Using Time - Dependent Sensitivity Analysis to Combat Tuberculosis

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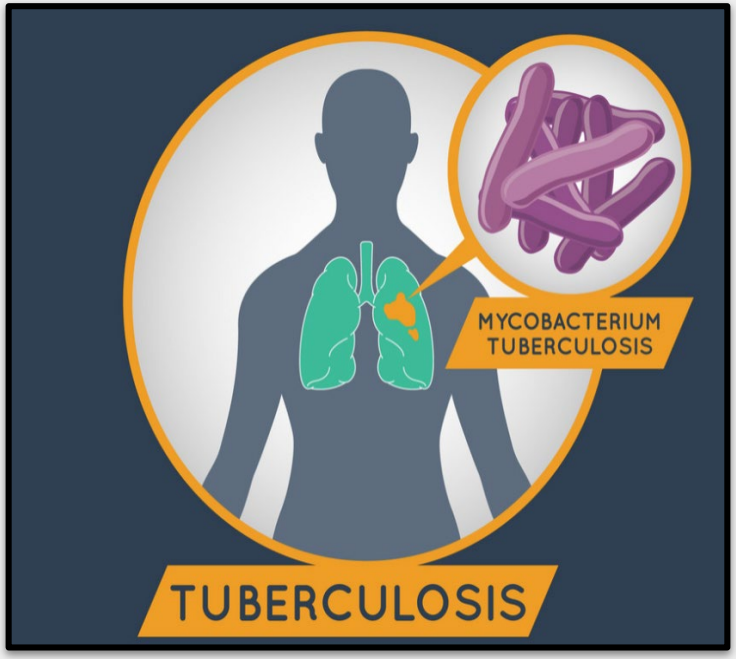
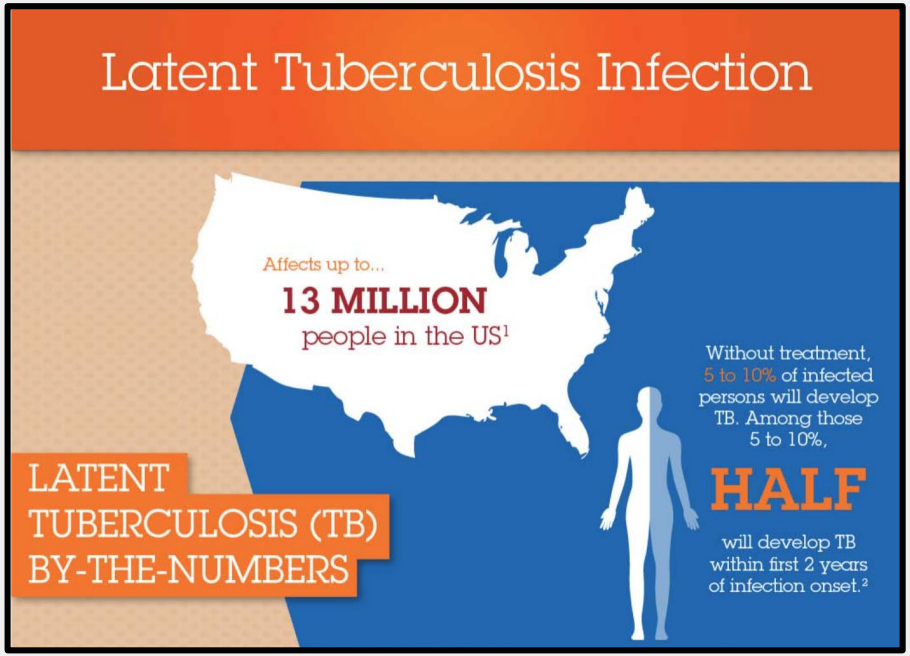
Why are we conducting this research?





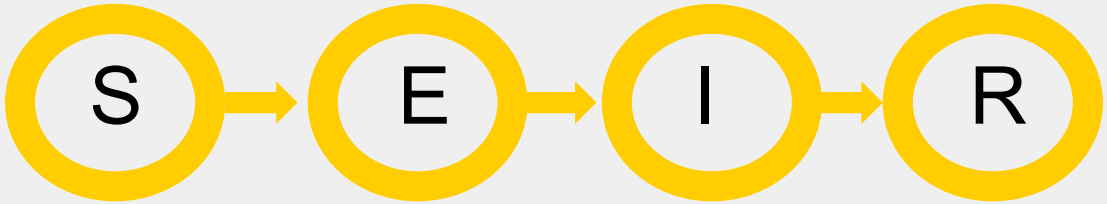
Tuberculosis."VectorStock.com.2018.https://www.vectorstock.cm/royalty - free- vector/tuberculosis - lgo - icon- vector - 17791903

http://www.multivu.com/players/English/7701451 - sanofi- priftin - rifapentine/

Our Research Goal

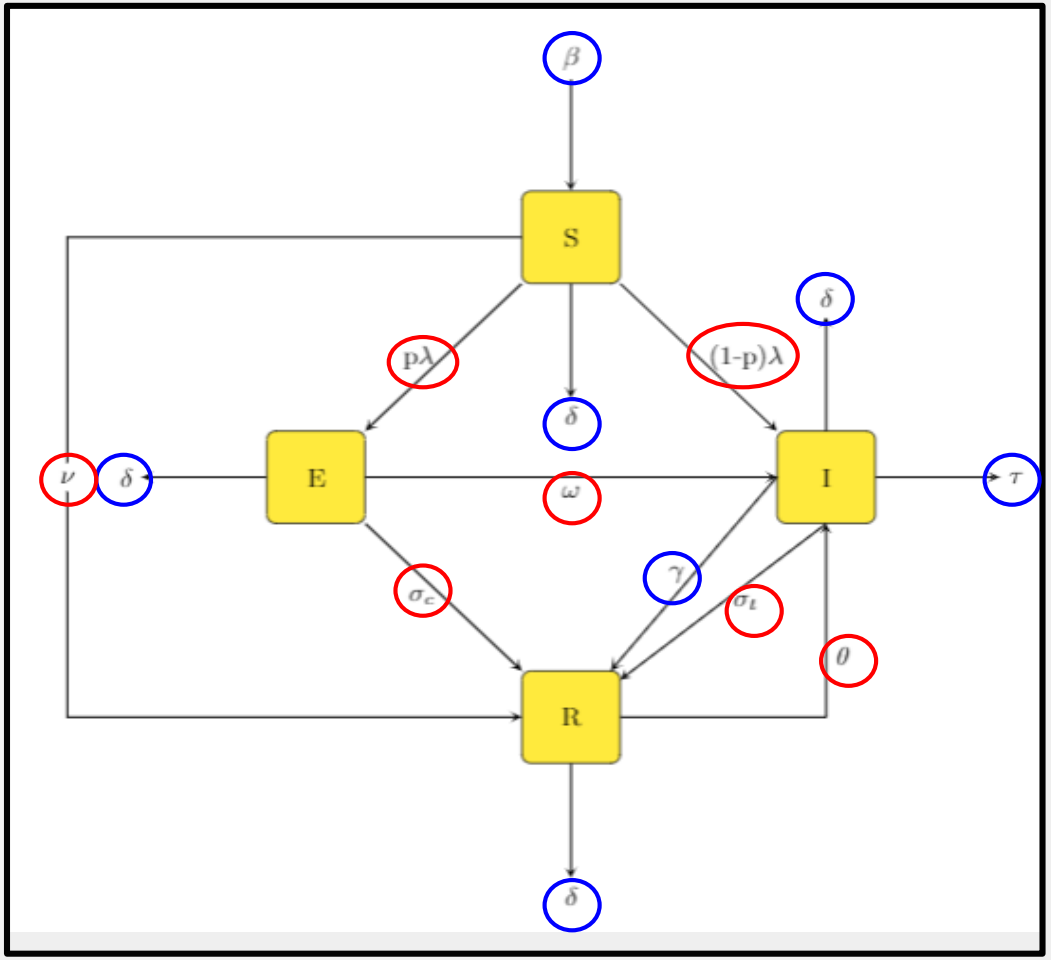
To use compartmental modeling and sensitivity analysis to determine the best method(s) of targeting tuberculosis (TB)

Setup and Assumptions for our Model



S- Susceptible
E- Exposed
I- Infectious
R- Recovered

- Everyone is born susceptible to TB.
- Vaccinations have a 50% efficacy.
- Drug treatment has a 100% efficacy.



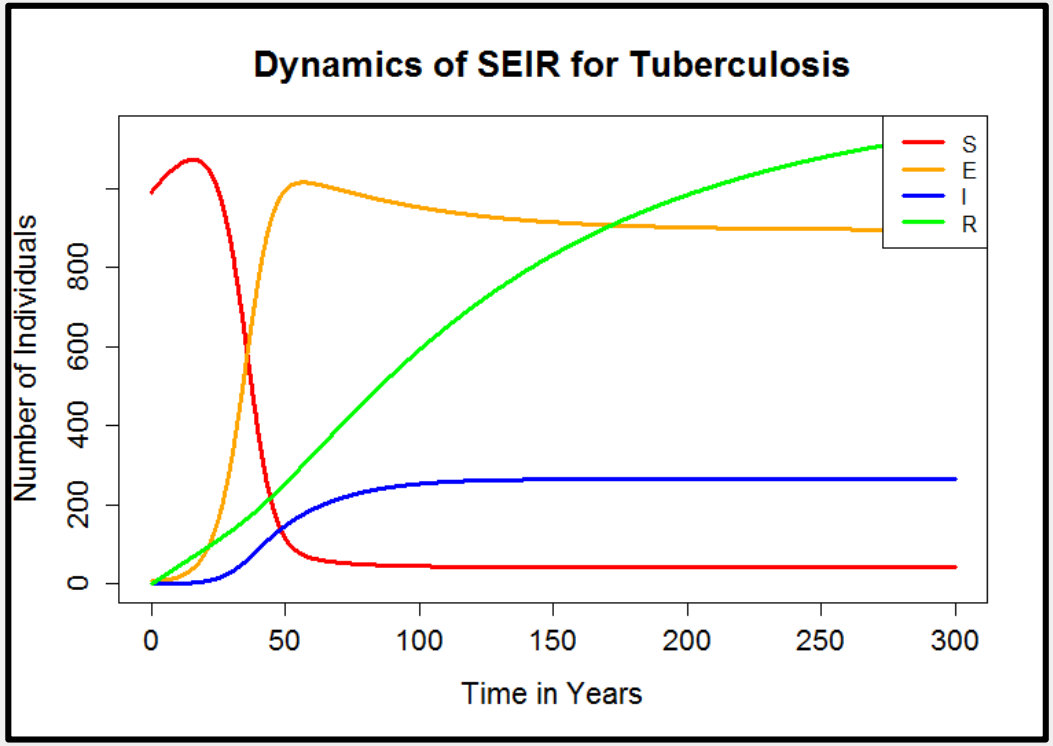
$$\frac{dS}{dt} = \beta - \lambda SI - \delta S - \nu S$$

$$\frac{dE}{dt} = p\lambda SI - (\omega + \sigma_c + \delta)E$$

$$\frac{dI}{dt} = (1-p)\lambda SI + \omega E - (\delta + \tau + \gamma + \sigma_t)I + \theta R$$

$$\frac{dR}{dt} = \nu S + \sigma_c E + (\sigma_t + \gamma)I - (\delta + \theta)R$$

SEIR Population Dynamics



- S—Susceptible (red)
- E—Exposed (orange)
- I—Infectious (blue)
- R—Recovered (green)

- Initial Population: 1000 people (990 susceptible, 10 exposed)

Sensitivity Analysis

- Proportion of the total population that is affected by tuberculosis

$$f(t) = \frac{E(t) + I(t)}{N(t)}$$

E : Exposed population
I : Infectious population
N : Total population

Active Subspace Example (Year 68)

Singular Value Decomposition formula:

$$G = U\Sigma V^T$$

Where is the first significant split in diagonal values?

$$\Sigma = \begin{bmatrix} 1.00 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.64 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.06 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.01 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.004 \end{bmatrix}$$

First two columns = active subspace

$$U = \begin{bmatrix} -0.26 & -0.90 & 0.33 & -0.07 & -0.02 & -0.07 \\ 0.31 & -0.25 & -0.32 & 0.40 & 0.76 & 0.01 \\ 0.04 & 0.06 & -0.04 & -0.11 & 0.06 & -0.99 \\ 0.74 & -0.31 & -0.34 & -0.25 & -0.42 & 0.02 \\ 0.12 & -0.01 & 0.18 & 0.87 & -0.43 & -0.13 \\ -0.52 & -0.14 & -0.80 & 0.12 & -0.23 & -0.03 \end{bmatrix}$$

Two Dimensional

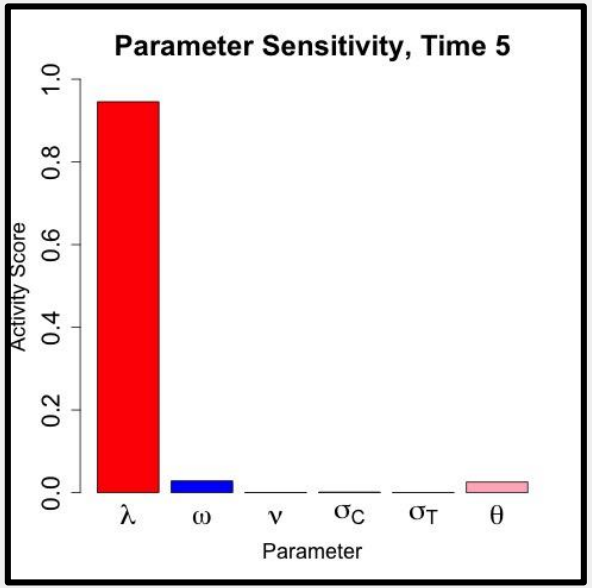
Activity Scoring

$$\alpha_i(n) = \sum_{j=1}^{n} (\sigma_j)^2 (u_{i,j})^2$$

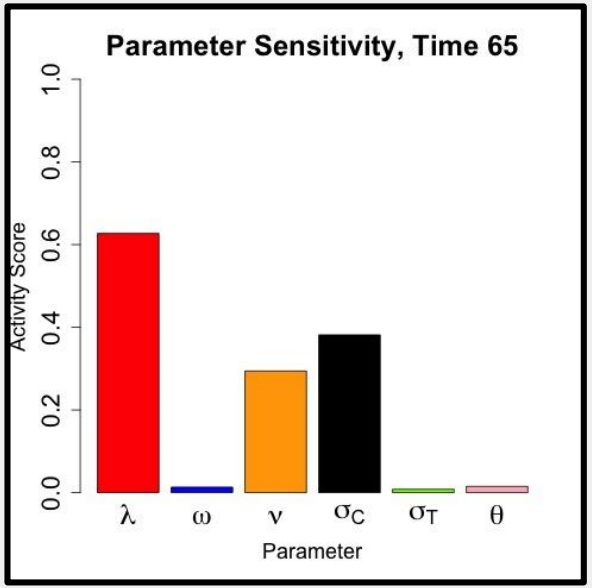
n = number of dimensions
i = any parameter i
σ_j = singular values
$u_{i,j}$ = (i,j)th entry of U_1

- Weighs the contributions from the important directions in the input s for each parameter according to its singular value

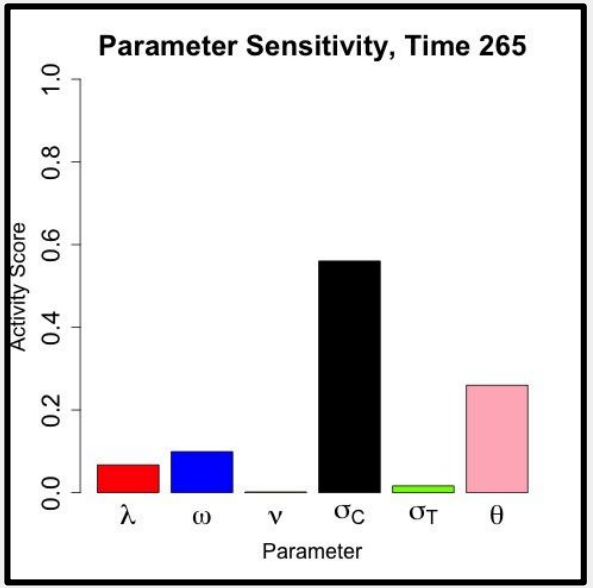
Methods of Targeting TB Based on our Results



Rate of infection (λ)



Latent treatment (σ_c)
Vaccination rate (ν)



Relapse Rate (θ)

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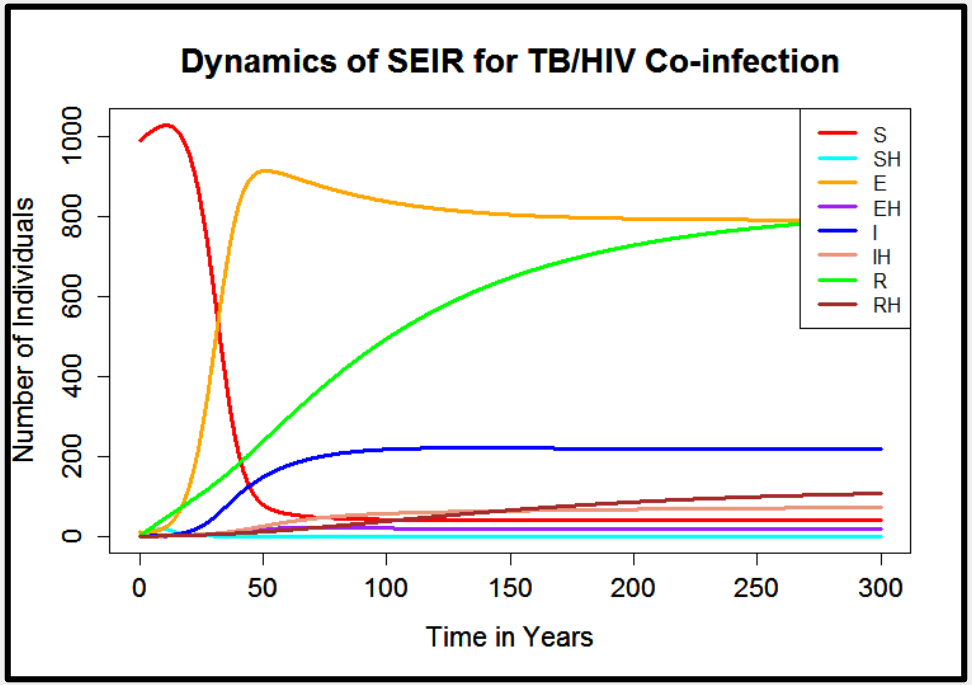
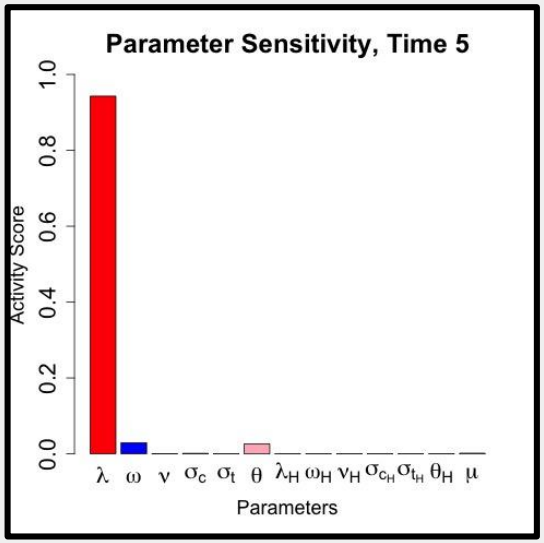
Table of Parameter Values

Parameter	Units	Nominal	Source
(ν) rate of vaccination	$\frac{1}{\text{time}}$	0.00445	WHO
(λ) rate of infection	$\frac{1}{\text{people*time}}$	0.0018	Estimated
(ω) rate of deterioration	$\frac{1}{\text{time}}$	0.0084	TB Facts
(σ_c) latent TB treatment	$\frac{1}{\text{time}}$	0.005	Estimated
(σ_t) active TB treatment	$\frac{1}{\text{time}}$	0.005	Estimated
(θ) rate of reinfection	$\frac{1}{\text{time}}$	0.0005	Blower

SEIR Dynamics (Co - infection Model)



Results For Our Co -infection Model



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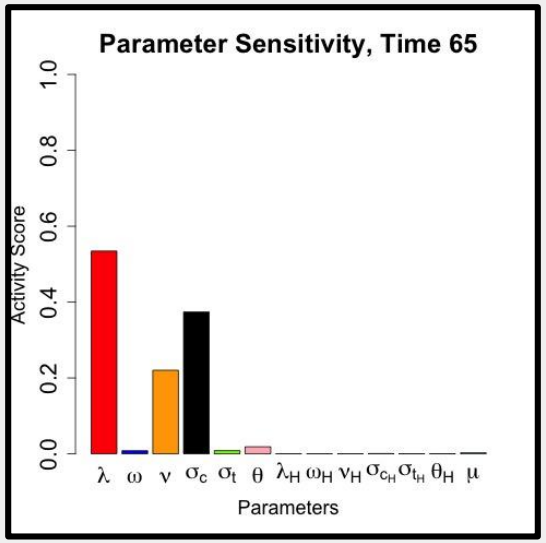

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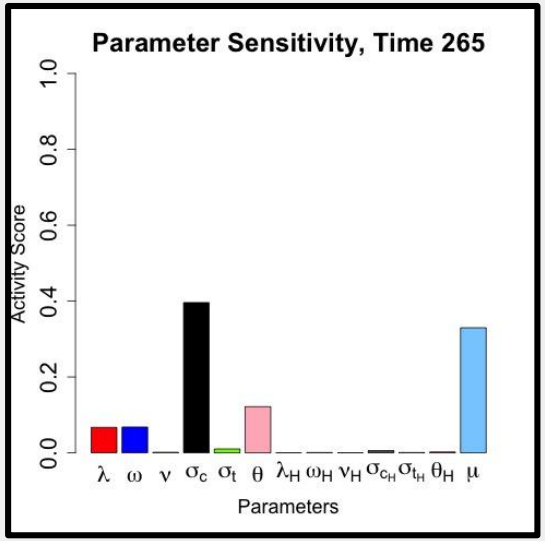
Vaccination rate (ν)



HIV incidence rate (μ)