Comparing Object Correlation Metrics for Effective Space Traffic Management

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Motivation

Figure 1. Catalogued Objects in Space Surveillance Network
Using a simulation framework, estimate the state of non-cooperative objects in orbit given a priori knowledge.
Overview

- Using a simulation framework, estimate the state of non-cooperative objects in orbit given a priori knowledge.

- Given sensor data with measurement errors, optimally assign these measurements to known objects using a certain distance metric.
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- Given sensor data with measurement errors, optimally assign these measurements to known objects using a certain distance metric.

- Comprehensively compare these likelihood-of-coincidence metrics.
Simulation Framework-Satellites

- Simulate the state (position and velocity) and orbits of satellite
  - Orbit found through numerically solving the defining ODE in orbital mechanics
  - Modeled forces such as oblateness of the Earth, atmospheric drag, solar radiation pressure, and third-body gravity along with 2-body gravity
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  \[(\mu^{(t_0)}, \Sigma^{(t_0)})\] at time \(t_0\).
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- Use the orbital dynamics to determine what the distribution will be at a future time \(t_1 > t_0\), which is assumed to still be Gaussian
Figure 2. Propagation of Mean and Covariance
Simulation Framework-Sensors

- Model ground-based (move only as Earth rotates) and space-based (move like a satellite) sensors that take measurements
  - Range: The Euclidean distance from the sensor to the satellite
  - Angle: Represented in azimuth and elevation (also called altitude)

*Figure 3. Sensor Movement*  
*Figure 4. Angle Diagram*
Figure 5. Object Correlation in Measurement Space
Distance Metrics

Let \( D_1 \overset{d}{=} N(\mu_1, \Sigma_1) \) and independent \( D_2 \overset{d}{=} N(\mu_2, \Sigma_2) \), with dimension \( k \).

**Mahalanobis:**

\[
d_M(D_1, D_2) = (\mu_2 - \mu_1)^T (\Sigma_1 + \Sigma_2)^{-1} (\mu_2 - \mu_1)
\]
Distance Metrics

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**Mahalanobis:**

$$d_M(D_1, D_2) = (\mu_2 - \mu_1)^T(\Sigma_1 + \Sigma_2)^{-1}(\mu_2 - \mu_1)$$

**Bhattacharyya:**

$$d_B(D_1, D_2) = \frac{1}{4}(\mu_2 - \mu_1)^T(\Sigma_1 + \Sigma_2)^{-1}(\mu_2 - \mu_1)$$

$$+ \frac{1}{2} \log \left( \frac{\det(\Sigma_1 + \Sigma_2)}{\sqrt{\det \Sigma_1 \det \Sigma_2}} \right) - \frac{k}{2} \log 2$$
Distance Metrics

*Kullback-Liebler divergence* is a nonsymmetric function, so we define two versions of it.

**KL1:**

\[
d_{KL}(D_1, D_2) = \frac{1}{2} (\mu_2 - \mu_1)^T \Sigma_2^{-1} (\mu_2 - \mu_1) \\
+ \frac{1}{2} \log \left( \frac{\det \Sigma_2}{\det \Sigma_1} \right) + \frac{1}{2} \text{Tr}(\Sigma_2^{-1} \Sigma_1) - \frac{k}{2}
\]

**KL2:**

\[
d_{KL}(D_2, D_1)
\]
Experiment Design

How do the metrics perform on clusters of satellites?

- **CLUSTER-OUT**: Satellites begin in a cluster at time $t_0$ and disperse as they are propagated to a later time $t_1$
Experiment Design

How do the metrics perform on clusters of satellites?

- **CLUSTER-OUT**: Satellites begin in a cluster at time $t_0$ and disperse as they are propagated to a later time $t_1$

![Diagram of CLUSTER-OUT]

- **CLUSTER-IN**: Satellites begin dispersed at time $t_0$ and become clustered as they are propagated to a later time $t_1$

![Diagram of CLUSTER-IN]
Specifications

- 10 space-based sensors at equal longitudinal intervals above the equator in geosynchronous orbit

- 20 ground-based sensors at major cities chosen to have a relatively even distribution of latitudes and longitudes

- Generate 30 clusters of 25 satellites
  - 15 clusters as in CLUSTER-OUT
  - 15 clusters as in CLUSTER-IN

- Generate 250 single satellites with randomly selected elliptical orbits for a total of 1000 satellites

- Run 30 trials of this test, averaging the success rates across trials for each metric and modality
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Parameter Values

- Observation Gap: 3600 seconds
- Altitude: 750 km - approximate height of the cluster
- Dispersal: $10^8$ m$^2$ - a measure of cluster tightness
- Satellite *a priori* uncertainty: $10^2$ m$^2$
- Sensor measurement error: $10^2$ m$^2$
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We ran also tests of 30 trials with 50 satellites in 1 cluster.

- Varied the 5 parameters listed above and calculated average success rate for each metric and modality
- For the top 2 performing metrics, used a $t$-test with $\alpha = 0.05$ to determine which metric performed significantly better than the others
SIMULATING-LEO: Results

Figure 9. SIMULATING-LEO

![Bar chart showing correlation success rates for different sensor modalities and metrics.]

- **Mahalanobis**
- **Bhattacharyya**
- **KL1**
- **KL2**

The chart compares the success rates of various correlation metrics (Mahalanobis, Bhattacharyya, KL1, KL2) across different sensor modalities (Range, Angle, Range and Angle). It illustrates the effectiveness of these metrics in the context of object correlation for effective Station-Track-Merge (STM) operations.
General Results

Single Cluster Tests

- Mahalanobis performs best for range measurements, and significantly better when the sensor noise is low.
- KL2 performs best for angle and range-and-angle measurements, and has significantly higher accuracy in cases of tighter clusters and higher sensor noise.

SIMULATING-LEO
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- **SIMULATING-LEO**
  - Mahalanobis is consistently the best metric.
  - Success rates for range measurements are overall lower than those of angle and range-and-angle measurements by \( \approx 1\% \).
Future Work

- Testing more existing metrics and experimenting with new metrics
- More realistic dynamical system
- Range and angle rate sensors
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Questions?
Need to determine most impactful parameters
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Set to these values when varying:

- Observation gap $t_1 - t_0$ (seconds): 300, 1800, 3600, 7200
- Altitude $alt_C$ (km): 350, 762.5, 1175, 1587.5, 2000
- Dispersal $\sigma_r^2$ (m$^2$): $10^6, 10^8, 10^{10}$
- Satellite *a priori* uncertainty $\sigma_{ap}^2$ (m$^2$): $10^0, 10^2, 10^4$
- Sensor measurement error $\sigma_{meas}^2$ (m$^2$): $10^0, 10^2, 10^4$
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- Altitude $\text{alt}_C$ (km): 350, 762.5, 1175, 1587.5, 2000
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- Satellite a priori uncertainty $\sigma_{ap}^2$ (m$^2$): 10$^0$, 10$^2$, 10$^4$
- Sensor measurement error $\sigma_{meas}^2$ (m$^2$): 10$^0$, 10$^2$, 10$^4$

Set to nominal values when not varying:
- Observation Gap: 3600 seconds
- Altitude: 750 km
- Dispersal: 10$^8$ m$^2$
- Satellite a priori uncertainty: 10$^2$ m$^2$
- Sensor measurement error: 10$^2$ m$^2$
If we pick 2 parameters to vary, we can vary them in higher resolution.
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Use same nominal values when fixed
Fine Parameter Analysis

- If we pick 2 parameters to vary, we can vary them in higher resolution.
- Use same nominal values when fixed.
- Vary across these ranges in 20 equal increments:
  - Observation gap: \([300, 7200]\) s
  - Altitude: \([350, 2000]\) km
  - Dispersal: \([10^4, 10^{10}]\) m\(^2\) (log scale)
  - Satellite *a priori* uncertainty: \([10^0, 10^4]\) m\(^2\) (log scale)
  - Sensor measurement error: \([10^0, 10^4]\) m\(^2\) (log scale)
Run 30 trials, with 50 satellites leaving a cluster
Run 30 trials, with 50 satellites leaving a cluster

For each metric and modality, use LASSO to determine 3 most impactful parameters
Run 30 trials, with 50 satellites leaving a cluster
For each metric and modality, use LASSO to determine 3 most impactful parameters
Will run finer pairwise investigation on observation gap, dispersal, and sensor variance
CLUSTER-OUT: Grid Plots

- Run 30 trials of each test
CLUSTER-OUT: Grid Plots

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- For each parameter pair, find the metric with the highest average success rate

Conduct a one-sided paired $t$-test with $\alpha = 0.05$ for determining whether the winning metric has a significantly higher mean than the metric with 2nd highest accuracy. If the null hypothesis is rejected, color the cell according to the metric. Else, color the cell gray since the win is insignificant.
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Figure 9. CLUSTER-OUT Test: Observation Gap vs Cluster Dispersal
Figure 10. CLUSTER-OUT: Observation Gap vs Sensor Variance
Figure 7. CLUSTER-OUT: Cluster Dispersal vs Sensor Variance
CLUSTER-OUT: Results

- Range measurements: Mahalanobis wins for sensor variance \( \log(\sigma^2_{meas}) < 1.5 \), and in other regions hard to describe
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Angle measurements: KL2 wins for dispersal $\log(\sigma_r^2) < 6.5$
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Angle measurements: KL2 wins for dispersal $\log(\sigma_r^2) < 6.5$

Range and angle measurements: KL2 wins for dispersal $\log(\sigma_r^2) < 5.5$ and generally higher sensor variance
Run 30 trials, with 50 satellites going towards a cluster
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For each metric and modality, use LASSO to determine 3 most impactful parameters
Run 30 trials, with 50 satellites going towards a cluster
For each metric and modality, use LASSO to determine 3 most impactful parameters
Dispersal and sensor variance appear in all models
Altitude appears 7 out of the 12 times, with observation gap appearing 5 times
CLUSTER-IN: Determining Impactful Variables

- Run 30 trials, with 50 satellites going towards a cluster
- For each metric and modality, use LASSO to determine 3 most impactful parameters
- Dispersal and sensor variance appear in all models
- Altitude appears 7 out of the 12 times, with observation gap appearing 5 times
- Will run finer pairwise investigation on altitude, dispersal, and sensor variance
Figure 12. CLUSTER-IN: Altitude vs Cluster Dispersal
Figure 13. CLUSTER-IN: Altitude vs Sensor Variance
Figure 8. CLUSTER-IN: Cluster Dispersal vs Sensor Variance
CLUSTER-IN: Results

- **Range measurements**: Mahalanobis wins for dispersal $\log(\sigma_r^2) > 7$, all altitudes, and sensor variance $\log(\sigma_{meas}^2) < 2$

- **Angle measurements**: KL2 wins for dispersal $\log(\sigma_r^2) < 7$, any altitude, and sensor variance $\log(\sigma_{meas}^2) > 2$

- **Range and angle measurements**: KL2 wins for dispersal $\log(\sigma_r^2) < 6$ and any altitude
CLUSTER-IN: Results

- **Range measurements:** Mahalanobis wins for dispersal $\log(\sigma^2_r) > 7$, all altitudes, and sensor variance $\log(\sigma^2_{meas}) < 2$

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CLUSTER-IN: Results

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- Angle measurements: KL2 wins for dispersal \( \log(\sigma_r^2) < 7\), any altitude, and sensor variance \( \log(\sigma_{meas}^2) > 2\)
- Range and angle measurements: KL2 wins for dispersal \( \log(\sigma_r^2) < 6\) and any altitude
References


Figures


Figure 4 Joshua Cesa (https://commons.wikimedia.org/wiki/File:Azimut_altitude.svg), “Azimut altitude”, Text, https://creativecommons.org/licenses/by/3.0/legalcode

Figures 2 and 5-9 were generated by our own simulation framework, using Plot.ly and R.