

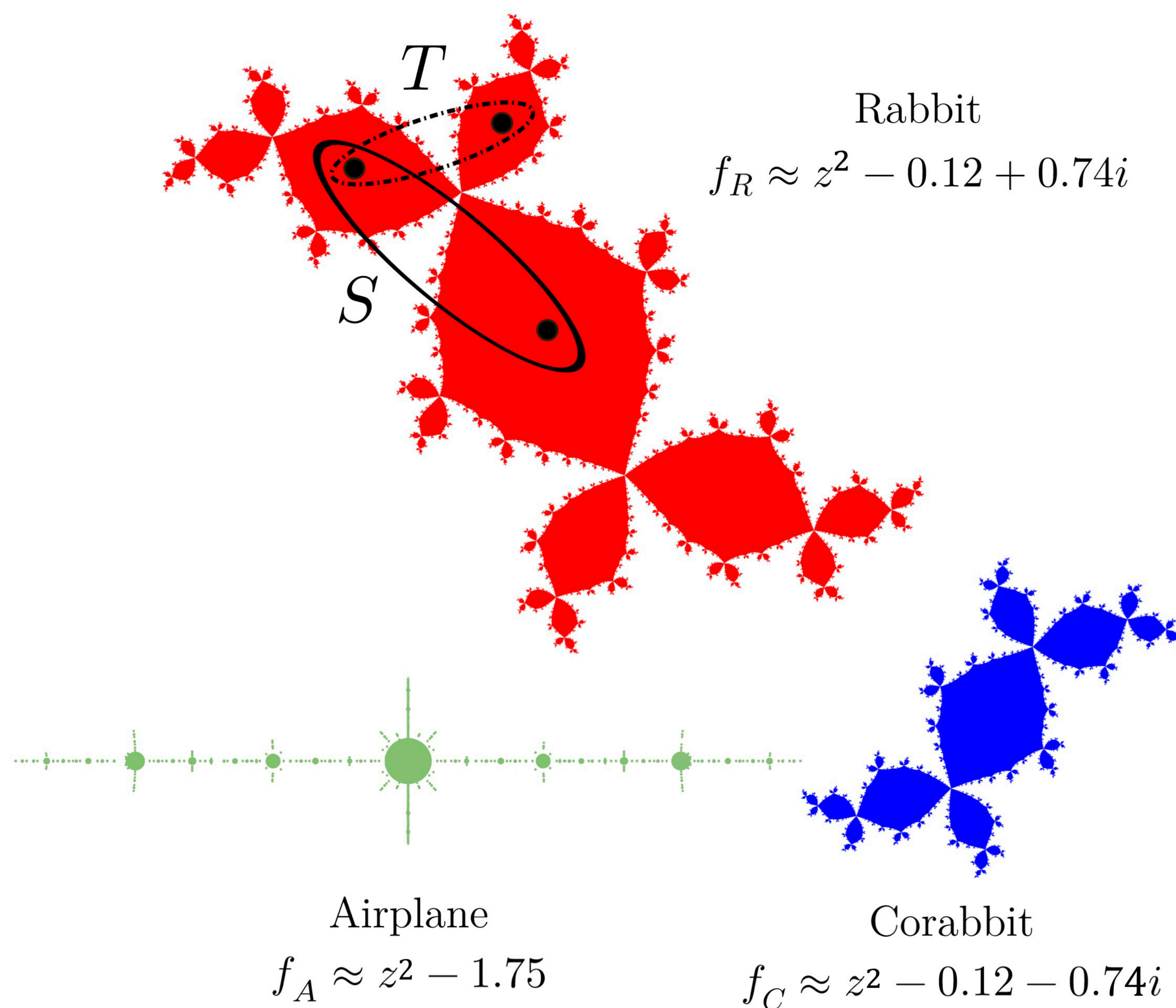


Twists of the Rabbit Polynomial

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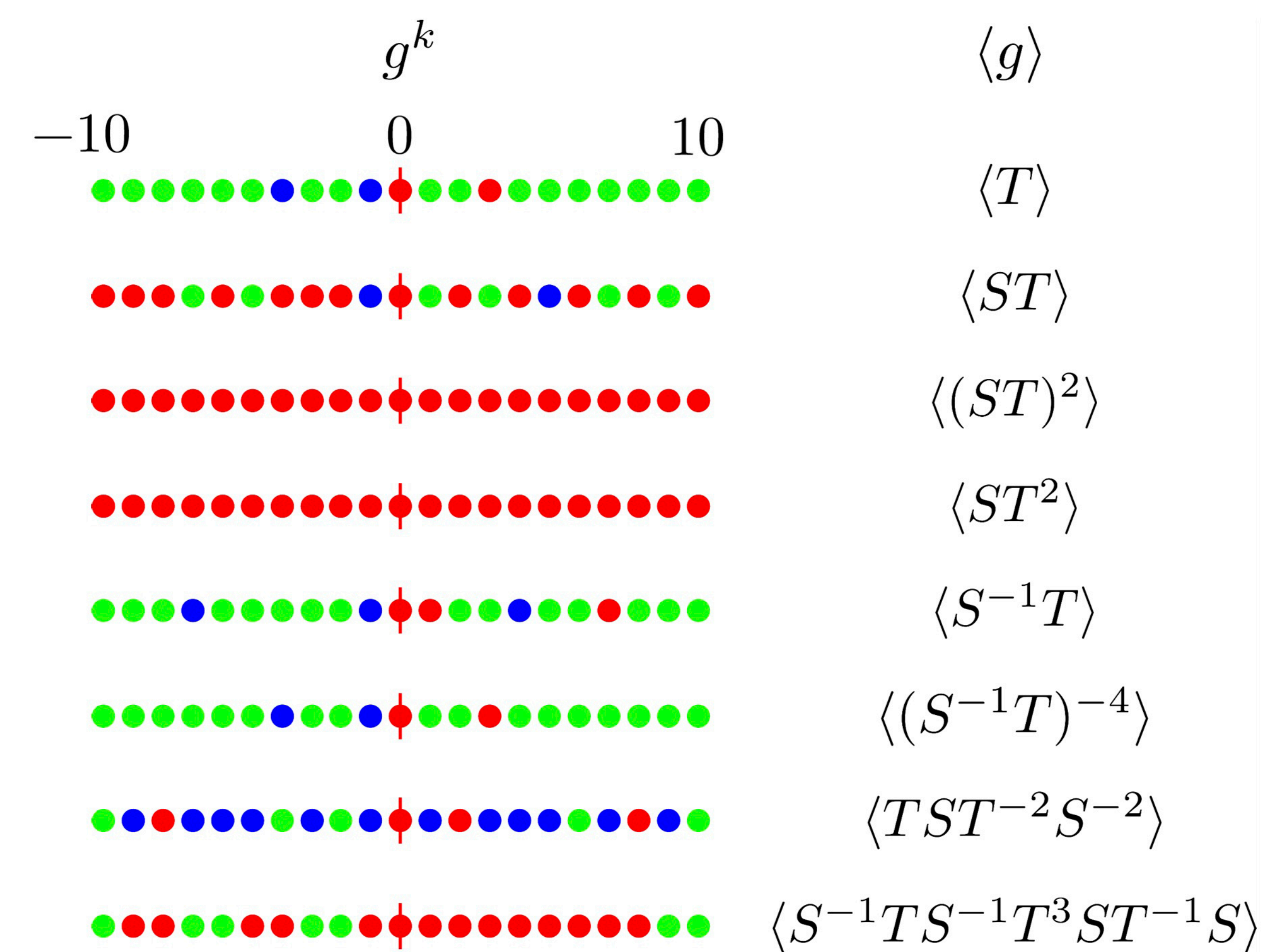
Three Polynomials



Question

Are there all-rabbit subgroups of $\langle S, T \rangle$?

Examples of Subgroups



Calculations

g	$\text{lift}(g)$
S	T
T^2	$S^{-1}T^{-1}$
TST^{-1}	1

Fact: $f_R \cdot g$ is equivalent to $f_R \cdot \text{lift}(g)$ when g is liftable

$$\begin{aligned}
 ST^2 &\sim \text{lift}(ST^2) \\
 &= \text{lift}(S)\text{lift}(T^2) \\
 &= T \cdot S^{-1}T^{-1} \\
 &\sim \text{lift}(TS^{-1}T^{-1}) \\
 &= 1
 \end{aligned}$$

Fact: $f_R \cdot g$ is equivalent to $f_R \cdot T \text{lift}(gT^{-1})$ when g is unliftable

$$\begin{aligned}
 S^{-1}T &\sim T \cdot \text{lift}(S^{-1}T \cdot T^{-1}) \\
 &= T \cdot T^{-1} \\
 &= 1
 \end{aligned}$$

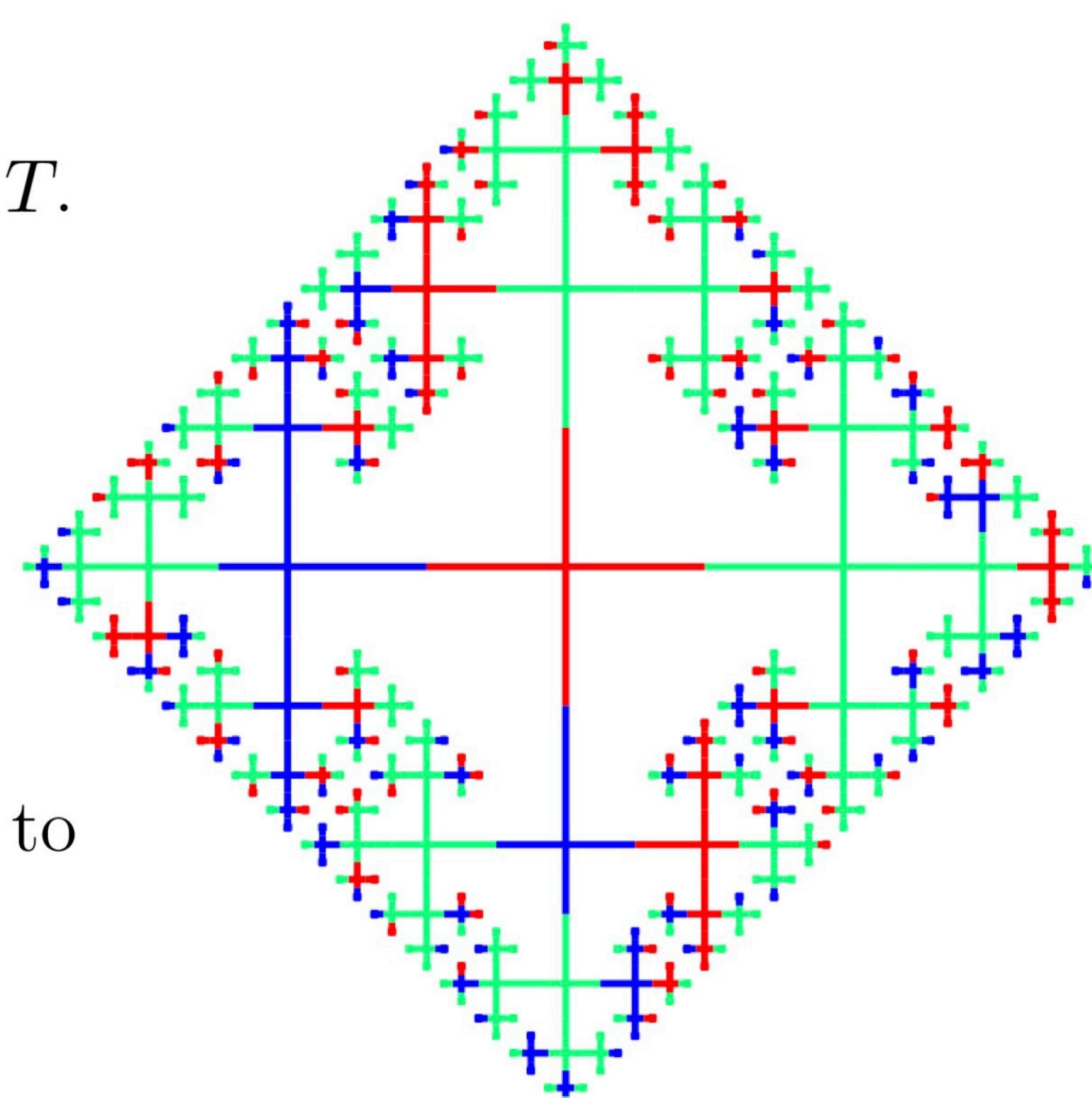
Twisted Rabbit Problem

Let g be a product in the Dehn twists about S and T . Since S and T generate a free group, we have:

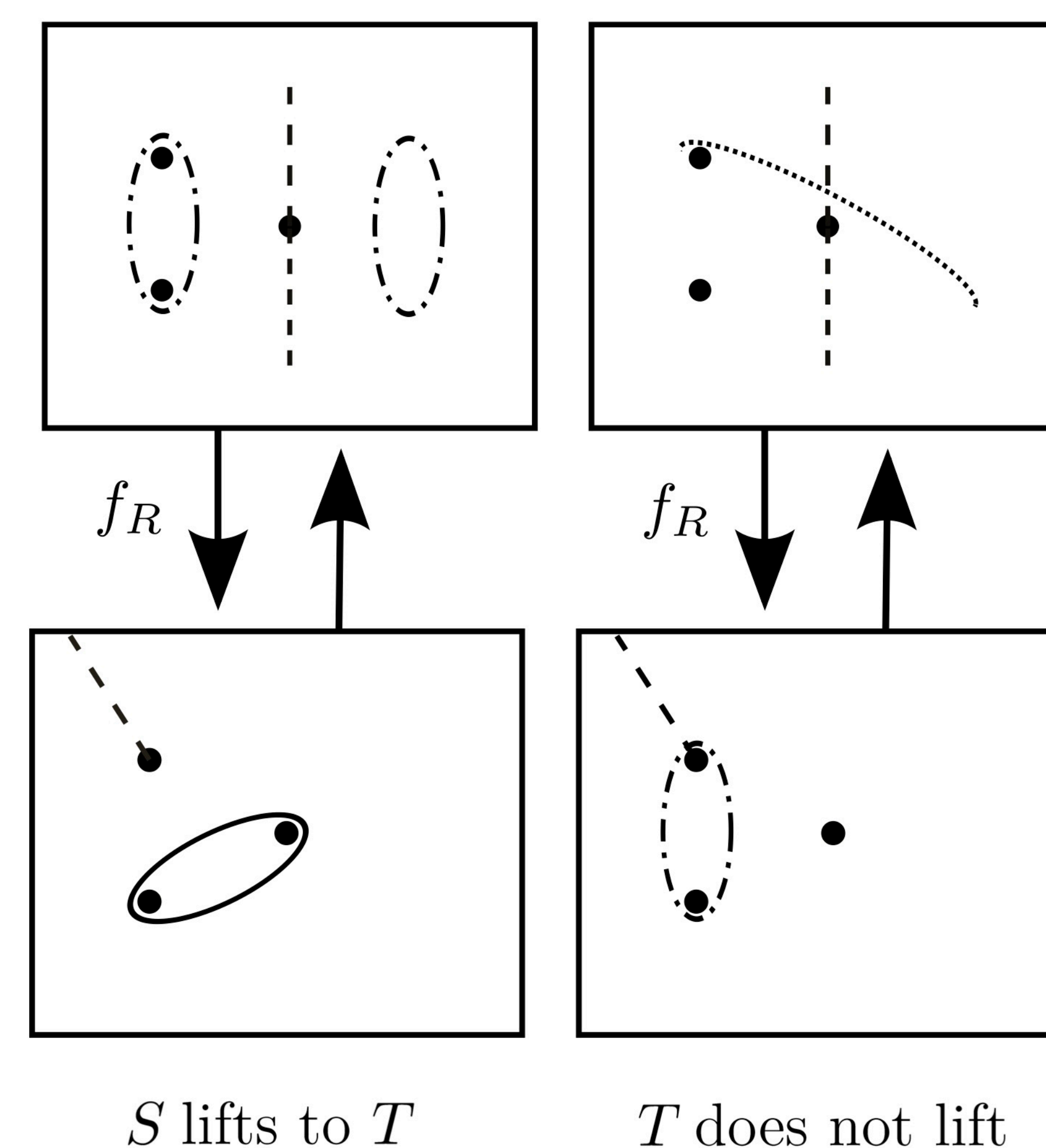
$$g \in \langle S, T \rangle \cong F_2$$

Fact: $f_R \cdot g$ is equivalent to one of f_R , f_C , or f_A .

Bartholdi and Nekrashevych (2006) gave an algorithm for determining which one.



Methods: Reduction by Lifting



Results

1. No non-trivial subgroup of $\langle T \rangle$ is all-rabbit.
2. Every infinitely liftable twist generates an all-rabbit subgroup.
3. In fact, the collection of all infinitely liftable twists generates an all-rabbit subgroup.
4. We developed a program that computes lifts of twists and creates visuals of subgroups.
5. No unliftable twist of word length 12 or less generates an all-rabbit subgroup.
6. We conjecture that no unliftable twist generates an all-rabbit subgroup.

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