Nonlocality in Shallow Quantum Circuits

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Introduction

Quantum supremacy is expected to drastically change modern computing, but physical implementation is difficult due to nature of qubits.

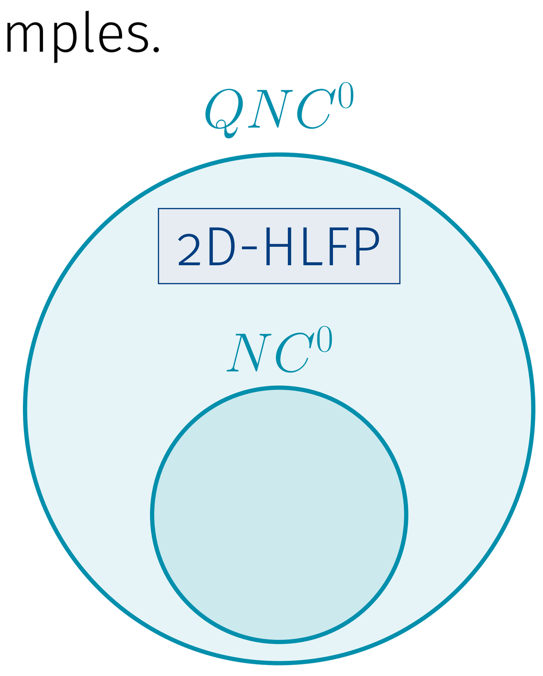
Due to limitations, there is increasing interest in Noisy Intermediate-Scale Quantum technology (50-100 qubits).

In 2017, Bravyi, Gosset, and König publish "Quantum Advantage of Shallow Circuits".

Authors prove quantum circuits are more powerful than classical ones, without complexity theory assumptions. In particular,

$$2D\text{-}HLFP \in QNC^0 \setminus NC^0.$$

We illustrated the importance of the nonlocality property in shallow quantum circuits by proving some of their results for some small examples.



$\mathbf{NC^0}$ = complexity class of all poly-size circuits of $O(1)$ depth with bounded fan-in gates

$\mathbf{QNC^0}$ = quantum analog of NC^0

Hidden Linear Function Problem

Hidden Linear Function Problem (HLFP):

Given: quadratic form $q : \mathbb{F}_2^n \to \mathbb{Z}_4$,

$$q(x) = 2 \sum_{1 \le i < j \le n} A_{ij} x_i x_j + \sum_{k=1}^{n} b_k x_k,$$

where $x_k, A_{ij}, b_k \in \mathbb{F}_2^n$.

Find: $z \in \mathbb{F}_2^n$ such that $q(x) = 2z^T x$, $\forall x \in \mathcal{L}_q$, where $\mathcal{L}_q$ denotes the set

$$\{x \in \mathbb{F}_2^n \mid q(x \oplus y) = q(x) + q(y), \forall y \in \mathbb{F}_2^n\}.$$

2D-HLFP:

Let $G = (V, E)$ denote the $N \times N$ grid graph and $A \in \{0,1\}^{|E|}$ be its adjacency matrix.

Given: quadratic form $q : \mathbb{F}_2^{|V|} \to \mathbb{Z}_4$

$$q(x) = 2 \sum_{(u,v) \in E} x_u x_v + \sum_{v \in V} b_v x_v,$$

where $b \in \{0,1\}^{|V|}$

Find: $z \in \mathbb{F}_2^{|V|}$ such that $q(x) = 2z^T x, \forall x \in \mathcal{L}_q$.

Background

Quantum logic gates used in circuit $\mathcal{Q}_N$:

$$\boxed{H} := \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}, \quad \boxed{S} := \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix},$$

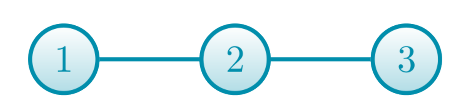
$$\boxed{X} := \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \boxed{Y} := \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix},$$

$$\boxed{Z} := \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \boxed{CZ} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Definition: Let $G = (V, E)$ be a finite simple graph with $|V| = n$ and $|E| = m$. Suppose that a qubit is associated with each vertex of G. The n-qubit graph state of G is

$$|\phi_G\rangle := \left(\prod_{(u,v) \in E} CZ_{uv} \right) H^{\otimes n} |0^n\rangle.$$

Example:



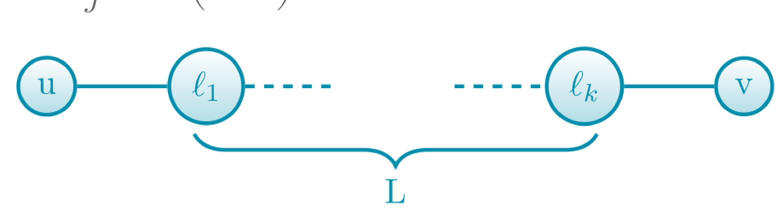
$V = \{1,2,3\}, E = \{(1,2),(2,3),(3,2),(2,1)\}$

$$|\phi_G\rangle = CZ_{12} CZ_{23} H^{\otimes 3} |0^3\rangle = \tfrac{1}{\sqrt{8}}(|000\rangle + |001\rangle + \dots + |111\rangle).$$

Claim: For finite simple graph $G = (V, E)$, $|\phi_G\rangle$ is a stabilizer state for the stabilizer group generated by the operators

$$g_v := X_v \left(\prod_{(u,v) \in E} Z_u \right), \forall\, v \in V.$$

Definition: Let $z \in \{0,1\}^*$. For a bit z_j of z, define $m_j := (-1)^{z_j}$. Describe G and L by



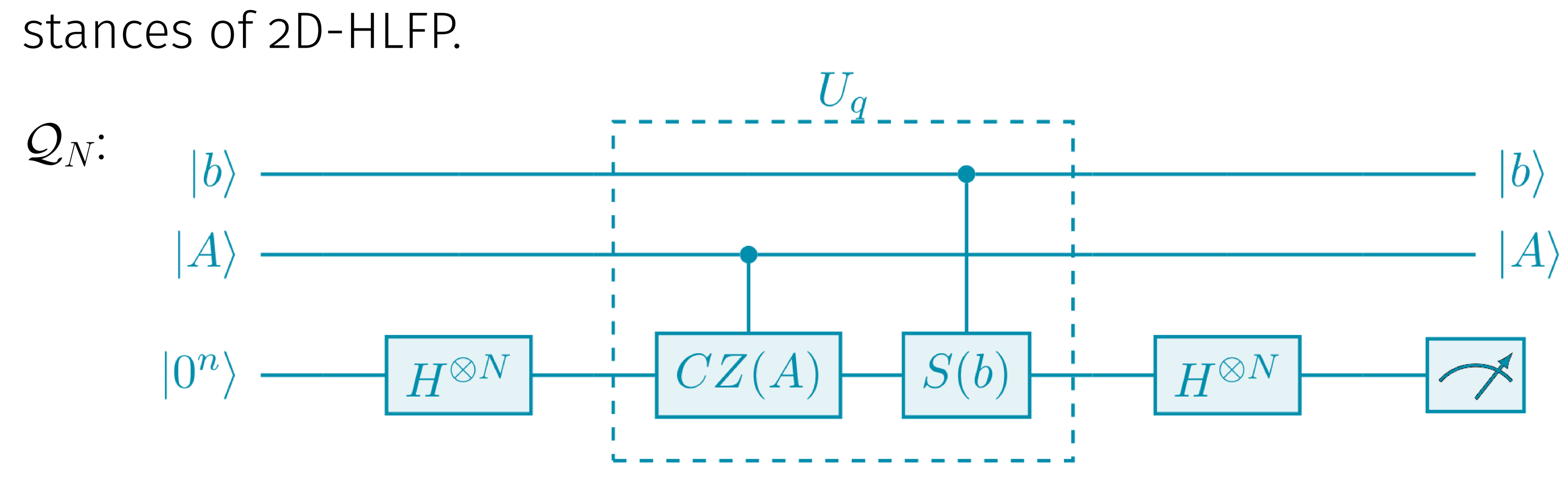
We define L_{odd} and similarly

$$L_{even} := \{\ell \in L \mid \delta(\ell, u) \equiv 0 \,(mod\, 2) \equiv \delta(\ell, v)\}.$$

Also define $m_L := \prod_{j \in L_{odd}} m_j$.

Quantum Circuit for 2D-HLFP

Theorem: For every $N \ge 2$, there exists a quantum circuit $\mathcal{Q}_N$ of depth $d = O(1)$ which deterministically solves size-N instances of 2D-HLFP.

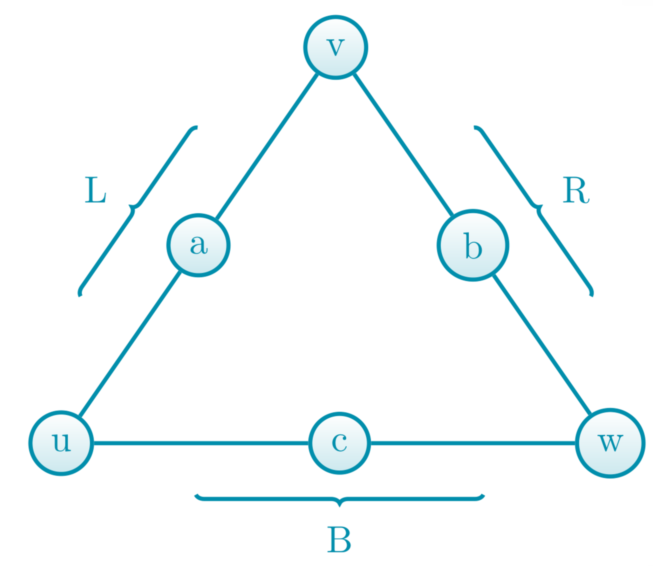


$$CZ(A) := \prod_{1 \le i < j \le N} CZ_{ij}^{A_{ij}} \text{ and } S(b) := \bigotimes_{j=1}^{N} S_j^{b_j}.$$

Nonlocality Property

Nonlocality: a form of correlation present in the measurement statistics of entangled quantum states that cannot be reproduced by local hidden variable models.

Example: Let G describe the graph below



Let $b := b_u b_v b_w \in \{0,1\}^3$ and define

$$\mathcal{T}(b) := \{z \in \{0,1\}^m \mid \langle z | H^{\otimes m} S_u^{b_u} S_v^{b_v} S_w^{b_w} |\phi_G\rangle \neq 0\}.$$

Claim: Let $b = b_u b_v b_w \in \{0,1\}^3$ and $z \in \mathcal{T}(b)$. Then $m_R m_B m_L = 1$. If $b_u \oplus b_v \oplus b_w = 0$, then $i^{b_u + b_v + b_w} m_u m_v m_w m_E m_R^{b_u} m_B^{b_v} m_L^{b_w} = 1$.

Lemma: The stabilizers of $|\phi_G\rangle$ are

$$X_u X_v X_w, -X_u Y_v Y_w X_a X_c, -Y_u X_v Y_w X_a X_b,$$
$$\text{and } -Y_u Y_v X_w X_b X_c.$$

Measurements on classical local circuits cannot simultaneously satisfy these nonlocality identities!

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[4] Corner image retrieved from: `https://www.hiclipart.com/`.