

Multiple
Regression

Sister Megan
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Tunink

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Applying Multiple Linear Regression to the Timing of Spring Migration in Hummingbirds

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Benedictine College

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Overview

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Purpose of this Research

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What external cues do hummingbirds use to determine the timing of their spring migration?

Can we create a model that uses external cues to predict arrival date in a given location?

The Ruby-throated Hummingbird

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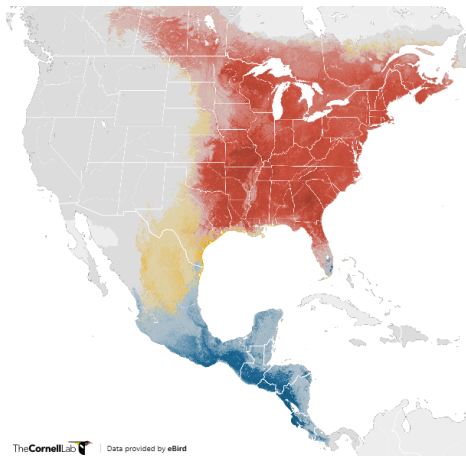
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RELATIVE ABUNDANCE birds per km/hr

Breeding season Jun 7 - Jul 27

Non-breeding season Nov 9 - Mar 1

Pre-breeding migratory season Mar 8 - May 31

Post-breeding migratory season Aug 3 - Nov 2

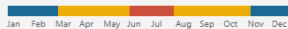
0.16

0.82

8.55

Note: Seasonal ranges overlap and are stacked in the order above; view full range in season maps.

SEASONS TIMELINE [Learn more](#)



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- Population regression line

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

- The least squares regression

$$\hat{Y}_i = b_0 + b_1 X_i$$

estimates the population regression line

- Minimizes

$$MSE = \frac{1}{N} \sum_i (Y_i - \hat{Y}_i)^2$$

Simple Regression Example

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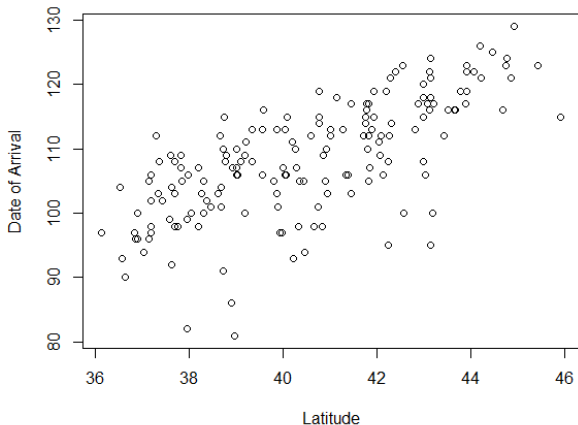
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Simple Regression of Latitude and Date of Arrival



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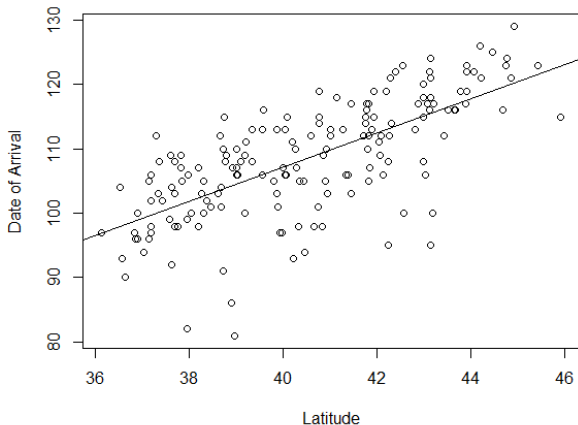
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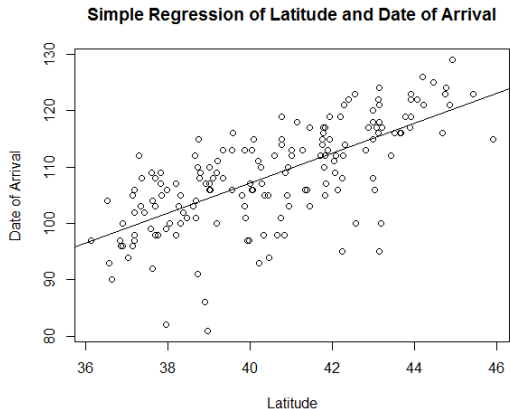
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$$\hat{Y}_i = 0.84 + 2.66X_i$$

$$R^2 = 0.476$$

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- Population regression line

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \cdots + \beta_p X_{ip} + \epsilon_i,$$

- The least squares regression

$$\hat{Y}_i = b_0 + b_1 X_{i1} + b_2 X_{i2} + \cdots + b_p X_{ip}$$

estimates the population regression line

- Minimizes

$$MSE = \frac{1}{N} \sum_i (Y_i - \hat{Y}_i)^2$$

Multiple Regression Example

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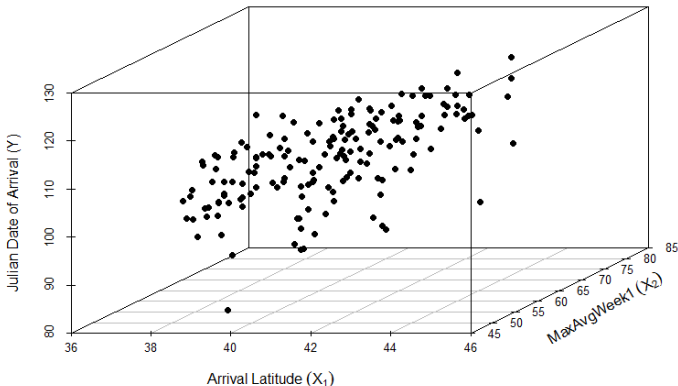
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Multiple Regression with Latitude and MaxAvgWeek1



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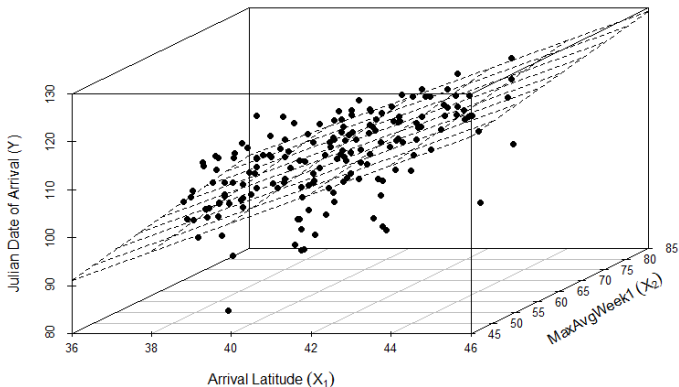
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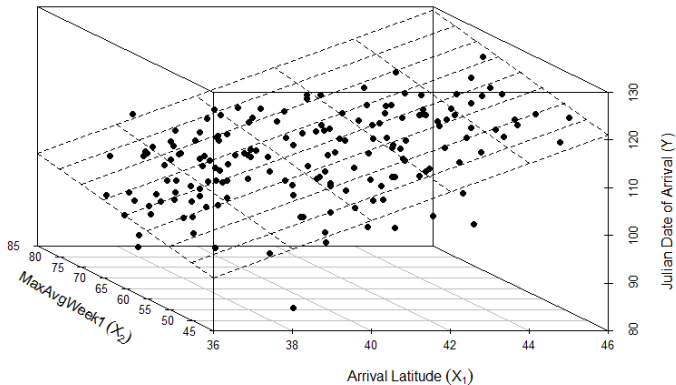
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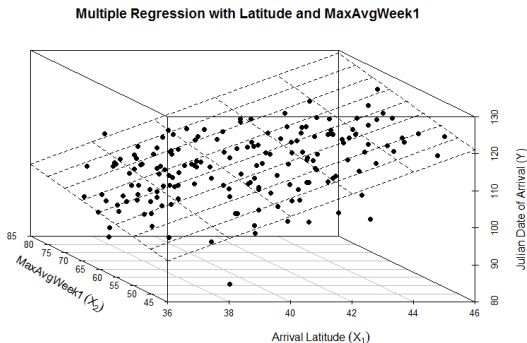
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$$\hat{Y}_i = -25.84 + 2.99X_{i1} + 0.21X_{i2}$$

$$R^2 = 0.495 \quad R^2_{\text{adjusted}} = 0.489$$

How do you create a model?

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- Many different methods
- Stepwise Regression
 - Adding and removing variables based on if they significantly contribute to the model

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$$\hat{Y}_i = -41.39 + 3.17X_{i1} + 0.32X_{i2} + 0.14X_{i3}$$

where

X_{i1} = Arrival Latitude

X_{i2} = Average Minimum Temperature in the Week
before Arrival Week

X_{i3} = Average Maximum Temperature in Arrival Week

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$$\hat{Y}_i = -41.39 + 3.17X_{i1} + 0.32X_{i2} + 0.14X_{i3}$$

where

X_{i1} = Arrival Latitude

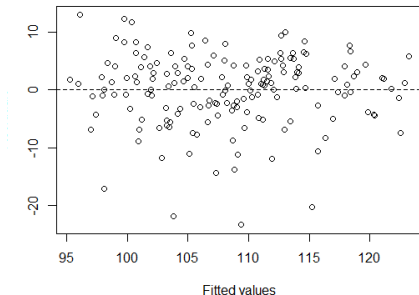
X_{i2} = Average Minimum Temperature in the Week
before Arrival Week

X_{i3} = Average Maximum Temperature in Arrival Week

$$R^2 = 0.542 \quad R^2_{\text{adjusted}} = 0.534$$

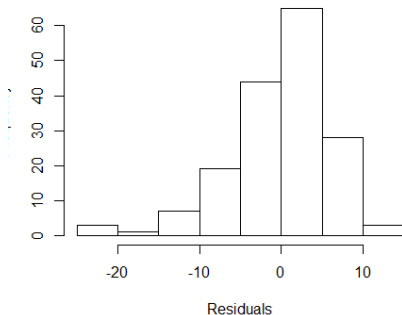
Assumptions of Multiple Regression

- 1 The errors, ϵ_i , are independent.
- 2 The errors, ϵ_i are normally distributed.
- 3 The errors, ϵ_i have equal variances.



Assumptions of Multiple Regression

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Future Research Directions

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- Data transformations
- Build multiple models using different methods and compare them
- Add variable(s) for conditions in Central America

Sampling of Sources

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Thank You!