Boundary Layer Transition/Separation in Turbulent Fluids

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The Evolution of Fluid Dynamics

- Euler equations (1755) - Leonhard Euler
  - Two coupled, nonlinear partial differential equations
  - Derived from Newton’s laws
  - Ignored viscosity
- Navier-Stokes equations (1822) - Claude-Louis Navier and George Stokes
  - Accounted for internal friction
  - Most accurate mathematical description of fluid flow to date
- First concept of boundary layer (1904) - Ludwig Prandtl
  - Fluid adjacent to the surface sticks to the surface, forming a very thin layer
  - Frictional effects only observed inside the boundary layer
  - Flow separation
  - Enabled us to study wake turbulence
Turbulent Flow Control for Drag Reduction


• The potential impact of turbulent flow control in many engineering applications:
  - Commercial airliners: up to 50% of the fuel consumption is associated with turbulent drag.
  - Alaska pipeline: adding small amount of long-chain polymers reduces friction loss of turbulent flows dramatically (1.44 million barrel/day → 2.14 million barrel/day; 50% increase).
  - Flow control technologies that reduce drag by 1% could save up $2 billion annually.

A350 XWB

Alaska pipeline for oil transport
Reynolds Number

\[ Re = \frac{\rho UL}{\mu} = \frac{\text{Inertial}}{\text{Viscous}} \]

- Dimensionless quantity
- Ratio of inertial forces to viscous forces

\( \rho \) - Density of the fluid
\( u \) - Velocity of the fluid with respect to the object
\( L \) - Characteristic length
\( \mu \) - Viscosity of the fluid
Boundary layer

Tollmien-Schlichting waves

Re\textsubscript{x} \sim 1
- leading edge region:
- full N-S equations

Re\textsubscript{x} \gg 1
- laminar B.L. equations valid; initial condition forgotten at \( x_0 \) required
- similarity; initial condition forgotten

\( \delta(x) \)

\( x_0 \)
- instability: disturbances grow and interact
- transition: flow becomes increasingly irregular downstream

Re\textsubscript{c} \sim 10^6
- First occurrence of growth of disturbance

\( U \)

Extent of viscous flow
Boundary layer equations

- Two-dimensional Navier-Stokes equations:

\[
\begin{align*}
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \\
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)
\end{align*}
\]

Continuity:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,
\]

Assumptions for boundary layer:

- \( u \gg v \)
- \( \frac{\partial}{\partial y} \gg \frac{\partial}{\partial x} \)
- \( Re \gg 1 \)
- \( U = \text{constant} \)
Boundary layer equations

- Boundary layer equations:

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2},
\]

\[
- \frac{1}{\rho} \frac{\partial p}{\partial y} = 0, \quad \Rightarrow \quad p = p(x)
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.
\]

- Steady laminar boundary layer equations on a flat surface (U=const)

  - Bernoulli equation:

    \[
    \frac{1}{2} U^2 + \frac{p}{\rho} = \text{const} \quad \Rightarrow \quad \frac{dp}{dx} = -\rho U \frac{dU}{dx} = 0
    \]

  - Finally we have the following equation:

    \[
    u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}
    \]

- Boundary conditions:

  \[
  \begin{align*}
  u &= 0 \text{ at } y = 0, \\
  v &= 0 \text{ at } y = 0, \\
  u &\rightarrow U \text{ as } y \rightarrow \infty
  \end{align*}
  \]
Boundary layer equations

• What we have left:

\[ \frac{u}{\partial x} \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \]

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \]

- Boundary conditions:

\[ u = 0 \text{ at } y = 0, \]
\[ v = 0 \text{ at } y = 0, \]
\[ u \to U \text{ as } y \to \infty. \]

• We’re going to use these two equations to derive the Blasius equation – a single ordinary differential equation for laminar boundary flow at zero pressure gradient.
Boundary layer equations

• Similarity solution for boundary layer equations
  - Similarity variable: \[ \eta = \frac{y}{\delta} = \frac{y}{(\nu x/U)^{1/2}} \]
  - Stream function: \[ \psi = U\delta f(\eta) \]
    \[ u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \]

- Plugging the similarity variable and stream function to BL equations and using the chain rule for derivatives:

\[ f''' + \frac{1}{2} f f'' = 0 \]

- Boundary conditions:

\[ f = f' = 0 \quad \text{at} \quad \eta = 0 \]

\[ f' = 1 \quad \text{as} \quad \eta \to \infty \]

Third-order ordinary differential equation (ODE) \( \rightarrow \) three first-order ODEs

➢ How to solve? We use Runge-Kutta method with secant method
Boundary layer on a flat surface

- Blasius boundary layer for a 1.0 m/s airflow over a 3-m-long surface
Boundary layer on a flat surface

- Blasius boundary layer for a 1.0 m/s airflow over a 3-m-long surface
  - Velocity profiles:
Boundary layer on a flat surface

- Wall shear stress

\[ \tau_w = \mu \frac{\delta u}{\delta y} |y = 0 \]

- Flow separation occurs when

\[ \tau_w = 0 \]
What about turbulence?
Direct Numerical Simulations

• Channel geometry
  - $x$: streamwise, $y$: wall-normal, $z$: spanwise

Navier-Stokes equations for an incompressible Newtonian fluid:

$$\nabla \cdot \mathbf{u} = 0,$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u}$$

Reynolds number: $Re = \frac{\rho U L}{\mu} = \frac{\text{Inertial}}{\text{Viscous}}$

• The Navier-Stokes equations are solved spectrally using the Fourier ($x$) – Chebyshev ($y$) – Fourier ($z$) spatial discretization

DNS Code: ChannelFlow (J. Gibson) – Modified for this Study
Transition to turbulence

- At around Re = 15,000, transition appears to occur.
Transition to turbulence
Acknowledgement

Funding

• NASA Nebraska Fellowship
• Nebraska EPSCoR Program
• Startup funds from the University of Nebraska-Lincoln

Computing Resources

• Holland Computing Center

Thank you!