An Agent-Based Model of Pollen Competition in *Arabidopsis thaliana*

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I. Introduction

Background:

- Pollen competition occurs when pollen from different accessions compete to fertilize ovules while consuming limited resources.
- When *Arabidopsis thaliana* is pollinated by Columbia and Landsberg accessions, Columbia sires disproportionately more seeds.
- Previous research has studied which pollen performance traits differ between Columbia and Landsberg that may account for this:
  - **Proportion of pollen germinated**- the proportion of pollen grains from each accession that grow pollen tubes.

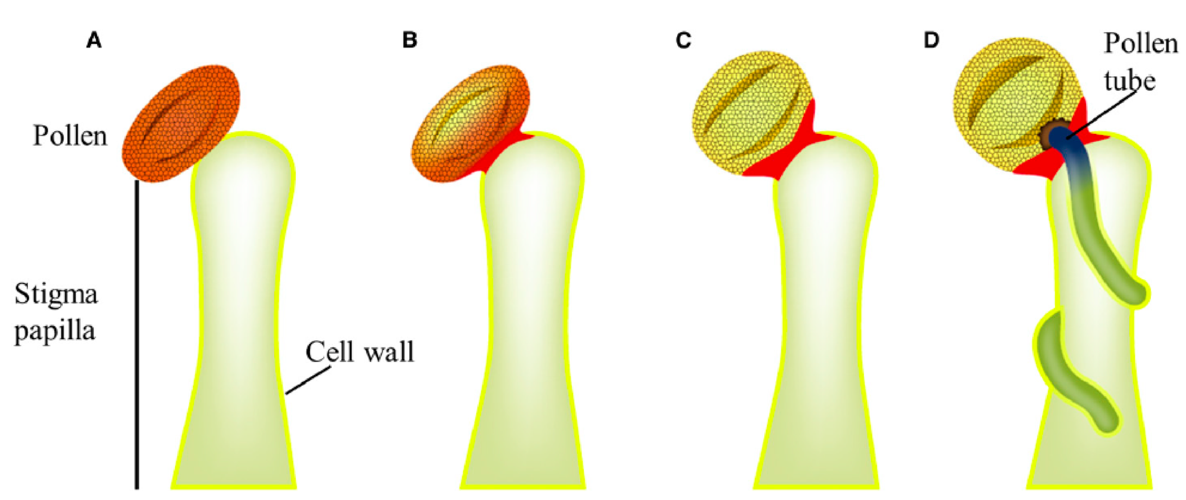


Figure 1: During pollen germination the pistil hydrates the pollen grain and a pollen tube emerges through holes in the pollen grain wall before growing down the ovary.

  - **Time to germination**- the amount of time it takes for pollen from each accession to germinate.
  - **Pollen tube growth rate**- how quickly pollen tubes grow down the transmitting tract.
  - **Bioenergetics**- how pollen tubes use and gain energy from the pistil during the process of fertilization.

Agent-based Modeling

- Agent-based models (ABMs) show how unique individuals interact with other agents and their local environment.
- Interactions between individuals cause properties to emerge in ABMs - the emergent property of interest in our model is the proportion of seeds sired by each accession.

NetLogo

- We created our model in NetLogo 6.1.0.
- One tick represents one minute and one patch represents the dimensions 0.01 mm × 0.01 mm.

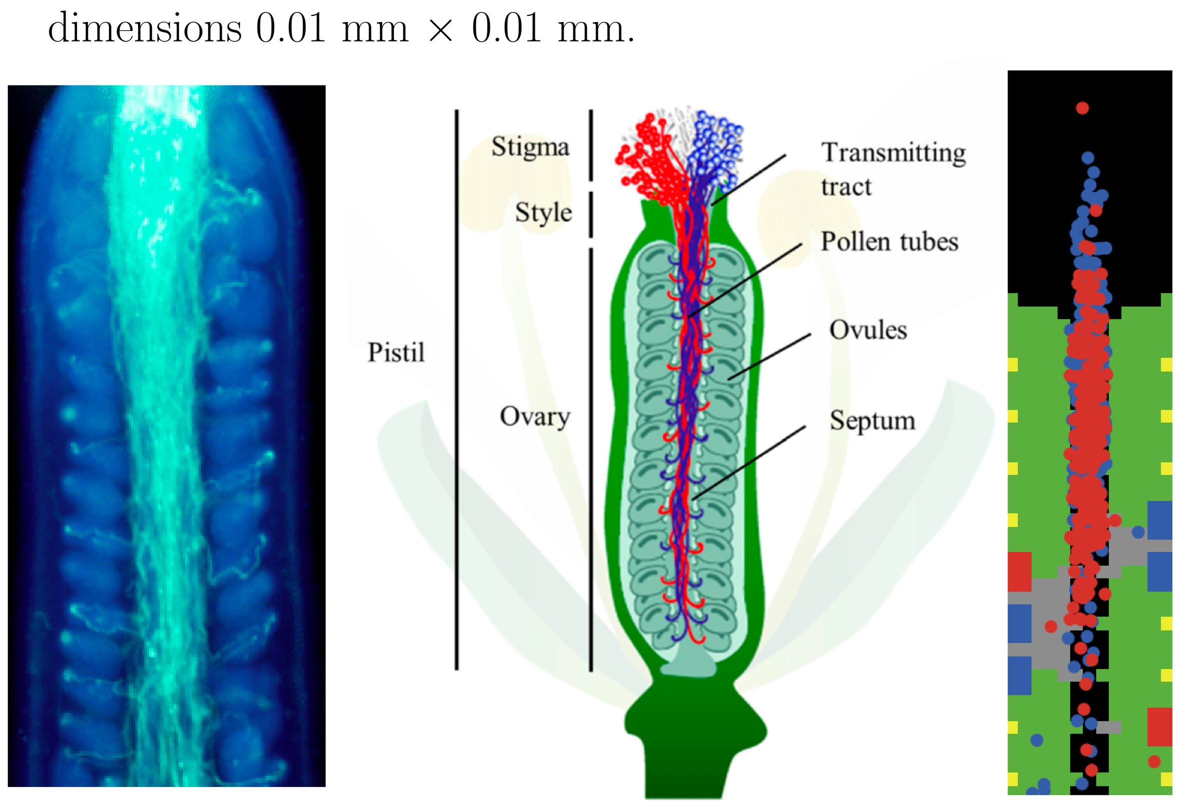


Figure 2: From left to right, an image of an *Arabidopsis thaliana* pistil, a diagram of an *Arabidopsis thaliana* pistil, and a screenshot of our model.

II. The Model

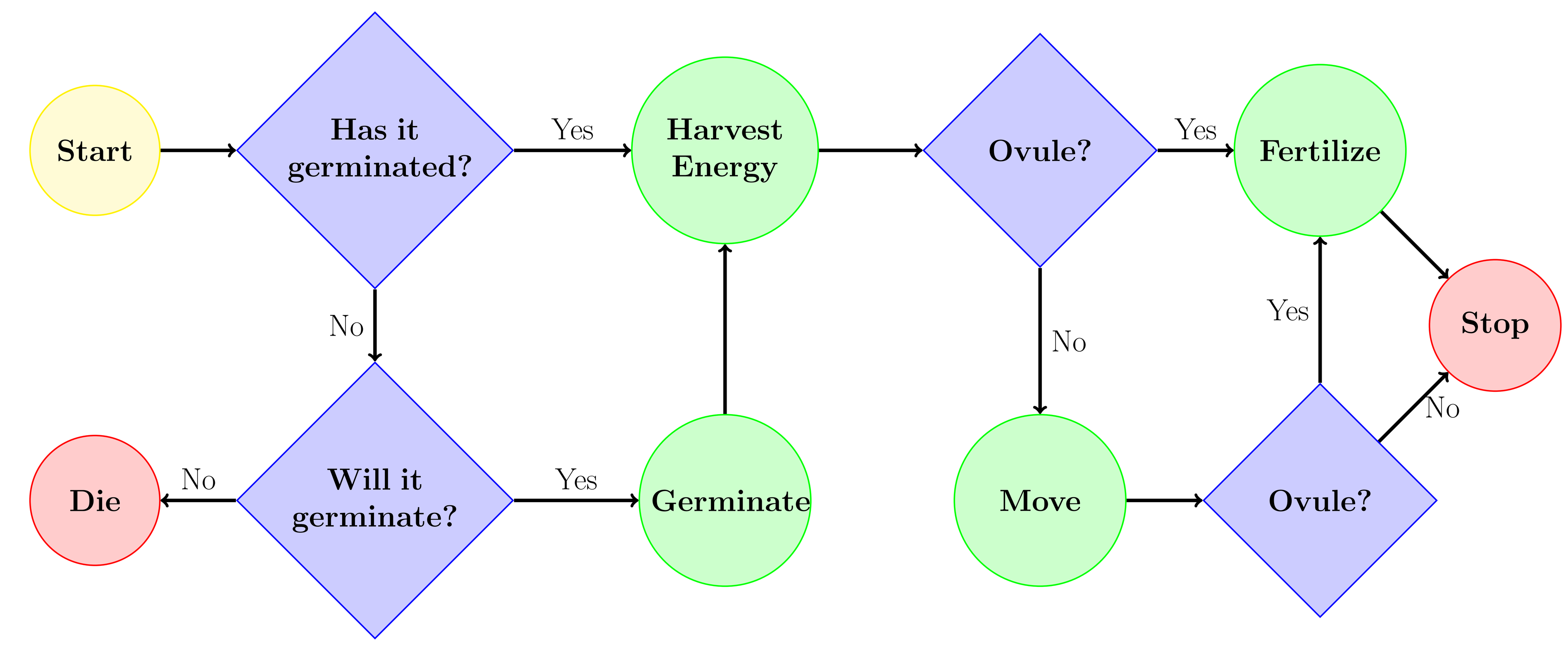


Figure 3: The decision tree describes the potential actions of a pollen tube tip during each tick of the model.

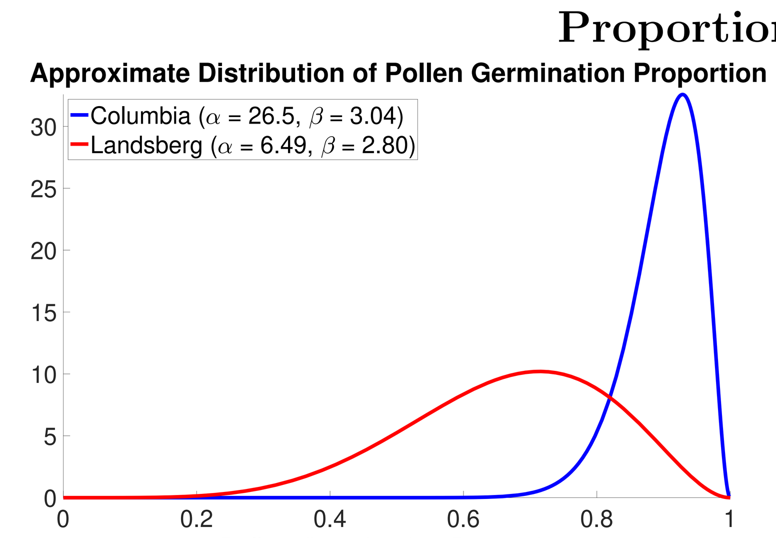
Attractants	`slope` `chemoattractant-radius`			

Bioenergetics	Agent	`step-size`	Blue:
			`blue-initial-pollen-energy`
			`blue-movement-cost`
			`blue-energy-from-patch`
			`blue-fertilization-threshold`
				Red:
				`red-initial-pollen-energy`
				`red-movement-cost`
				`red-energy-from-patch`
				`red-fertilization-threshold`

	Patches	`initial-starch`	`replenish-starch:`
			`replenish-only-empty`
			`frequency-starch-replenish`
			`proportion-starch-replenish`
			`unfertilized-ovule-replenish-cutoff`	

Table 1: The variables in the model.

Proportion of pollen germinated

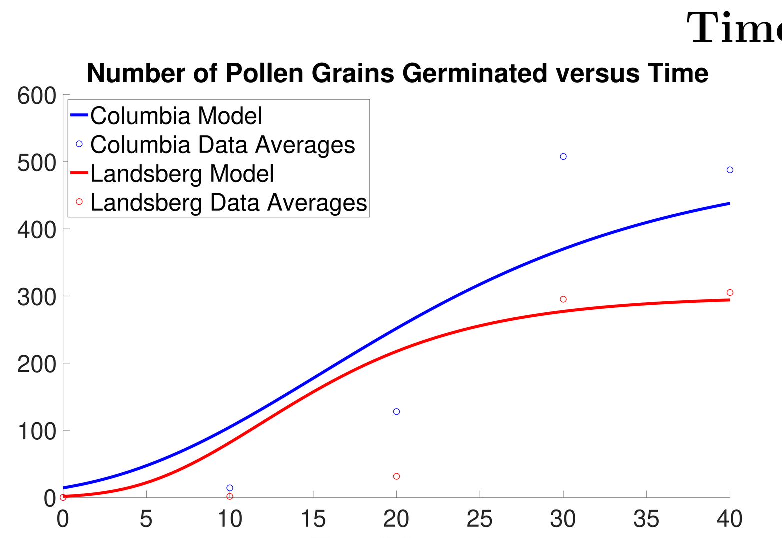


We assumed the pollen germination proportion of each accession followed a Beta distribution. We calibrated the parameters of the Beta distribution to literature values.

If $\sigma^2 < \mu(1-\mu)$, the parameters for the Beta distribution are calculated using the equations:
$\alpha = \mu\left(\frac{\mu(1-\mu)}{\sigma^2} - 1\right)$ and $\beta = (1-\mu)\left(\frac{\mu(1-\mu)}{\sigma^2} - 1\right)$.

Figure 4: Distributions of proportion of pollen germinated for the two accessions.

Time to germination



We compared families of functions to represent the differential germination time of each accession. The most parsimonious of these was the Gompertz function.

The number of pollen grains which germinate $N(t)$ at a given tick t is modeled as the derivative of these functions.

$$N_{\text{Col}}(t) = 14.22\exp\left(3.560\left(1-\exp\left(-0.0823t\right)\right)\right)$$
$$N_{\text{Ler}}(t) = 1.5556\exp\left(5.2619\left(1-\exp\left(-0.0914t\right)\right)\right)$$

Figure 5: Number of pollen grains germinated over time for the two accessions.

Chemoattractants

We modeled the strength of chemoattractants by $A_i(x,y,t) = F(t)\left(\frac{\texttt{chemoattractant-radius}-d(x,y)}{\texttt{chemoattractant-radius}}\right)$ where `d(x,y)` is the distance from the ovule and $F(t)$ determines the saturation rate of chemoattractant strength.

$$F(t) = \begin{cases} 0 & \text{if } 0 \le t < t_0 \\ K\left(t - t_0\right) & \text{if } t_0 \le t < t_0 + \frac{1}{K} \\ 1 & \text{if } t_0 + \frac{1}{K} \le t \end{cases}$$

III. Results

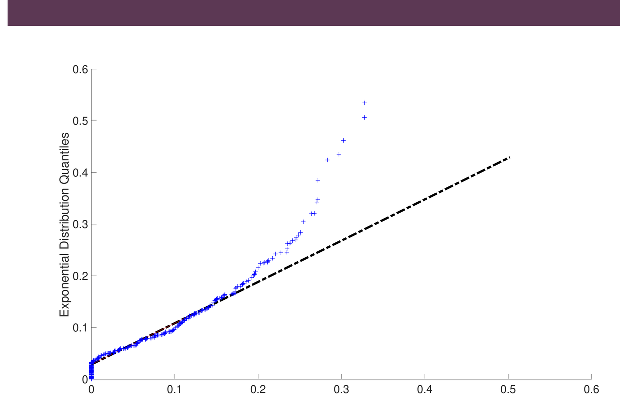


Figure 6: Q-Q plot for Col at 6 hours with the following parameter combinations: step-size=0.5, slope=0.5, chemoattractant-radius=4, blue-energy-from-patch=0.5, red-energy-from-patch=0.5, blue-movement-cost=2, red-movement-cost=0.5, and no starch regeneration.

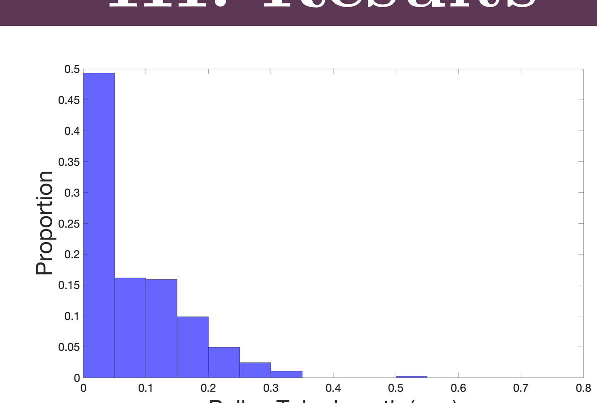


Figure 8: Histogram of Col pollen tube lengths at 6 hours in the ABM.

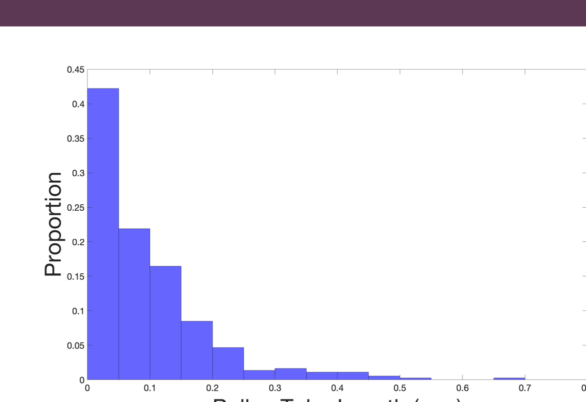


Figure 10: Histogram of Col pollen tube lengths at 6 hours using exponential distribution ($\mu = 0.144$) from literature.

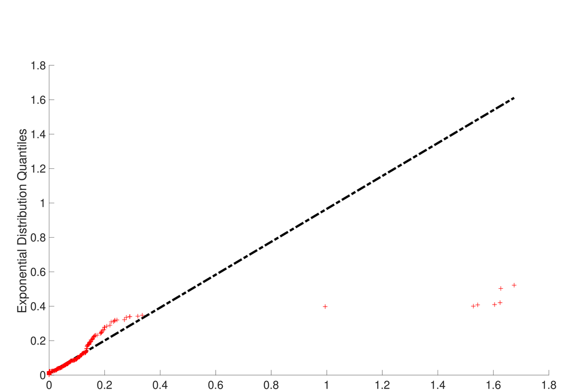


Figure 7: Q-Q plot for Ler at 6 hours with the following parameter combination: step-size=0.5, slope=0.05, chemoattractant-radius=5, blue-energy-from-patch=2, red-energy-from-patch=2, blue-movement-cost=2, red-movement-cost=2, and no starch regeneration.

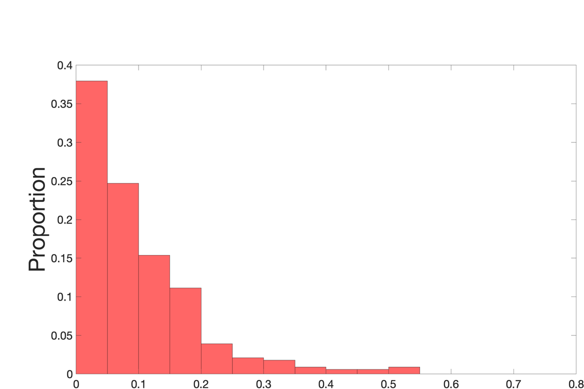


Figure 9: Histogram of Ler pollen tube lengths at 6 hours in the ABM.

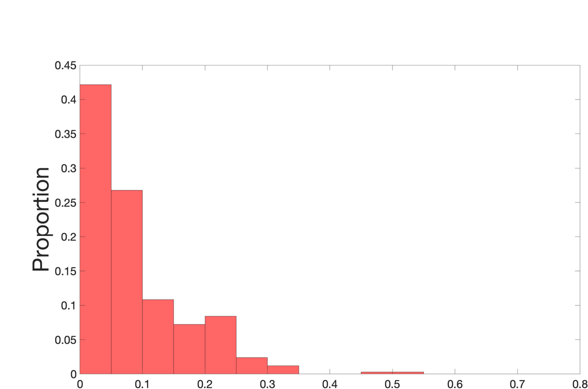


Figure 11: Histogram of Ler pollen tube lengths at 6 hours using exponential distribution ($\mu = 0.094$) from literature.

IV. Future Work

- Run further experiments exploring a larger region of parameter space to better calibrate our model parameters.
- Since the ABM pollen tube growth rates do not follow an exponential distribution as closely as desired, try comparing them to a Gamma distribution.
- Once the model parameters determining pollen tube growth rates have been calibrated, compare the proportion of seeds sired by each accession to empirical data.

V. Acknowledgments

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