Can 1 Clue Define a Puzzle?

We begin by considering puzzles with a single clue. Since Hanson and Nash have already proven that one clue cannot define puzzles with an even dimension, we only need to examine odd \( \times \) odd puzzles.

- **Clues on Lines of Symmetry**
  Given any clue on a line of symmetry, if there exists a solution, another solution can be found by reflecting the puzzle over the line of symmetry.

- **Border Clues**
  Given a clue on the border of the puzzle, we can draw a path around the border to be filled in either direction, providing 2 unique solutions.

- **Other Clues**
  We can extend the above technique by connecting the border path to clues inside the puzzle.

Diagonal Numbrix Puzzles

Diagonal Numbrix puzzles are puzzles which allow for all normal Numbrix moves as well as diagonal moves in any of the four diagonal directions.

Puzzles defined under normal Numbrix rules may not be defined as diagonal Numbrix puzzles.

Checkerboard Arrangement

**Theorem (Hensley & Peper):** The minimum number of clues needed to define a diagonal Numbrix puzzle is at most \( \left\lceil \frac{mn}{2} \right\rceil \).

Another arrangement of a checkerboard puzzle is shown below:

This arrangement is minimal, but not minimum!

Upper Bound for the Minimum

**Theorem (Hensley & Peper):** The minimum number of clues needed to define a diagonal Numbrix puzzle is at most \( \left\lceil \frac{mn}{2} \right\rceil (m + 1) \).

Can 1 Clue Define a Diagonal Puzzle?

A Hamiltonian circuit can be drawn through any diagonal Numbrix puzzle with \( m, n > 1 \).

Numbrix on a Flat Torus

When playing on a torus we can move from a border cell to the cell on the opposite side in a jump move.

Can 1 Clue Define a Torus Puzzle?

Every \( 1 \times n \) torus puzzle has a Hamiltonian circuit. In fact, a Hamiltonian circuit can be drawn in any Numbrix puzzle on a torus.

**Theorem (Hensley & Peper):** One clue cannot define a Numbrix puzzle on a torus larger than \( 1 \times 2 \) or \( 2 \times 1 \).

Further Research

- Prove that 2 clues do not define an \( m \times n \) Numbrix puzzle with \( m, n \geq 5 \).
- Explore minimum number of clues to define a Numbrix puzzle on a torus.

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