Boundary Layer Transition/Separation in Turbulent Fluids

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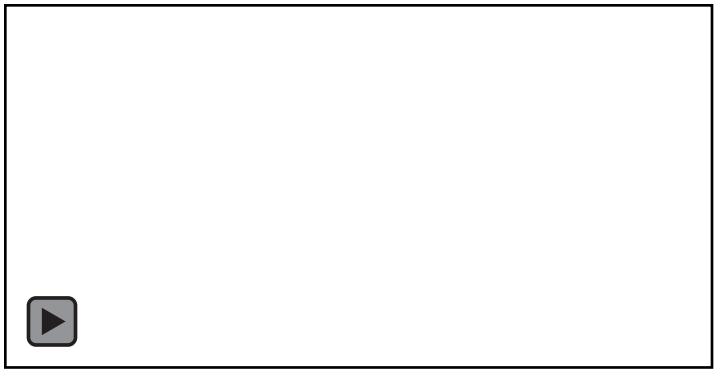


The Evolution of Fluid Dynamics

- Euler equations (1755) - Leonhard Euler
  - Two coupled, nonlinear partial differential equations
  - Derived from Newton's laws
  - Ignored viscosity
- Navier-Stokes equations (1822) - Claude-Louis Navier and George Stokes
  - Accounted for internal friction
  - Most accurate mathematical description of fluid flow to date
- **First concept of boundary layer (1904) - Ludwig Prandtl**
  - Fluid adjacent to the surface sticks to the surface, forming a very thin layer
  - Frictional effects only observed inside the boundary layer
  - Flow separation
  - Enabled us to study wake turbulence

Ludwig Prandtl

Turbulent Flow Control for Drag Reduction

McKeon, *et al. Phys. Fluids* (2013); Toms, *Proc. of In. Cong. On Rheology (1948)*

- The potential impact of turbulent flow control in many engineering applications:
  - Commercial airliners: up to 50% of the fuel consumption is associated with turbulent drag.
  - Alaska pipeline: adding small amount of long-chain polymers reduces friction loss of turbulent flows dramatically (1.44 million barrel/day → 2.14 million barrel/day; 50% increase).
  - Flow control technologies that reduce drag by 1% could save up $2 billion annually.

Reynolds Number

$$Re = \frac{\rho U L}{\mu} = \frac{\text{Inertial}}{\text{Viscous}}$$

- Dimensionless quantity
- Ratio of inertial forces to viscous forces

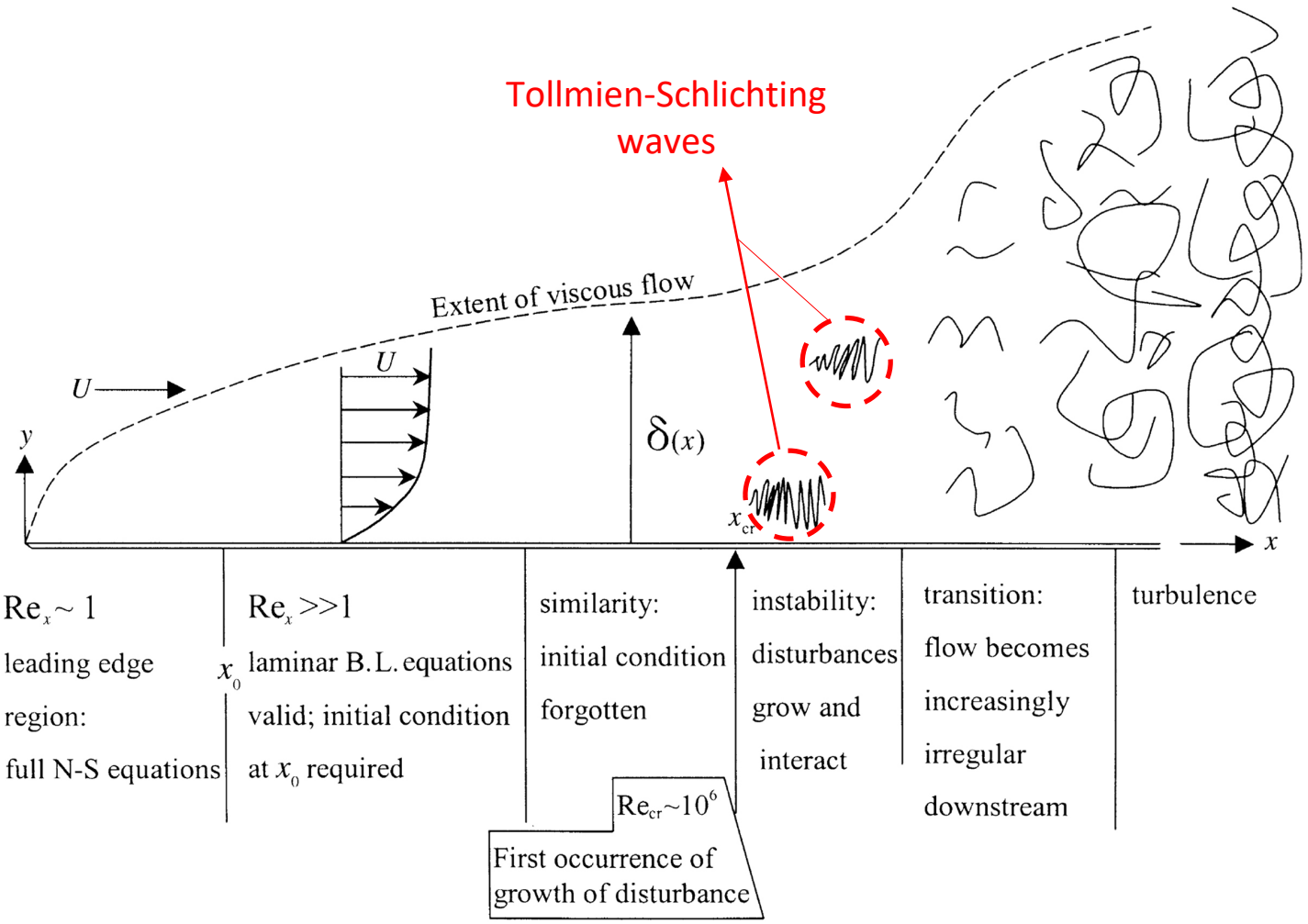
ρ - Density of the fluid

u - Velocity of the fluid with respect to the object

L - Characteristic length

μ - Viscosity of the fluid

Boundary layer



Boundary layer equations

- Two-dimensional Navier-Stokes equations:

X: $$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

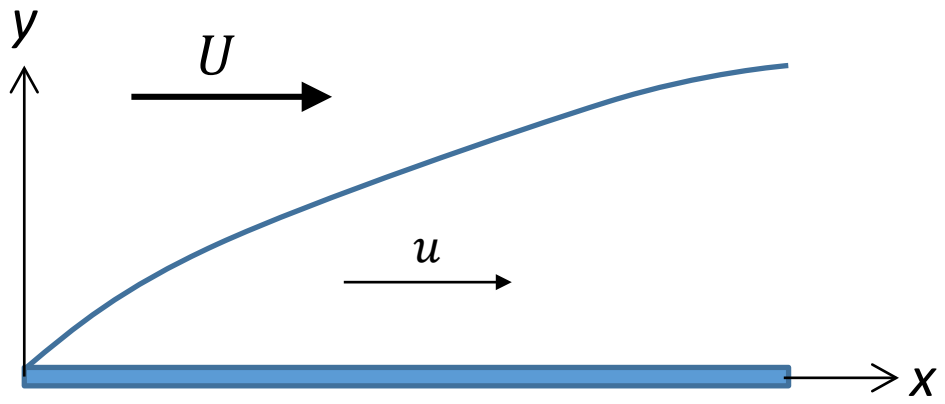
Y: $$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

Continuity: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

p = pressure
ρ = density
ν = viscosity
(u, v) = velocity components

Assumptions for boundary layer:

- $u \gg v$
- $\frac{\partial}{\partial y} \gg \frac{\partial}{\partial x}$
- $Re \gg 1$
- U = constant



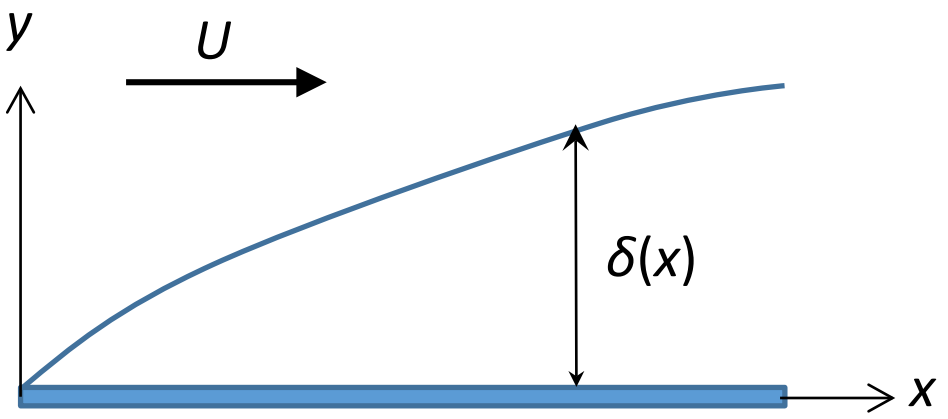
Boundary layer equations

- Boundary layer equations:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2},$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial y}=0, \longrightarrow p=p(x)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0.$$



- Steady laminar boundary layer equations on a flat surface (*U*=const)

  - Bernoulli equation: $\frac{1}{2}U^2+\frac{p}{\rho}=\text{const} \longrightarrow \frac{dp}{dx}=-\rho U\frac{dU}{dx}=0$

  - Finally we have the following equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}$$

- Boundary conditions:

$$\begin{cases} u=0 \text{ at } y=0 \\ v=0 \text{ at } y=0 \\ u\to U \text{ as } y\to\infty \end{cases}$$

Boundary layer equations

- What we have left:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$$

- Boundary conditions:

$$\begin{cases} u = 0 \text{ at } y = 0. \\ v = 0 \text{ at } y = 0. \\ u \to U \text{ as } y \to \infty \end{cases}$$

- We're going to use these two equations to derive the Blasius equation – a single ordinary differential equation for laminar boundary flow at zero pressure gradient

Boundary layer equations

- Similarity solution for boundary layer equations

  - Similarity variable: $\eta = \frac{y}{\delta} = \frac{y}{(\nu x/U)^{1/2}}$

  - Stream function: $\psi = U\delta f(\eta)$ ⟹ $u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$

  - Plugging the similarity variable and stream function to BL equations and using the chain rule for derivatives:

$$f''' + \frac{1}{2} f f'' = 0$$

- Boundary conditions:

$$f = f' = 0 \quad \text{at} \quad \eta = 0$$

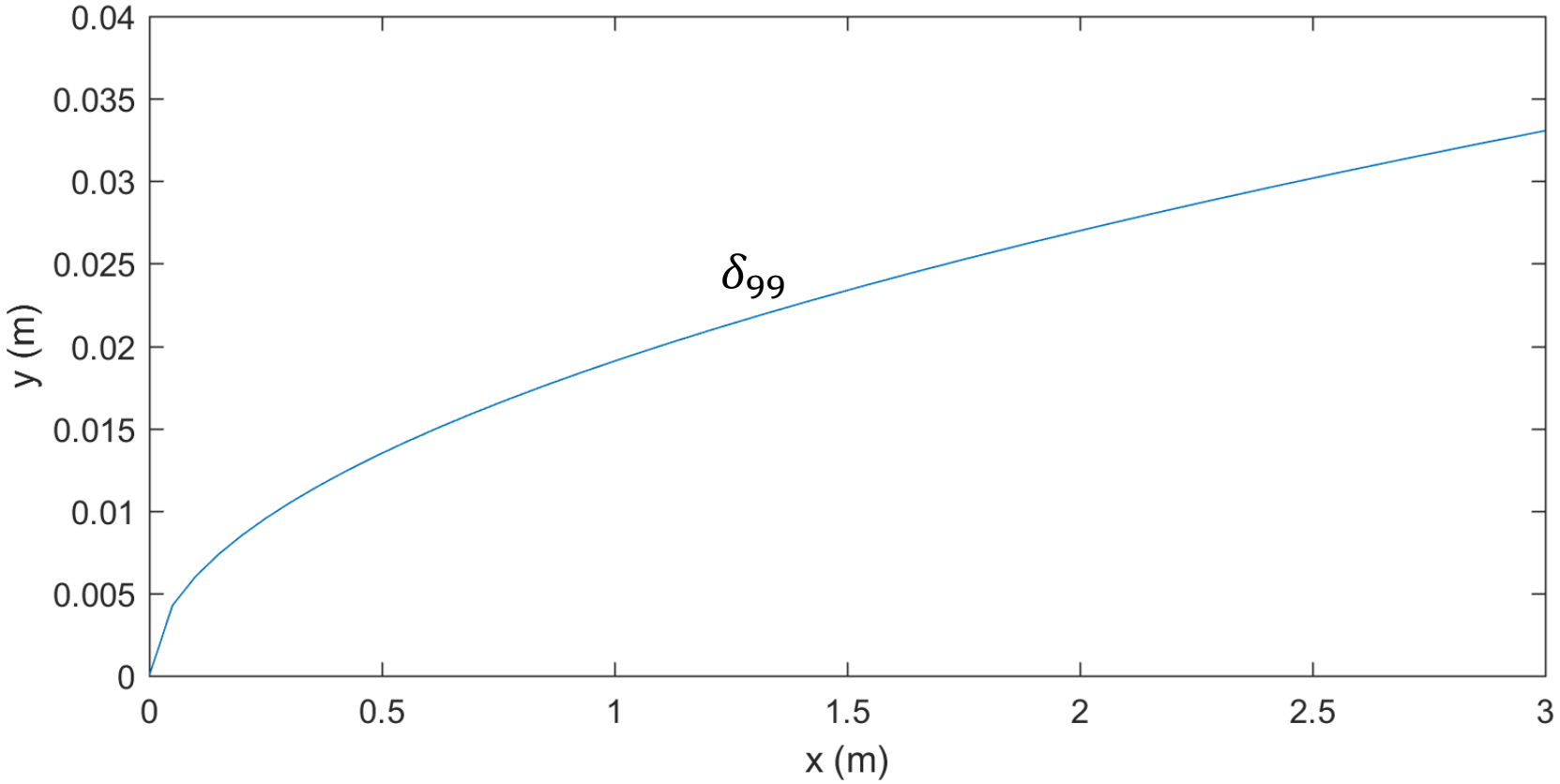
$$f' = 1 \quad \text{as} \quad \eta \to \infty$$

Third-order ordinary differential equation (ODE) → three first-order ODEs

➢ How to solve? We use Runge-Kutta method with secant method

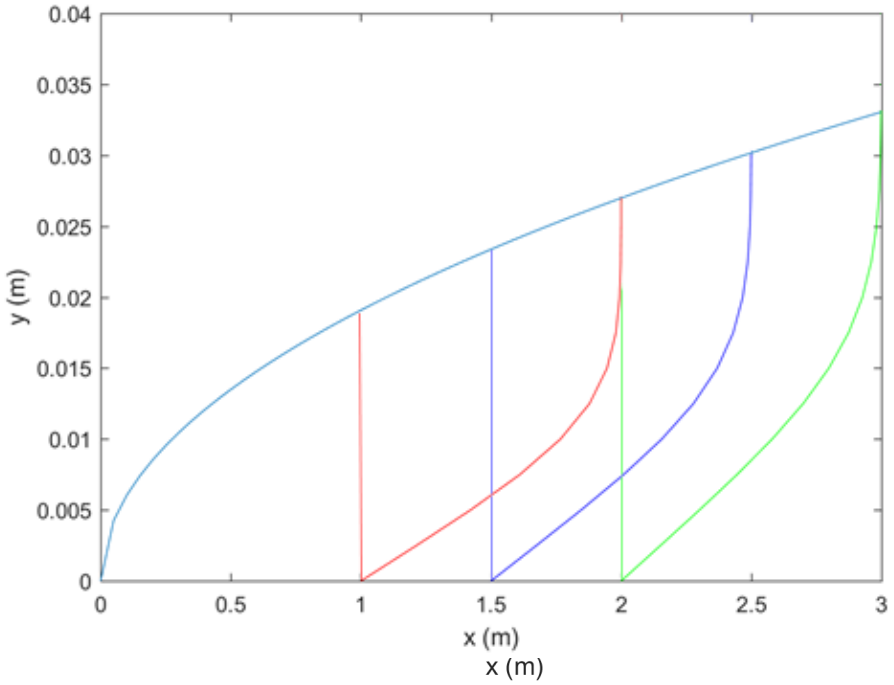
Boundary layer on a flat surface

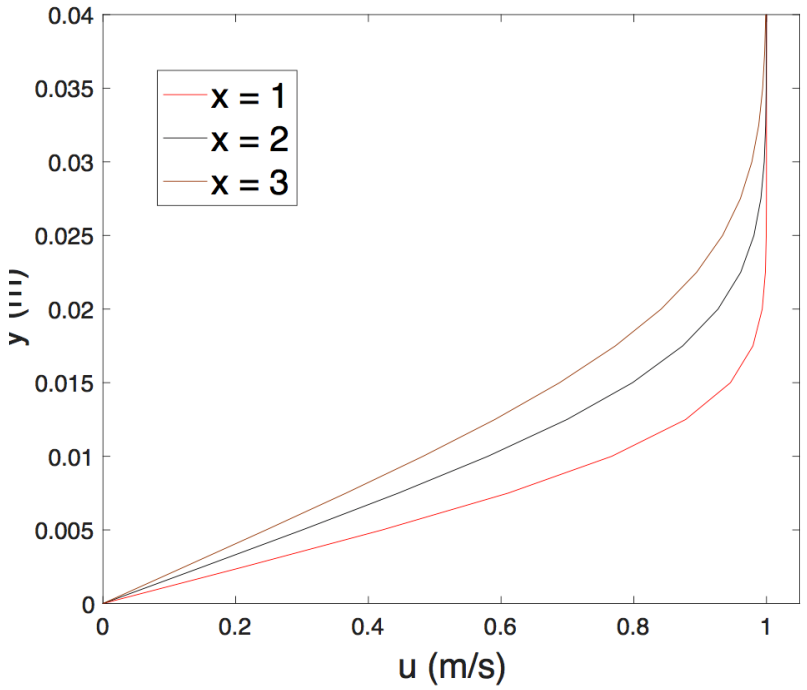
- Blasius boundary layer for a 1.0 m/s airflow over a 3-m-long surface



Boundary layer on a flat surface

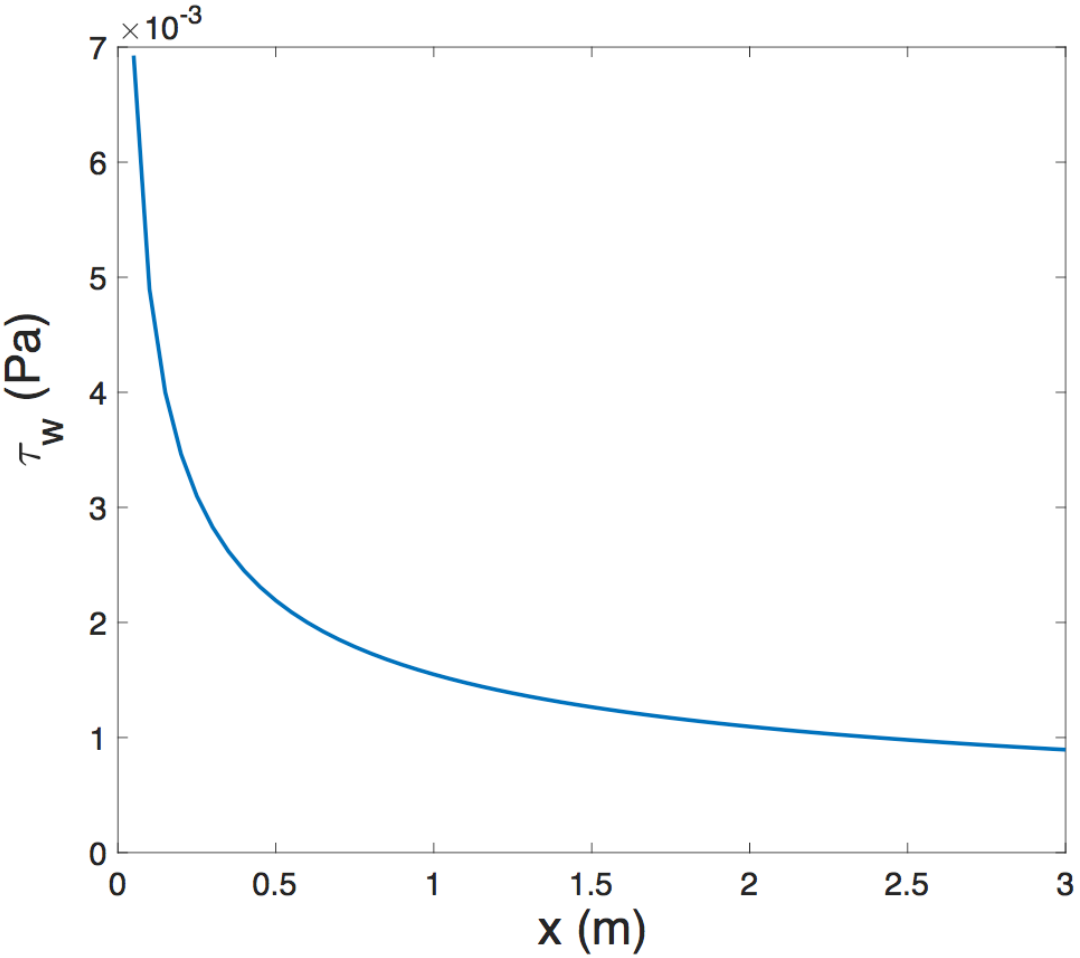
- Blasius boundary layer for a 1.0 m/s airflow over a 3-m-long surface
  - Velocity profiles:





Boundary layer on a flat surface

- Wall shear stress



$$\tau_\omega = \mu \frac{\delta u}{\delta y} \,|y = 0$$

- Flow separation occurs when

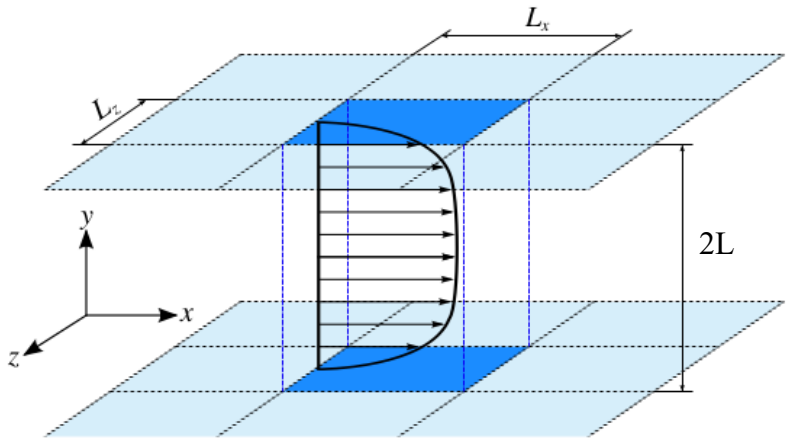
$$\tau_w = 0$$

What about turbulence?

Direct Numerical Simulations

- Channel geometry

  - *x*: streamwise, *y*: wall-normal, *z*: spanwise



Navier-Stokes equations for an incompressible Newtonian fluid:

$$\nabla \cdot \mathbf{u} = 0,$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u}$$

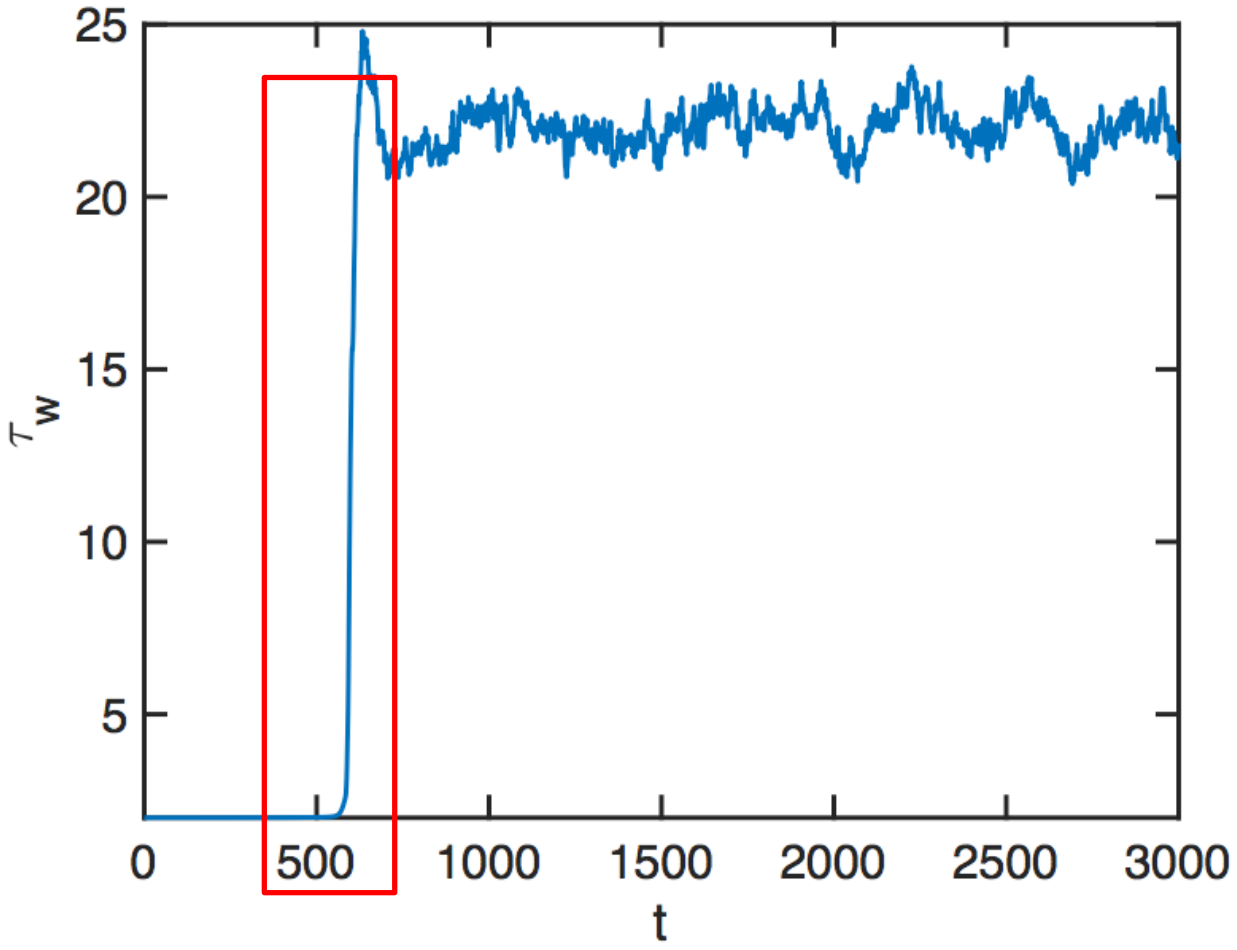
Reynolds number: $Re = \frac{\rho U L}{\mu}$ = $\frac{\text{Inertial}}{\text{Viscous}}$

- The Navier-Stokes equations are solved spectrally using the Fourier (x) – Chebyshev (y) – Fourier (z) spatial discretization

DNS Code : ChannelFlow (J. Gibson) – Modified for this Study

Transition to turbulence

- At around Re = 15,000, transition appears to occur.



Transition to turbulence

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Computing Resources

- Holland Computing Center

Thank you!