. Li. et al. found that a graph must contain a rainbow triangle if $e(G) + c(G) \geq \binom{n}{2} + n$. L. Li. and X. Li. conjectured a lower bound of $e(G) + c(G)$ such that G must contain proper mK_3. In this presentation, we provide a construction that disproves the conjecture. We also introduce a result that guarantees the existence of m copies of rainbow cliques in general graphs and a sharp result on the existence of proper mK_3 in complete graphs.

Sangeetha Ramanuj, Oberlin College
Using Natural Language Processing to Interpret Music Evoked Auto-biographical Memories

Music is special in its ability to evoke auto-biographical memories. We are unaware of what exactly drives music-evoked autobiographical memories (MEAMs) beyond its age or lyrical content. We want to explore how acoustic features of the music itself contribute to the content of the memory. We curated a corpus of 50 pop songs and their cover versions offering diverse musical variations. Cover songs offer a rich space to test our question because they preserve the higher-level structure or "identity" of the original music, but vary in low-level features like tempo, timbre, texture, genre, key, gender of the artist, etc. Participants listened to both versions of the song, in separate sessions, describing their memories, if any. Each song pair (original and cover) had a similarity rating collected from a different experiment based on their low-level features. However, we have no way of quantifying similarity between memory descriptions for these original/cover pairs. To unravel mechanisms underlying this phenomenon, we employed natural language processing (NLP) techniques. Leveraging the 'word2vec' model, we obtained 300-dimensional real-number vector embeddings for each word from the memory descriptions. Using PCA and K-means clustering, we identified five distinct semantic clusters, revealing their orientation. We used the human-generated similarity ratings to predict similarity between memory descriptions for these pairs. This was achieved using a linear mixed-effects model.

Camryn Rhude, Winona State University
Bat-Math! Modeling Mexican Free-tailed Bat Populations with Integrodifference Equations

In this talk we develop an integrodifference equation model of metapopulations in point-patch habitats (roosts). The first part of the model captures periodic dispersal of individuals from roosts to forage or find mates using a climate sensitive dispersal kernel. In the second part of the model (homing), individuals use olfactory chemotaxis via odor concentration gradients to find new patches for the next time step. This model is appropriate for a wide variety of organisms whose colonies are located on different isolated landscape points such as bat caves, bee hives, and ant colonies. We use a separation of scales method to derive analytic approximations for subpopulation equilibria. These approximations allow for an analysis of the effects of climate change on the long-term state of a metapopulation.

Olivia Roberts, University of South Dakota
Musical Systems with $\mathbb{Z}_n$-Cayley Graphs

It is well established that music theory uses mathematics to explain how music is created. The chromatic scale and pitch frequencies can be explained by modulo 12 arithmetic and geometric ratios. Using group theory, we interpret concepts from Western music theory. We show that in $\mathbb{Z}_{12}$, chords, scales, the circle of fifths, and the first species of counterpoint can be explained using a Cayley graph with respect to generators 3 and 4. Using $\mathbb{Z}_{12}$ as a model, we generalize to $\mathbb{Z}_n$ where n is a product of two relatively prime numbers. Most major and minor chords can be constructed using paths on the oriented Cayley graph, beginning on the root of the chord. The circle of fifths can be explained by adding both generators at once. The unoriented Cayley graph gives way to a weaker form of counterpoint, with minimum distance elements forming a set of consonant elements. Then, we create partitions of consonant and dissonant elements using affine transformations to create a full generalization of counterpoint. We assume equal-tempered tuning for various $\mathbb{Z}_n$ systems. As application, we have written code in Maple to hear chords, scales, and counterpoint in these musical systems.

Molly Sager, Northeastern University
Hyperbolic Surfaces and Teichmüller Theory

Hyperbolic surfaces are surfaces with constant negative curvature, and the study of its geometric properties has various different applications. The focus of this project will be the study of compact hyperbolic surfaces and Teichmüller theory, which by uniformization theorem is also related to the moduli spaces of Riemann surfaces. We hope to explore classifications and deformation of Riemann surfaces and make progress in understanding related geometric and analytic properties.

Janee Schrader, University of Wisconsin-Eau Claire
Exploring the Combinatorics of Pretzel Links Through Grid Diagrams

Using grid diagrams, we establish a methodology to encode and analyze pretzel links, enabling a systematic approach of their topological and algebraic properties. By employing a combination of knot theory, graph theory, and combinatorial techniques, we uncover relationships between the structure of pretzel links and their corresponding grid diagram representations. Additionally, we present computational techniques for efficiently generating and analyzing invariants of these specific grid diagrams. We present a comprehensive array of findings along with intriguing open questions.

Emily Shambaugh, Dickinson College
Ghost Series and a Motivated Proof of the Bressoud-Göllnitz-Gordon Identities

We give a "motivated proof" of the Bressoud-Göllnitz-Gordon partition identities combining techniques from earlier papers containing similar "motivated proofs" involving "ghost series". A motivated proof is a proof method that uses only the product sides of a partition identity to prove they are equal to combinatorial sum sides. A ghost series is a series with similar combinatorial properties to our original desired sum-sides, but has a flip in its parity condition, and is an essential ingredient in our motivated proof. Similar motivated proofs have been provided for other integer partition identities, such as the Rogers-Ramanujan, Andrews-Gordon, and Göllnitz-Gordon-Andrews integer partition identities. This work was funded by NSF grant 1851948. This work was carried out as part of Ursinus College's NSF REU program during Summer 2023.

Lillian Stolberg, University of Rochester
Discrete Neural Networks

We explore the concept of a neural network on a discrete universe. Activation functions within nodes are polymorphisms. To create a learning algorithm, we simulate the notion of distance and continuity among polymorphisms by using neighbor functions of a clone, which we combine with a loss function to train the network and tweak the activation functions. We have also constructed a relation class to represent input images, which allows us to perform the polymorphisms on coordinates of chosen pixels instead of entire images, improving efficiency. We are working toward deconstructing more complex structures by implementing higher arity polymorphisms, including one of the k-ary Hamming weight graph.

Katerina Stuopis, University of Wisconsin-Madison
see **Erica Choi**

Zixu Wang, Wellesley College
Political Structures and the Topology of Hypergraphs

My research employs hypergraphs to model political structures. In this framework, agents are represented as vertices, and hyperedges signify viable agent configurations. Hypergraphs outperform some previously used models, such as simplicial complexes, due to their lack of closure property under subset operations. This allows them to capture complex situations, such as when two agents are unwilling to cooperate directly but are open to collaborating simultaneously with a third agent. We apply various topological operations, such as wedge, cone, colapse, and homology, to hypergraphs, enabling the representation of merging structures, the introduction of mediators, and delegation within political systems. We evaluate the effects of these operations on the viability, stability, and local stability of hypergraphs to measure their influence on stability, power concentration, and other features in political structures. Furthermore, we explore the application of concepts like clustering coefficients within hypergraphs and propose possible future research directions.

Mayla Ward, Western Washington University
see **Sogol Cyrusian**

Madeleine Whalen, Indiana University - Purdue University Indianapolis
Using Theoretical Modeling to Predict Ideal Therapeutic Strategies for Organ Transplants

Organ transplants are lifesaving procedures but are still met with serious risks, including organ rejection, infection, and a compromised immune system. This study explores new treatments that yield transplant acceptance while reducing the reliance on immunosuppression. We adapted an ordinary differential equation model of the immune response to a transplant to analyze the effect of immunosuppression in combination with the adoptive transfer of regulatory T cells. First, we simulated the known mechanisms through which current immunosuppressive drugs preserve the transplant. The model is used to show that the combined use of adoptive transfer provides little additional benefit. Next, we investigated the key immune interactions to target to prevent rejection while minimizing immunosuppression. Results suggest that a drug inhibiting all T cell activation and the maturation of antigen-presenting cells could be an ideal therapy for transplant patients. In particular, long-term graft survival was achieved by targeting these mechanisms while gradually tapering immunosuppression and administering a single adoptive transfer dose. In conclusion, we show the utility of mathematical modeling in guiding future experimental studies of new therapeutic strategies in transplant immunology. By identifying the most promising therapeutic interventions, our approach provides a framework to optimize organ transplantation outcomes.

Victoria Wiest, California State University, Fresno
An Extension of the sl(n) Polynomial to Knotted 4-valent Graphs

We consider a special type of knotted oriented 4-valent graphs with rigid vertices and construct an invariant of such graphs. The resulting invariant is an extension of the well-known sl(n) polynomial invariant for knots and links. Our approach uses a version of the Murakami-Ohtsuki-Yamada state model for the sl(n) polynomial via 4-valent planar graphs and graphical relations among such graphs.

Qianqian Wu, Grinnell College
Minimal Hypergroups with Non-normal Structure

The concept of hypergroups is an extension of group theory. In hypergroups, the distinctive operation yields a non-empty set, rather than a single element, belonging to the hypergroup. Motivated by the fact that normality in group theory is a special property that helps understanding group structure, I investigated the presence of non-normal closed subsets within hypergroups. By examining the structural details of the minimal hypergroups containing a non-normal closed subset of a fixed size, my research moves on to constructs and proves the existence of two classes of the hypergroups in a general case. Each class of hypergroups is the minimal under certain conditions involving non-normal assumption and shares specific structural properties.

Kathy Xing, Amherst College
see **Avalon Blaser**

Zoey Yelsky, Northeastern University
Finite Group Quotients with Small Distortion

We consider the action of a finite group G on a real vector space V by orthogonal transformations. The ring of invariants corresponds to a geometric quotient V/G. This project studies the relation between the distance between G-orbits and their distance in the quotient space. Our goal is to modify the quotient in order to minimize the distortion of the distance between orbits after going to the quotient space. The hope is to preserve distance up to a constant. We analyze the special case of the cyclic group of order n acting by rotations in the plane.

Sarah Zaske, Grand Valley State University
see **Mallory Price**

Lingran Zhang, Fairfield University
see **Julia Jammalo**

Tara Zurick, Northern Arizona University
see **Natalie Burton**