Exploring Retinal Blood Vessels Through Fractal Dimension

Elaina Chapnell- Westminster College (James Anthony, advisor)
chapen22@wclive.westminster.edu

Abstract

Diabetic retinopathy is a vascular disease in the retina that can cause vision loss. Severe cases of this disease are characterized by neovascularization of blood vessels in the eye. In this study, we will use fractal dimension to attempt to determine a difference in the complexity of retinal blood vessels between individuals who have diabetic retinopathy and those without the disease.

Fractals

A fractal is an image that typically maintains its complexity on an infinite number of scales. A classic example of a fractal is the Sierpinski Triangle (right). This shape is formed by dividing a triangle into four sections and removing the middle section, and this process is repeated an infinite number of times for the remaining sections. If you zoom in on one section of the triangle, the shape will always maintain its complexity, no matter how far you zoom in. The complexity of a fractal can be calculated by determining its fractal dimension. For any planar figure, the fractal dimension will be between 0 and 2. The fractal dimension increases as the complexity of the fractal increases. One way of computing fractal dimension is through the box-counting method. In this method, you count the number of boxes, denoted N, that cover a fractal on multiple scales, denoted s. Then, you create a scatter plot with $\ln(1/s)$ on the x-axis and $\ln(N(1/s))$ on the y-axis and find the best fit line. The slope of the resulting best fit line is the fractal dimension.
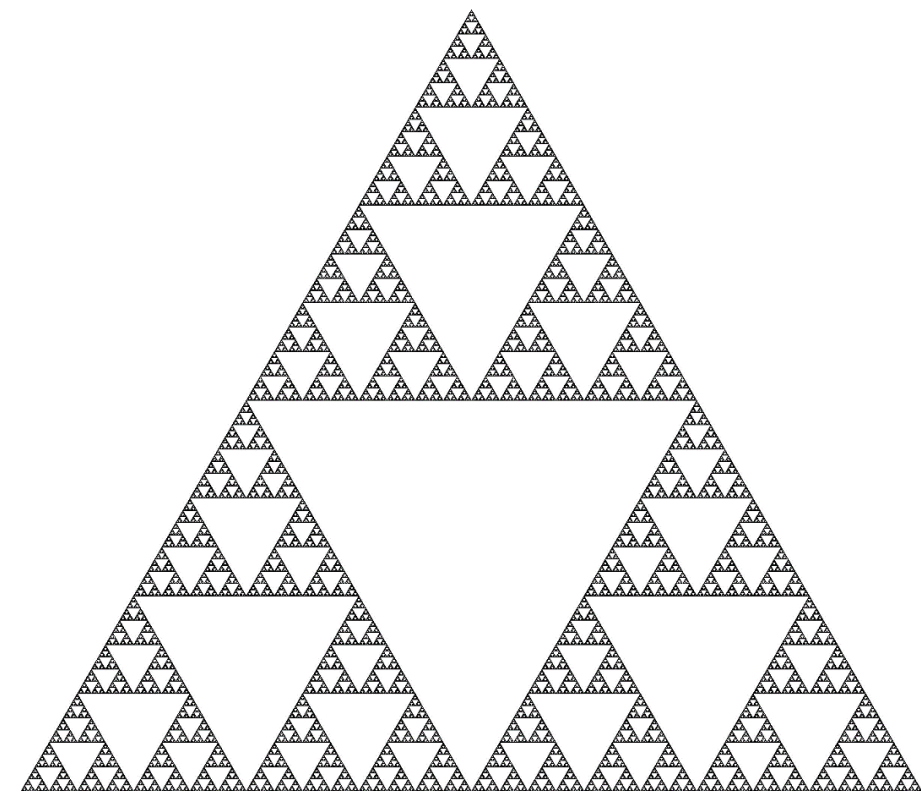

Figure 1. Sierpinski Triangle

Diabetic Retinopathy

Diabetic retinopathy is a disease caused by damage to retinal blood vessels that occurs in people with diabetes. This disease is nonproliferative in its early stages, but as it progresses, it can potentially become proliferative. Proliferative diabetic retinopathy is characterized by neovascularization, the formation of additional blood vessels. Neovascularization occurs to make up for the function of the damaged vessels, but the newly formed vessels are much weaker and susceptible to tearing, which could lead to hemorrhage and subsequent vision loss.

Method

The structure of blood vessels is a fractal because blood vessels continue to grow smaller as they branch out while still maintaining their complexity. We wanted to discover if there was a difference in fractal dimension between healthy eyes and eyes with diabetic retinopathy. The images we used in this study had a grade of either 0 or 3; eyes that were grade 0 did not have retinopathy, and eyes that were grade 3 had severe cases of retinopathy. We used the box-counting method to compute the fractal dimension of ten images each of grade 0 and grade 3 retinopathy. We focused on the area of the optic nerve and placed boxes on the blood vessels using a computer program from a scaling factor of 1/8 to a scaling factor of 1/128.

Determining Fractal Dimension

Figure 2. Grade 0 retinopathy
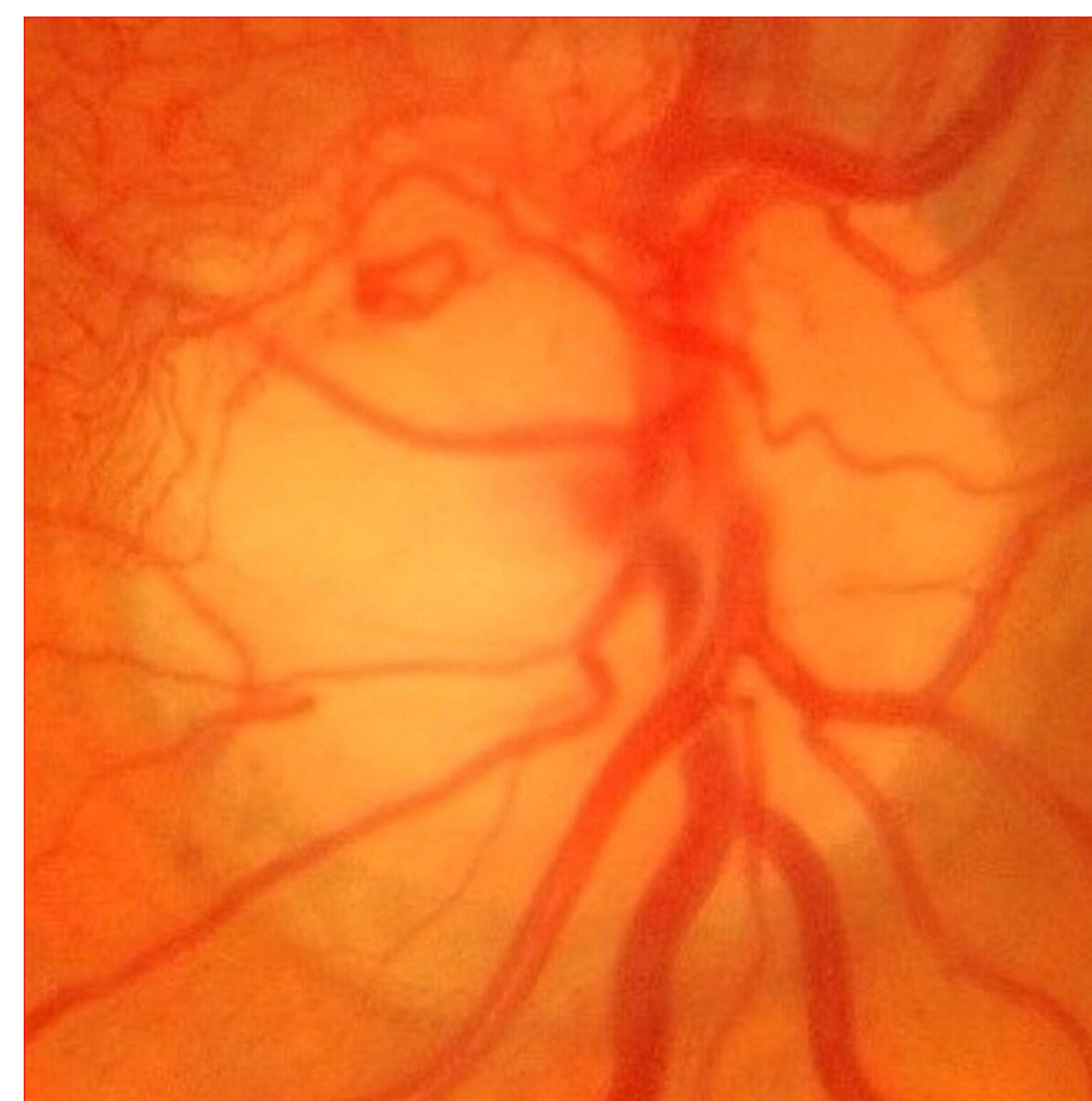

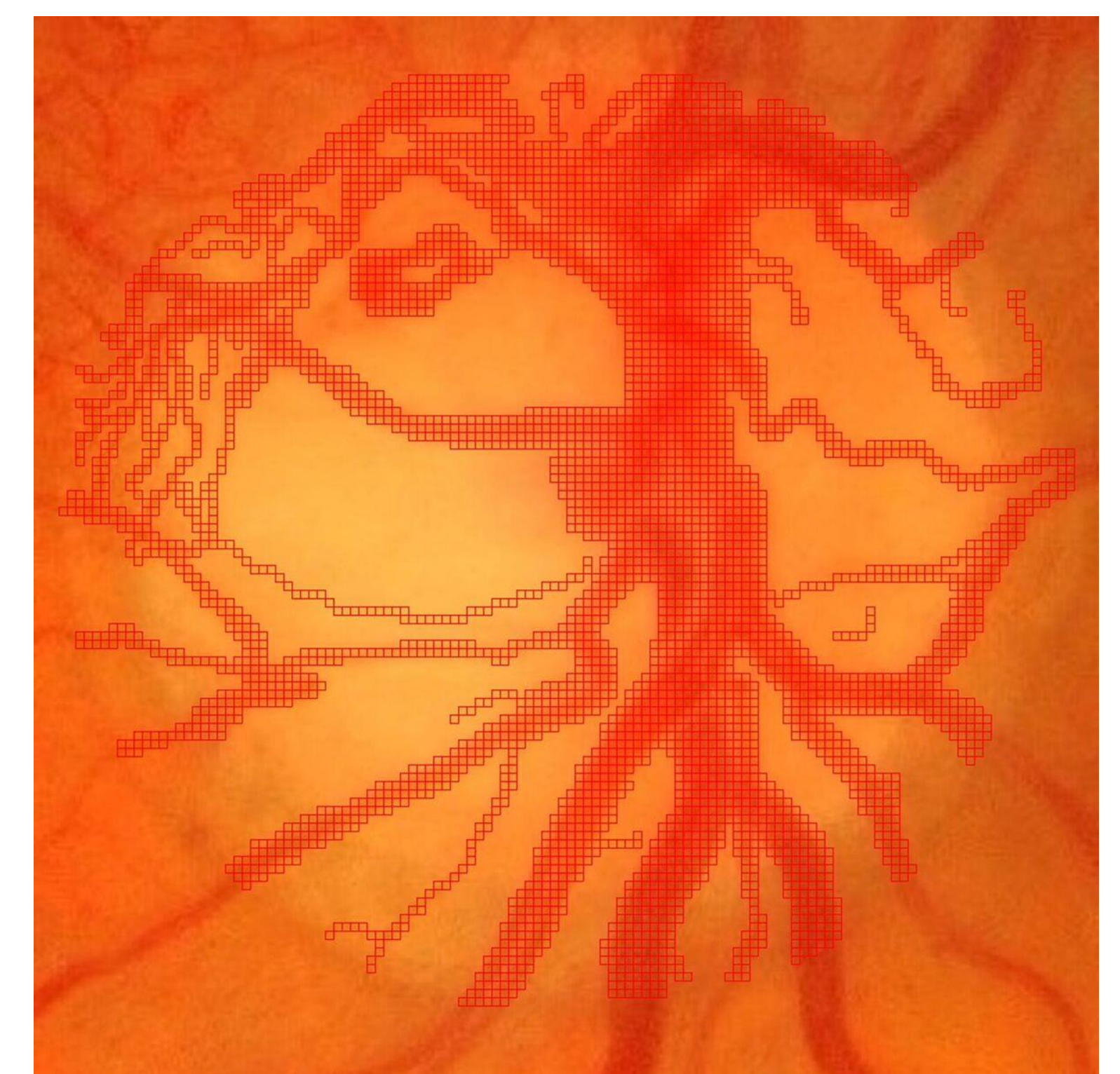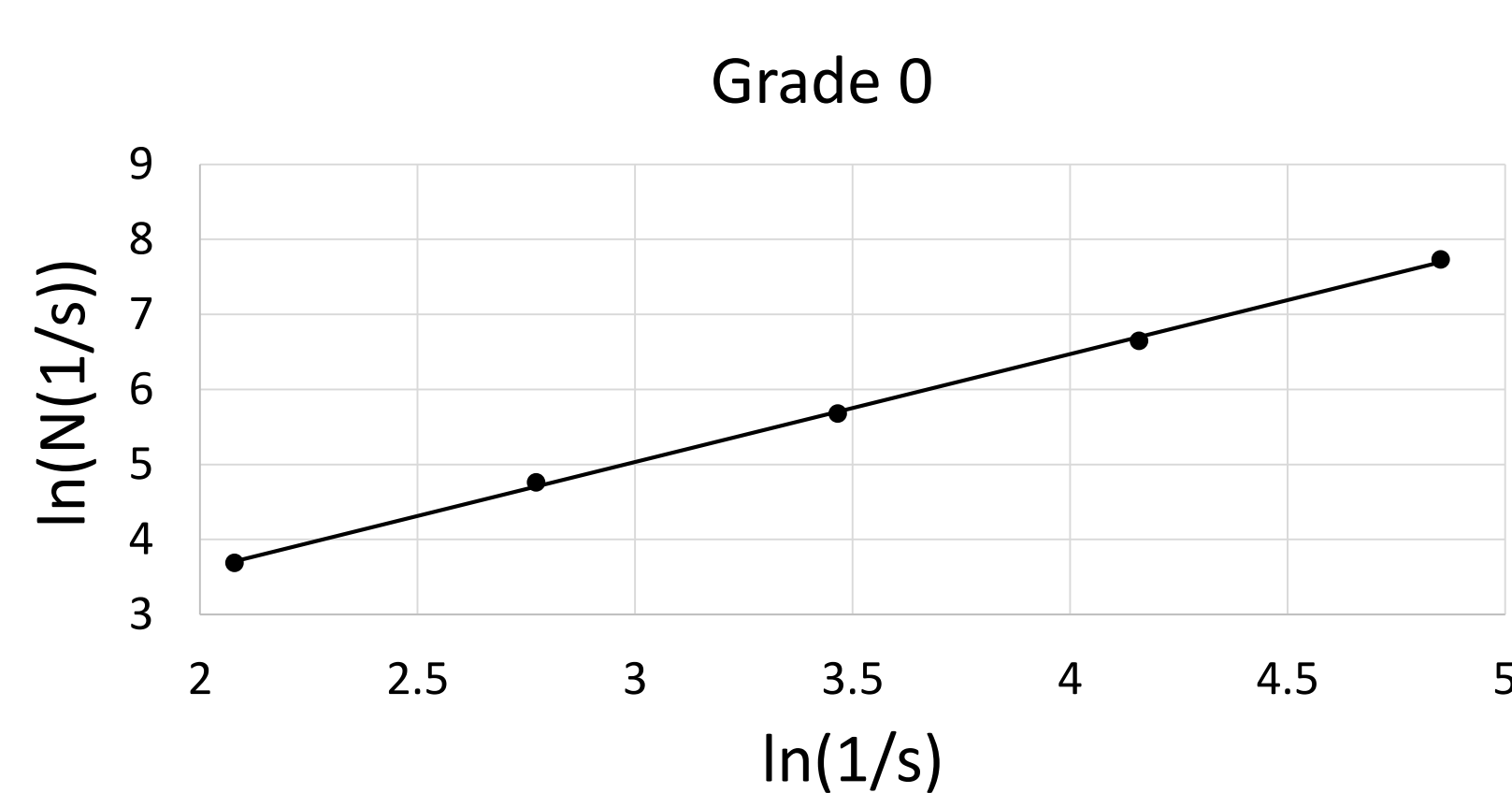Figure 3. Grade 0 retinopathy with boxes
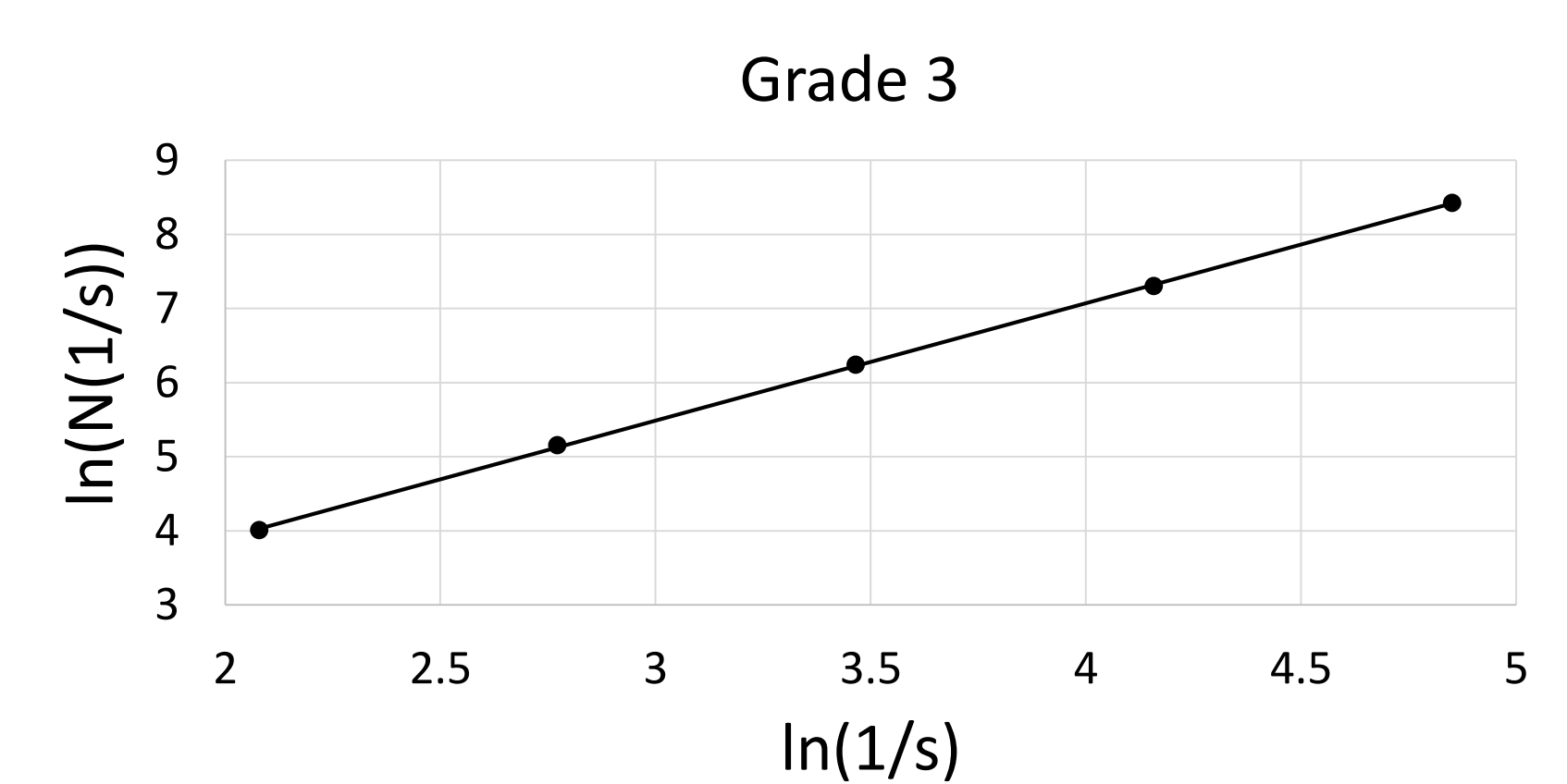

Figure 4. Grade 3 retinopathy

Figure 5. Grade 3 retinopathy with boxes

Grade 0			
1/s	N	ln(1/s)	ln(N(1/s))
8	40	2.079442	3.688880
16	117	2.772589	4.762174
32	293	3.465736	5.680172
64	771	4.158883	6.647688
128	2277	4.852030	7.730614

Grade 3			
1/s	N	ln(1/s)	ln(N(1/s))
8	55	2.079442	4.007333
16	173	2.772589	5.153292
32	512	3.465736	6.238325
64	1483	4.158883	7.301822
128	4563	4.852030	8.425735

Results

When we compared the two means, we obtained a p value less than 0.05, which means there is a statistically significant difference in the fractal dimension of the retinal blood vessel structures between eyes with grade 0 and grade 3 diabetic retinopathy.
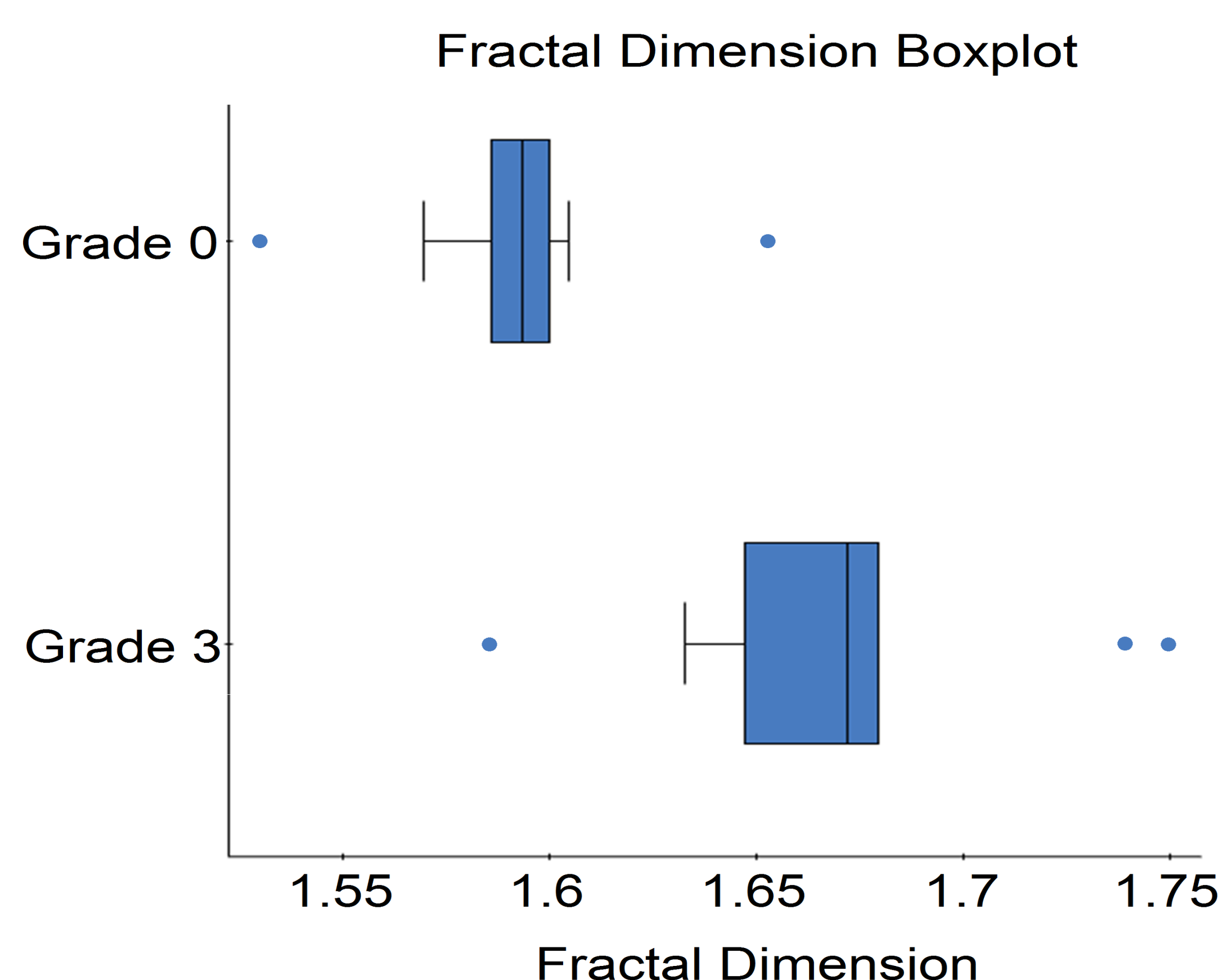
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Acknowledgements