The Hidden Information in Infinite Series Arising from Graphs

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Definition: A graph is a collection of edges and vertices.

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Definition: A graph is \textit{directed} if the edges have orientation.
Example:

$C_6 = \begin{array}{cc}
\end{array}$

$C_n = \text{cycle on } n \text{ vertices.}$
Definition: A *bouquet* is a collection of $C_n$’s glued at a vertex ($n$ can vary).

Example:
Definition (by example):
Definition (by example):
Definition (by example):
number of closed walks starting at vertex 1 of length 0
number of closed walks starting at vertex 1 of length 1
number of closed walks starting at vertex 1 of length 2
\[ 1, 0, 2, 1, 4, \ldots \]
\[ \leftrightarrow 1, 0, 2, 1, 4, \ldots \]

\[ \leftrightarrow 1, 0, 1, 1, 2, \ldots \]
\[1, 0, 2, 1, 4, \ldots\]

\[1, 0, 1, 1, 2, \ldots\]

\[1, 0, 2, 0, 4, \ldots\]
1, 0, 2, 1, 4, ...
\[ H(t) = 1 + 0t + 2t^2 + t^3 + 4t^4 + \ldots \]
Definition: Let $B$ be a bouquet with center 1 and set

\[ h_i = \text{number of closed walks of length } i \]

starting at vertex 1.

Then the generating function of $B$ is

\[ H(t) = \sum_{i=0}^{\infty} h_i t^i. \]
Theorem (-, Myers)

Let a bouquet have $n$ cycles of lengths $r_1, \ldots, r_n$. Then:

$$H(t) = \frac{1}{1 - tr_1 - \cdots - tr_n}$$
\[ H(t) = 1 + 0t + 2t^2 + t^3 + 4t^4 + \ldots \]
\[ H(t) = 1 + 0t + 2t^2 + 1t^3 + 4t^4 + \ldots \]
\[ H(t) = 1 + 0t + 2t^2 + 0t^3 + 1t^4 + 4t^5 + \ldots \]
\[ H(t) = 1 + 0t + 2t^2 + 1t^3 + 4t^4 + \ldots \]
\[ \begin{align*}
H(t) &= 1 + 2t^2 H(t) + t^3 H(t) \\
H(t) &= \frac{1}{1 + 0t + 2t^2 + t^3 + 4t^4 + \ldots} \\
&= \frac{t^2}{t^2 + 0t^3 + 2t^4 + \ldots} + \frac{t^2}{t^2 + 0t^3 + 2t^4 + \ldots} + \frac{t^3}{t^3 + 0t^4 + \ldots} \\
\end{align*} \]
\[
H(t) = 1 + 2t^2 H(t) + t^3 H(t)
\]

\[
H(t) = \frac{1}{1 - 2t^2 - t^3}
\]
Future Work
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