The Hidden Information in Infinite Series Arising from Graphs

Juliann Geraci

SUNY Oswego

Definition: A *graph* is a collection of edges and vertices.

Definition: A graph is *directed* if the edges have orientation.

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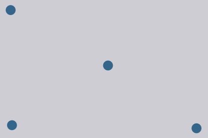
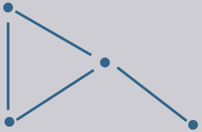


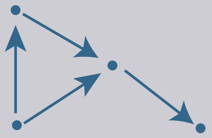
Definition: A graph is *directed* if the edges have orientation.

Definition: A *graph* is a collection of edges and vertices.



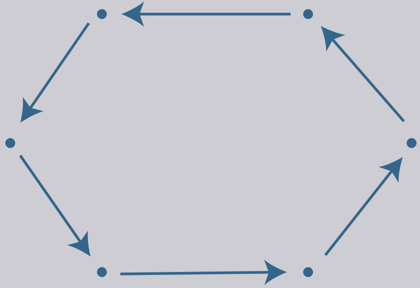
Definition: A graph is *directed* if the edges have orientation.

Definition: A *graph* is a collection of edges and vertices.



Definition: A graph is *directed* if the edges have orientation.

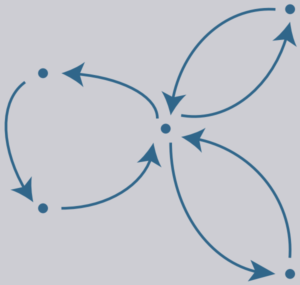
Example:

$C_6 =$ 

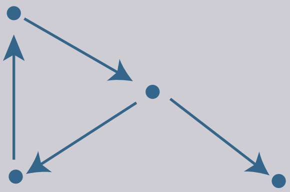
$C_n =$ cycle on n vertices.

Definition: A *bouquet* is a collection of C_n's glued at a vertex (n can vary).

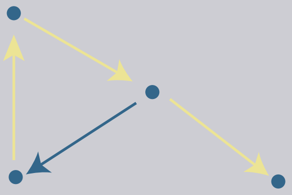
Example:



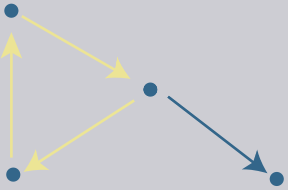
Definition (by example):

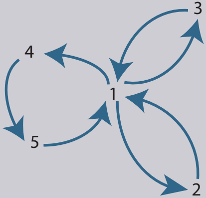


Definition (by example):



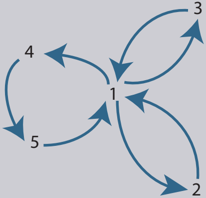
Definition (by example):





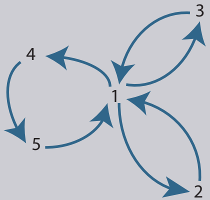
$\longleftrightarrow$ 1

number of closed walks starting at vertex 1 of length 0



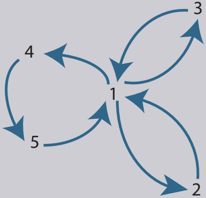
$\leftrightarrow$ 1, 0

number of closed walks starting at vertex 1 of length 1

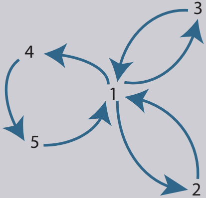


$\longleftrightarrow$ 1, 0, 2

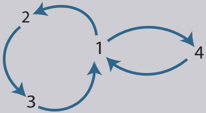
number of closed walks starting at vertex 1 of length 2



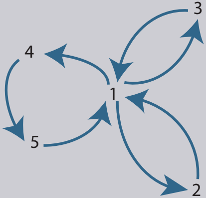
$\longleftrightarrow$ 1, 0, 2, 1, 4, …



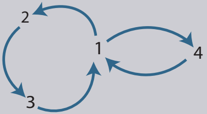
↔ 1, 0, 2, 1, 4, ...



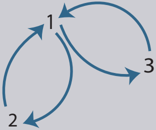
↔ 1, 0, 1, 1, 2, ...



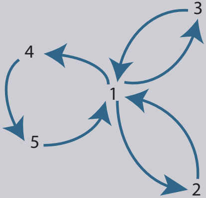
↔ 1, 0, 2, 1, 4, ...



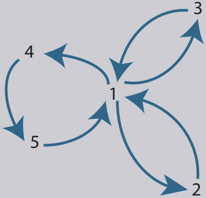
↔ 1, 0, 1, 1, 2, ...



↔ 1, 0, 2, 0, 4, ...



↔ 1, 0, 2, 1, 4, ...



$\leftrightarrow$ 1, 0, 2, 1, 4, ...

$$H(t) = 1 + 0t + 2t^2 + t^3 + 4t^4 + \ldots$$

Definition: Let B be a bouquet with center 1 and set

h_i = number of closed walks of length i starting at vertex 1.

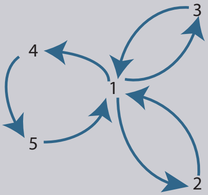
Then the *generating function of* B is

$$H(t) = \sum_{i=0}^{\infty} h_i t^i.$$

Theorem (-, Myers)

Let a bouquet have n cycles of lengths $r_1, \ldots, r_n$. Then:

$$H(t) = \frac{1}{1 - t^{r_1} - \cdots - t^{r_n}}$$



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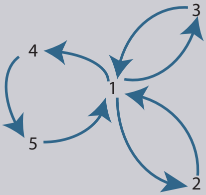

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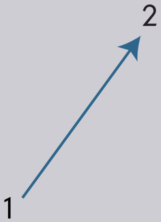

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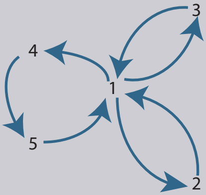
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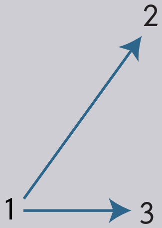
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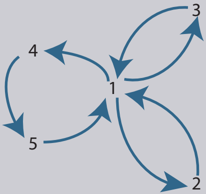

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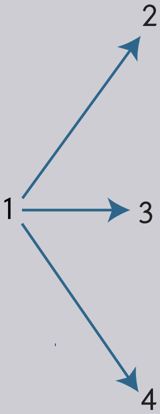

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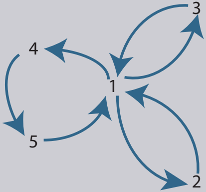
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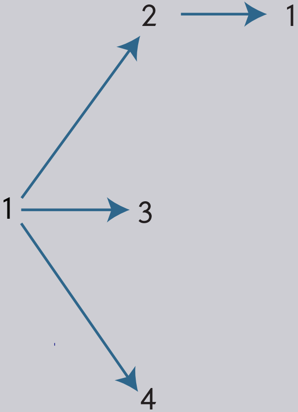
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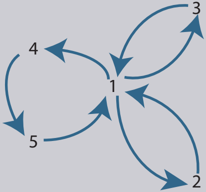
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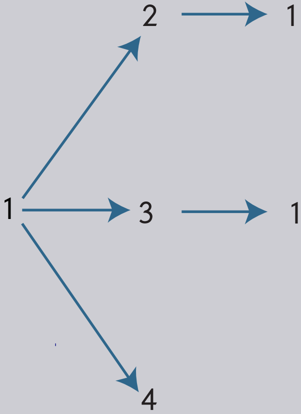
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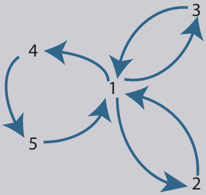
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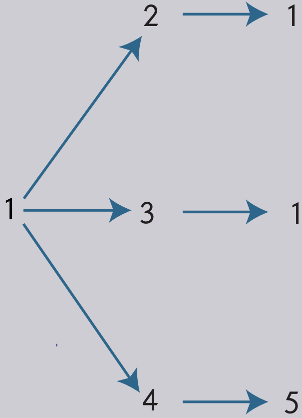
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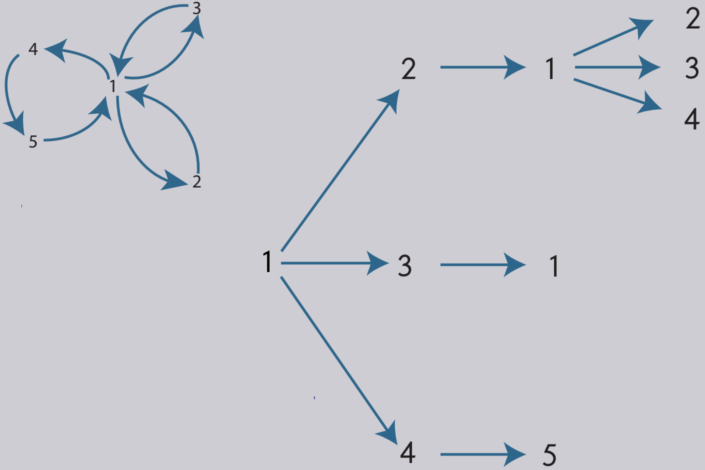

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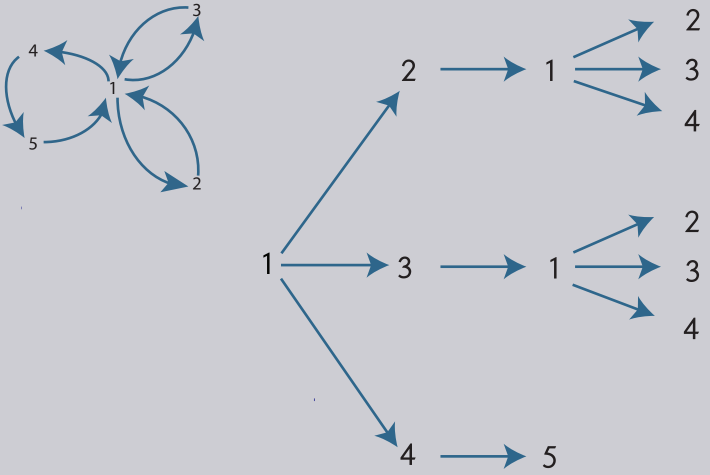

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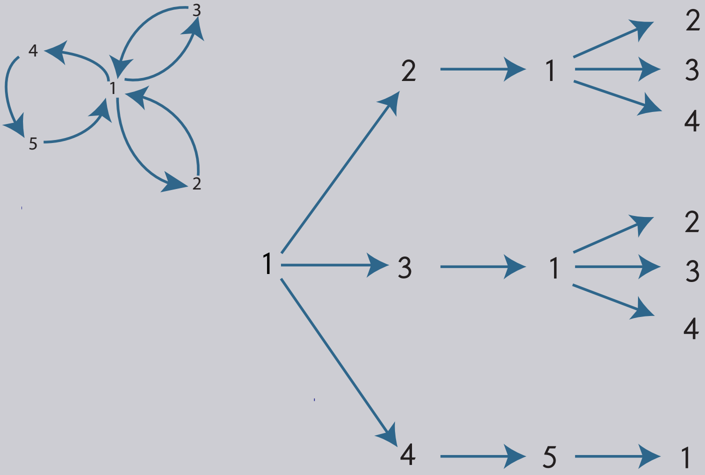
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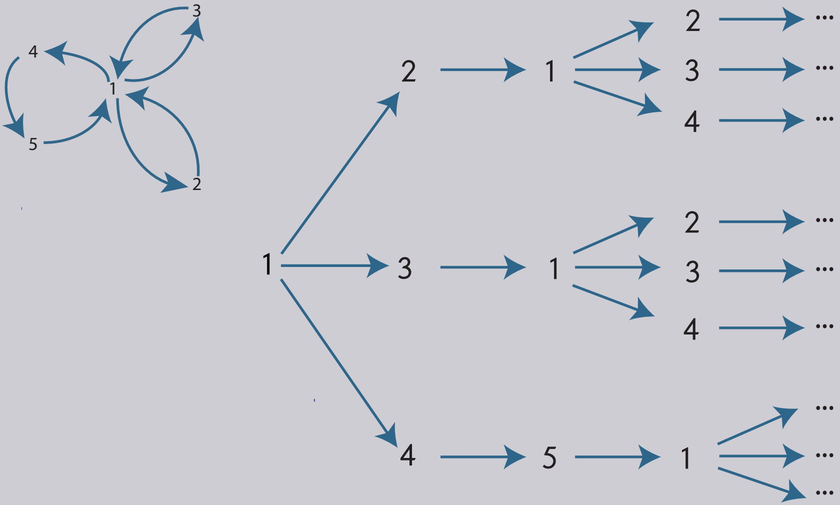

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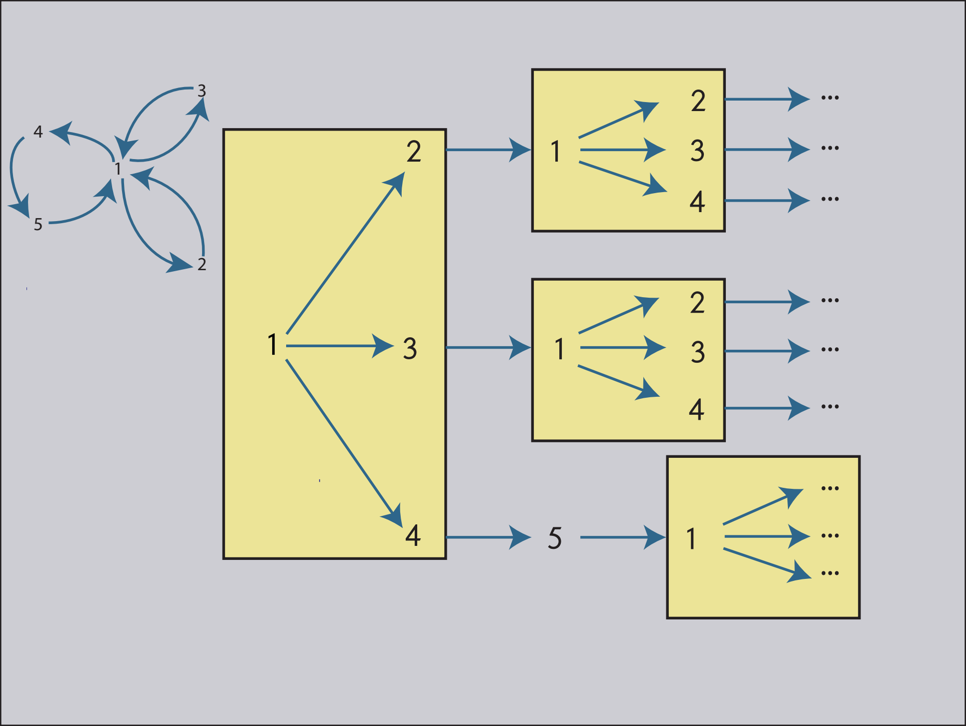

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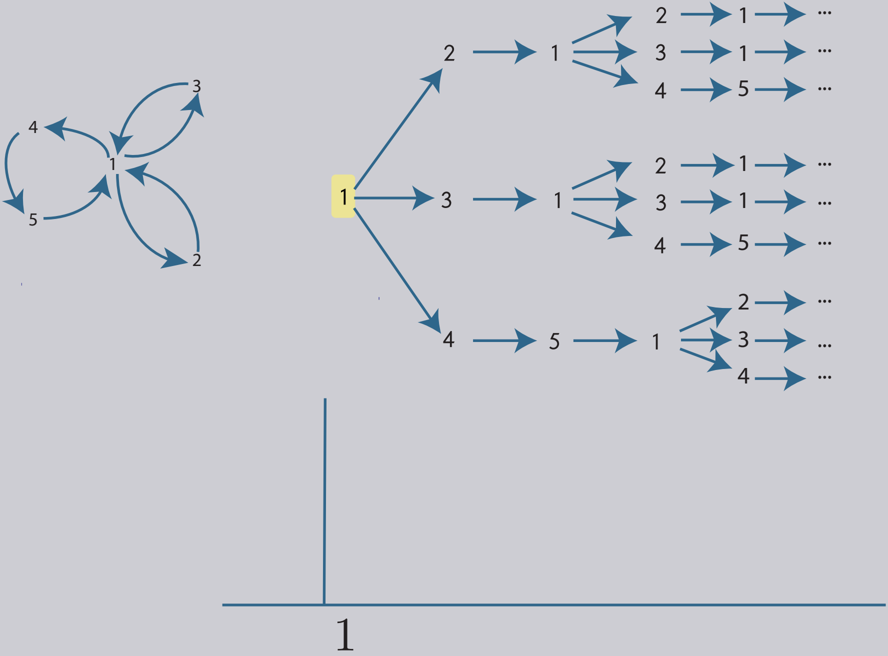
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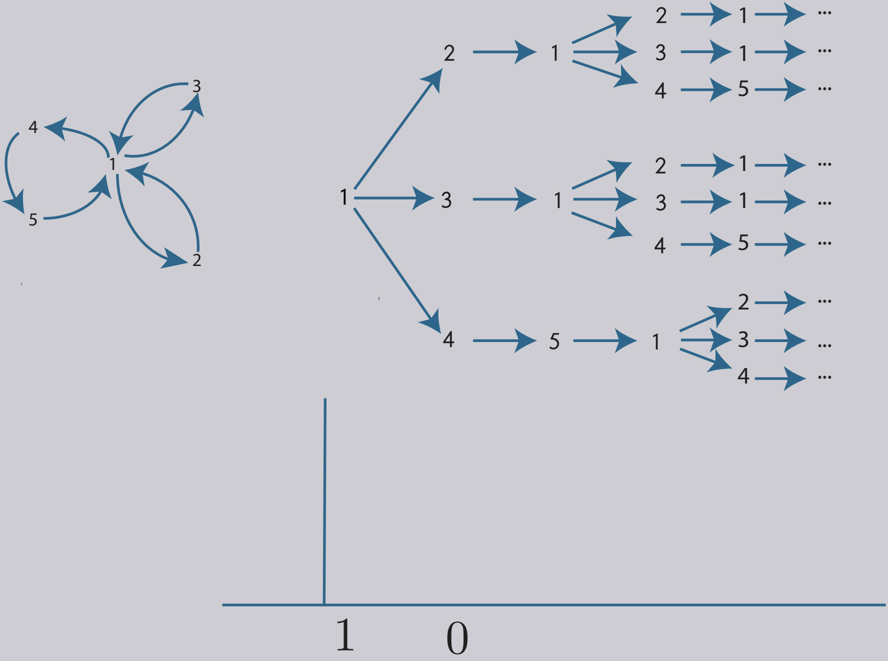

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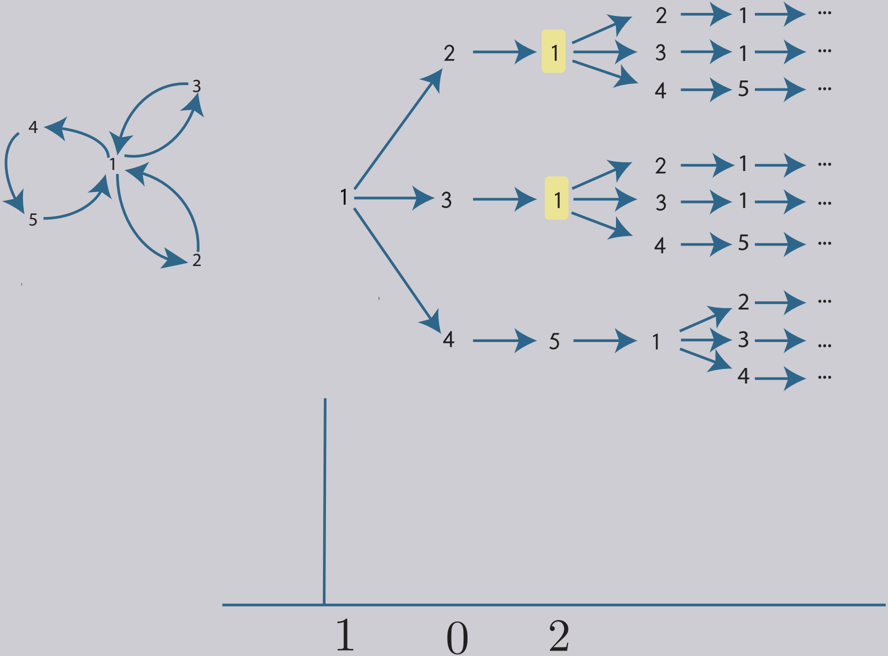
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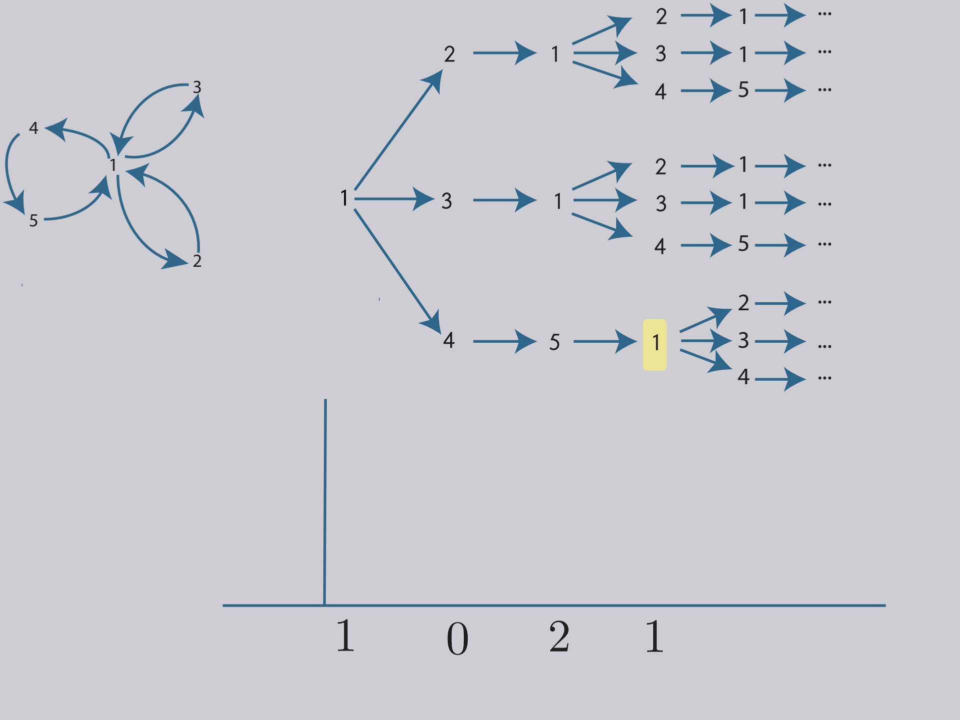

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1 2 3 4 5 1 2 3 4 5
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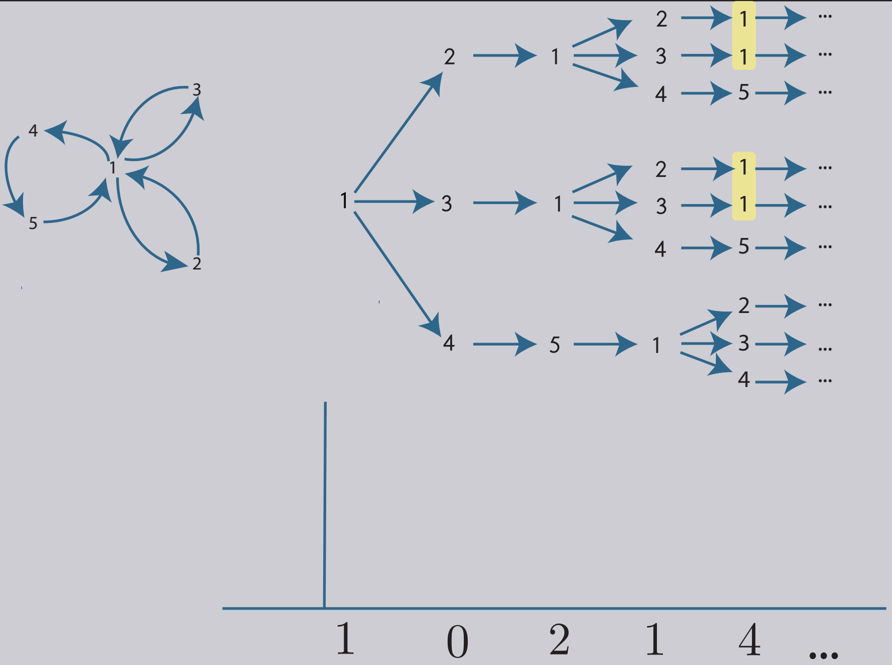
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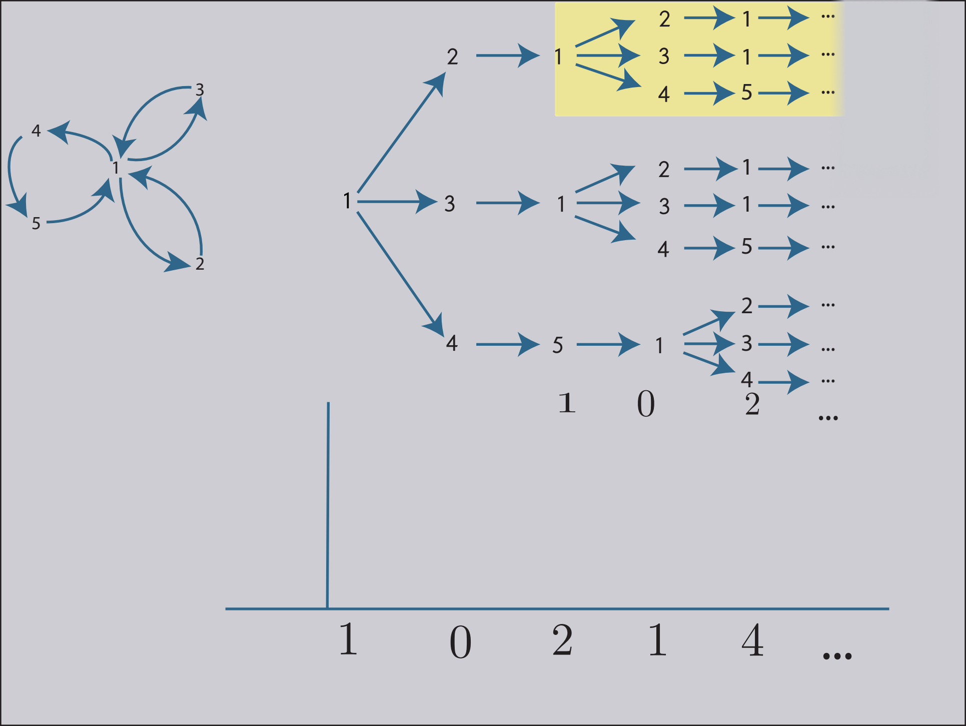

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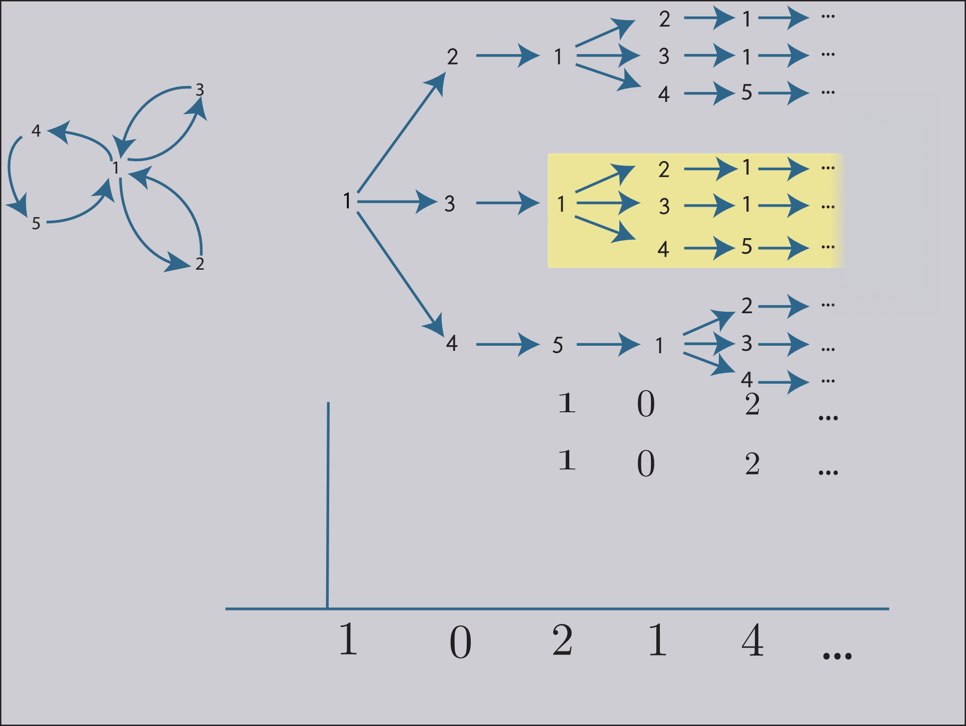
1 0 2 1 4 …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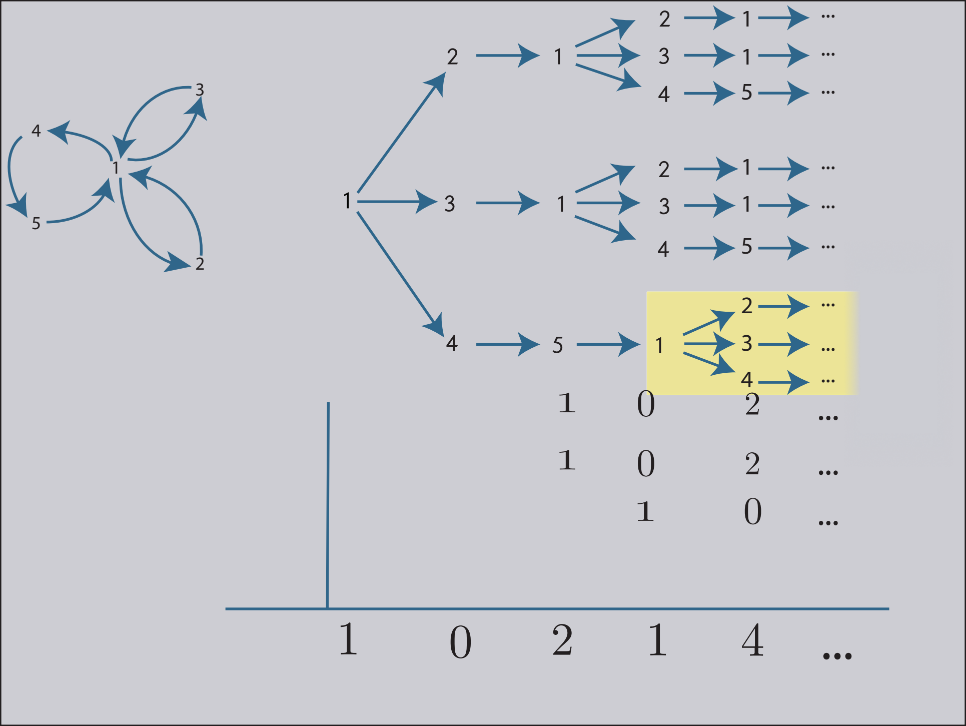

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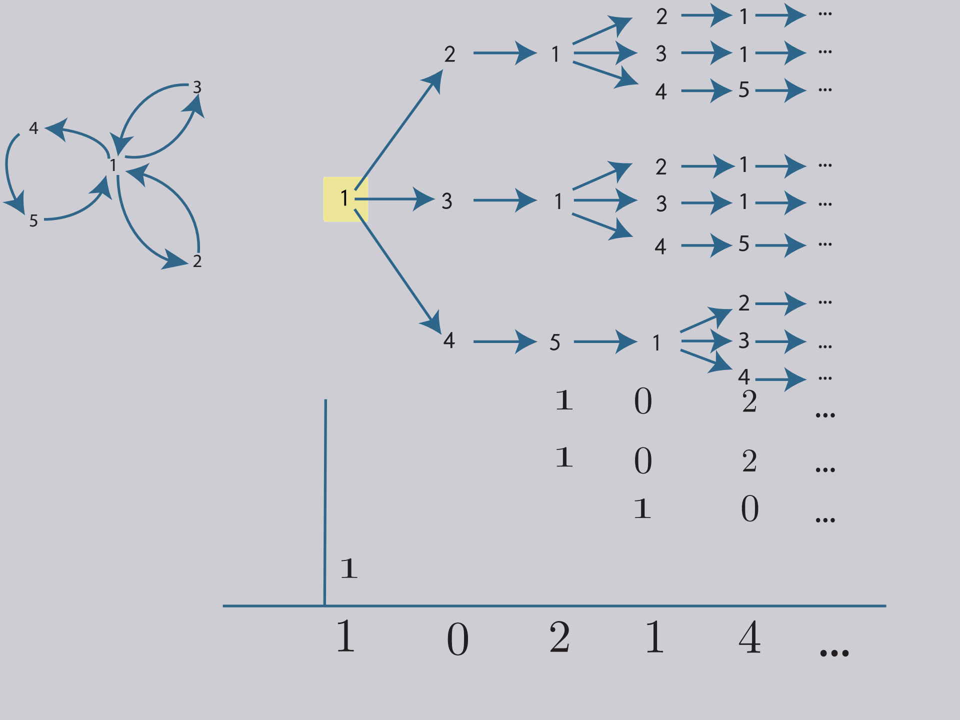
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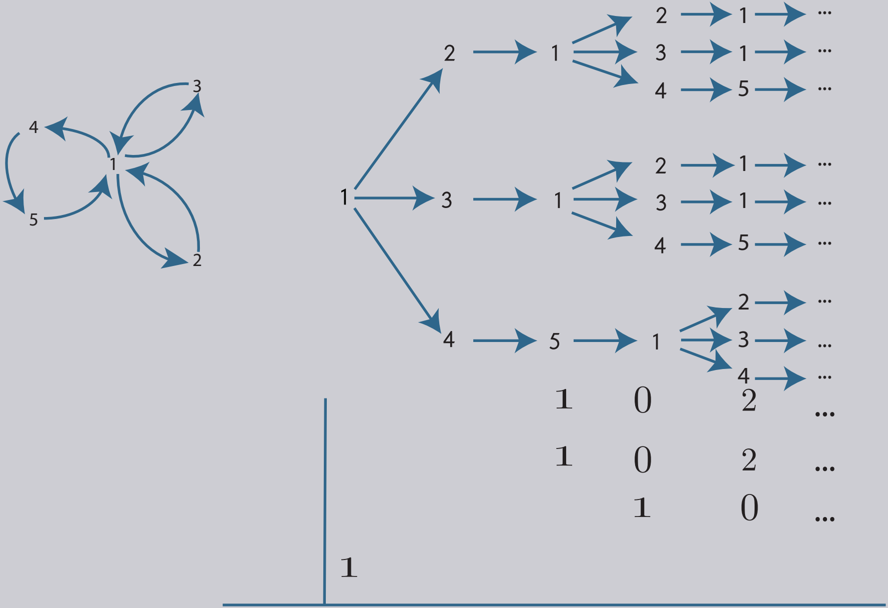
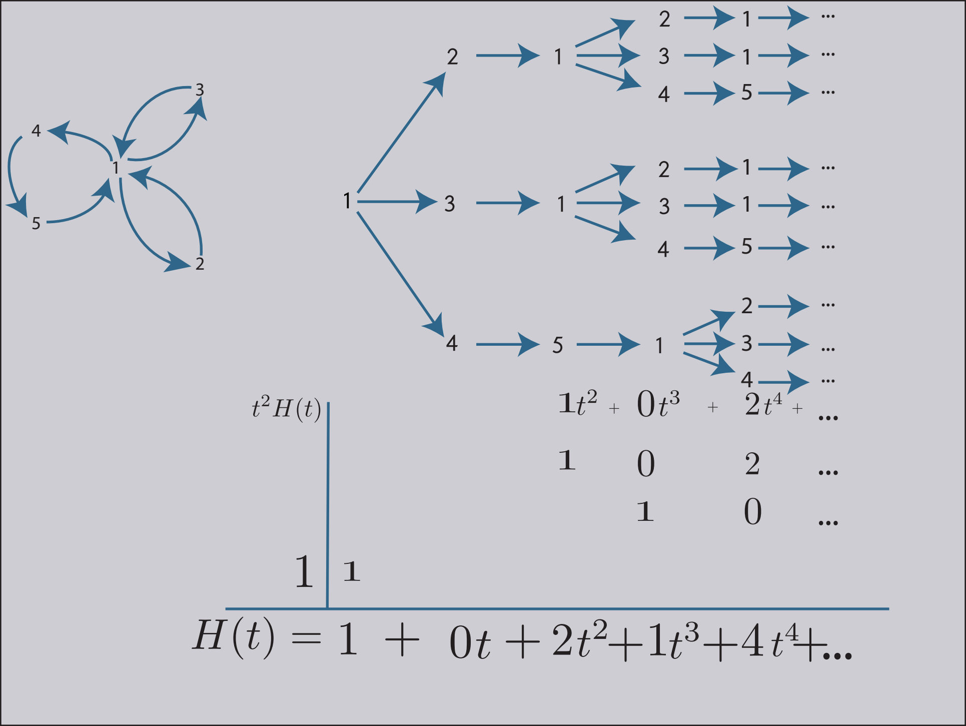
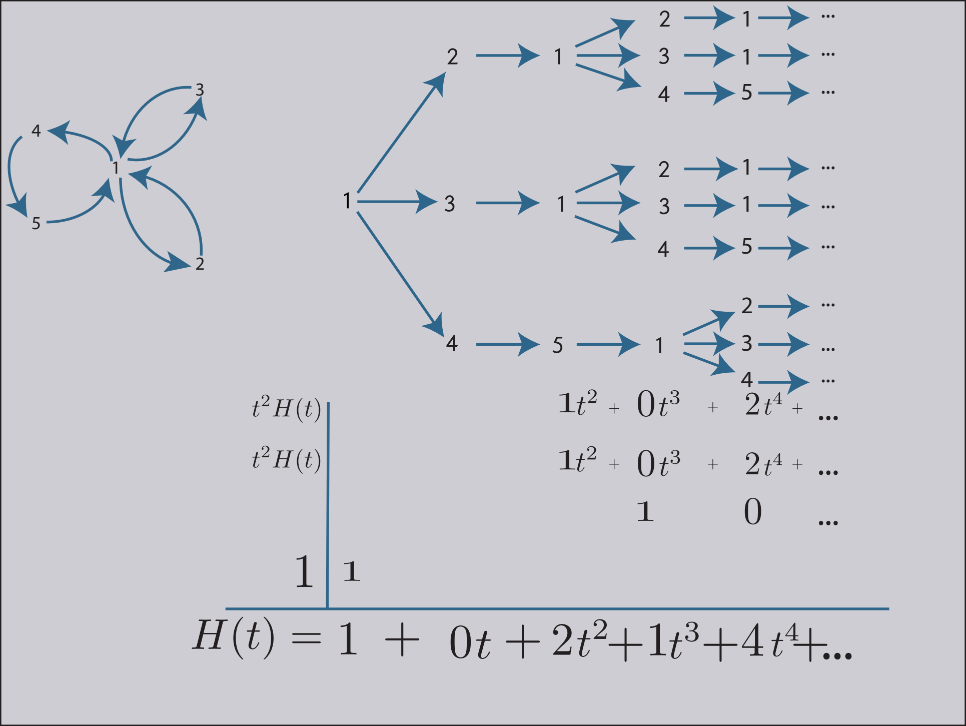
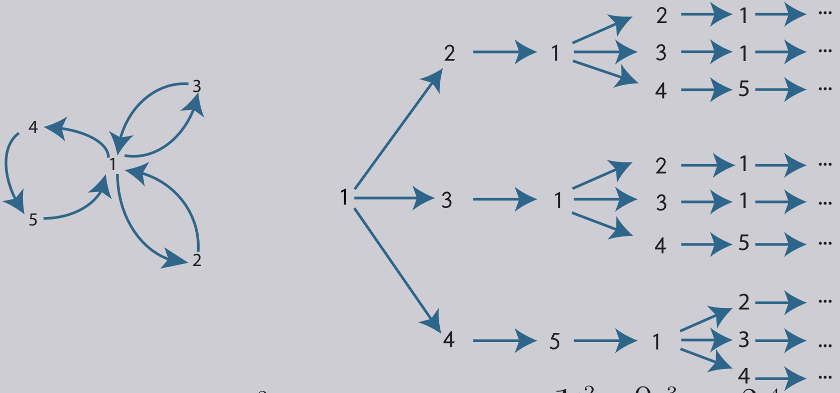


$$H(t) = 1 + 0t + 2t^2 + 1t^3 + 4t^4 + \dots$$



1 2 3 4 5
t²H(t)
1t² + 0t³ + 2t⁴ + ...
1 0 2 ...
1 0 ...
1 1
H(t) = 1 + 0t + 2t² + 1t³ + 4t⁴ + ...


$t^2H(t)$
$t^2H(t)$
$1t^2 + 0t^3 + 2t^4 + \dots$
$1t^2 + 0t^3 + 2t^4 + \dots$
$1 \quad 0 \quad \dots$
$1 \quad 1$
$H(t) = 1 + 0t + 2t^2 + 1t^3 + 4t^4 + \dots$



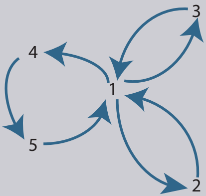
$$t^2H(t) \quad 1t^2 + 0t^3 + 2t^4 + \dots$$

$$t^2H(t) \quad 1t^2 + 0t^3 + 2t^4 + \dots$$

$$t^3H(t) \quad 1t^3 + 0t^4 + \dots$$

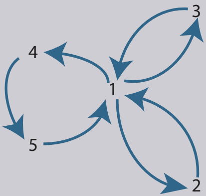
$$1 \quad 1$$

$$H(t) = 1 + 0t + 2t^2 + 1t^3 + 4t^4 + \dots$$



$$\begin{array}{r|l}
t^2H(t) & t^2+0t^3+2t^4+\ldots \\
+\ t^2H(t) & t^2+0t^3+2t^4+\ldots \\
+\ t^3H(t) & t^3+0t^4+\ldots \\
+\ 1 & 1 \\
\hline
H(t) & 1+0t+2t^2+t^3+4t^4+\ldots
\end{array}$$

$$H(t)=1+2t^2H(t)+t^3H(t)$$

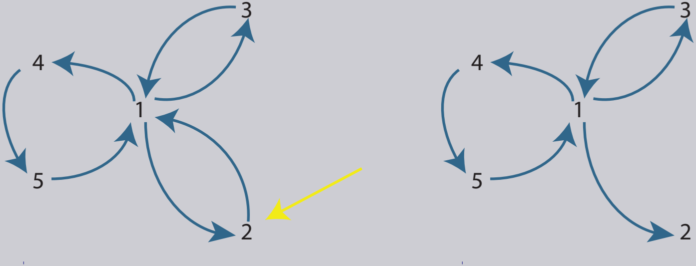


$$\begin{array}{r|l} t^2H(t) & t^2+0t^3+2t^4+\dots \\ +\ t^2H(t) & t^2+0t^3+2t^4+\dots \\ +\ t^3H(t) & t^3+0t^4+\dots \\ +\ 1 & 1 \\ \hline H(t) & 1+0t+2t^2+t^3+4t^4+\dots \end{array}$$

$$H(t) = 1 + 2t^2H(t) + t^3H(t)$$

$$H(t) = \frac{1}{1-2t^2-t^3}$$

Future Work



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