# Theoretical Model of Blood Flow in the Human Retinal Vasculature

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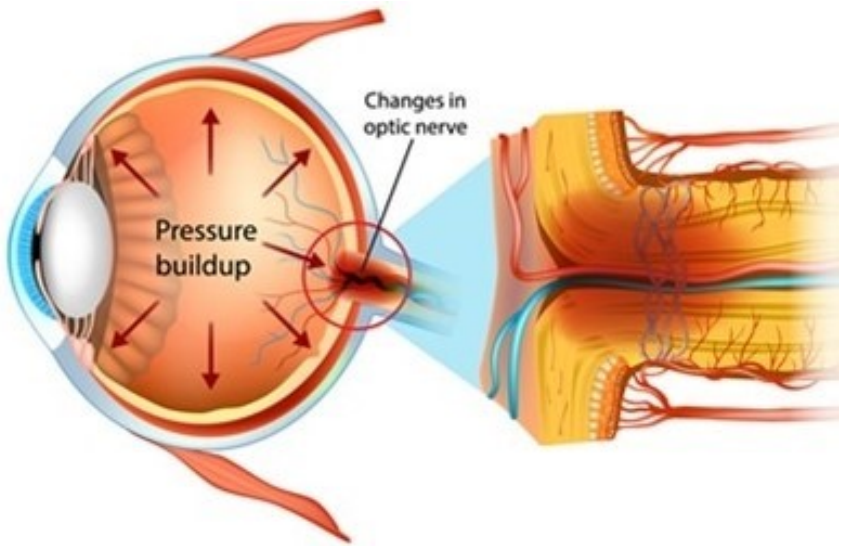
# Background

# Biological Background

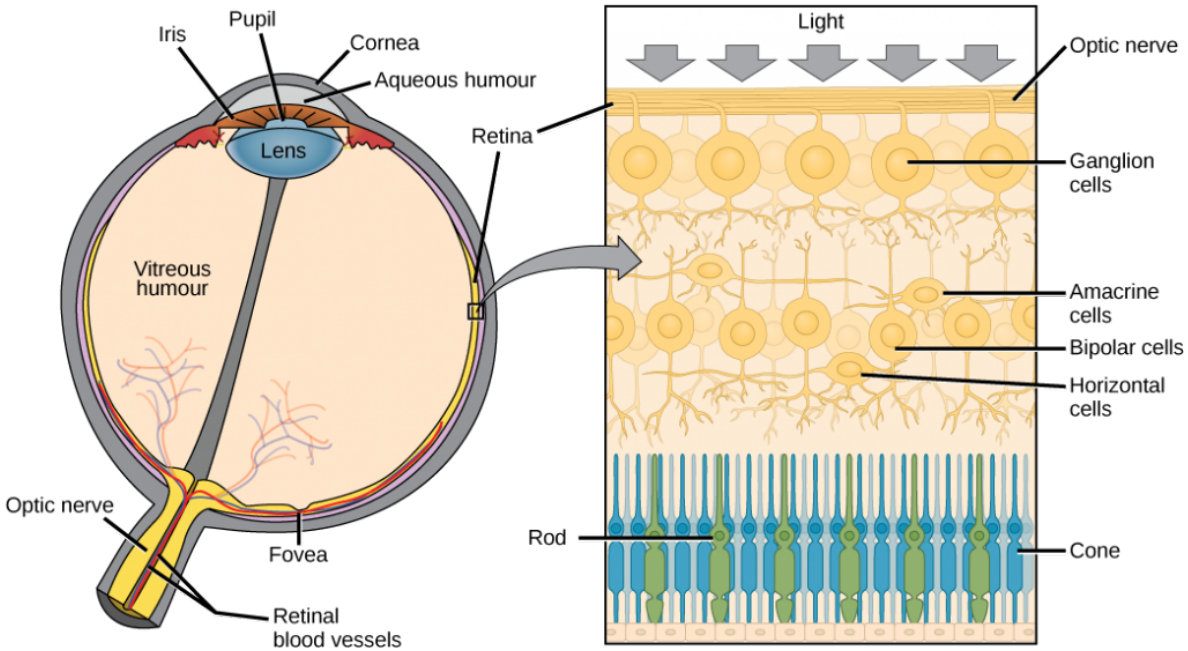
- Glaucoma is a serious ocular disease that results in irreversible vision loss
- Increased intraocular pressure damages the optic nerve
- Almost one third of glaucoma patients do not exhibit elevated pressures
- There are other factors that cause continuing damage




*Source: Murata Eyecare Optometry*

# Biological Motivation

- Possible correlation between impaired blood flow and glaucoma-related damage
- Impaired oxygenation could be a **cause** or an **effect** of retinal ganglion cell death and loss of retinal function




*Source: Concepts of Biology*

# Mathematical Model

# Heterogeneous Model

- Provides more accurate depiction of oxygen delivery
- A previous heterogeneous model predicts retinal oxygenation in the mouse
- Differences between mouse and man:
  - vessel size
  - four arterial branches vs. six
  - fovea

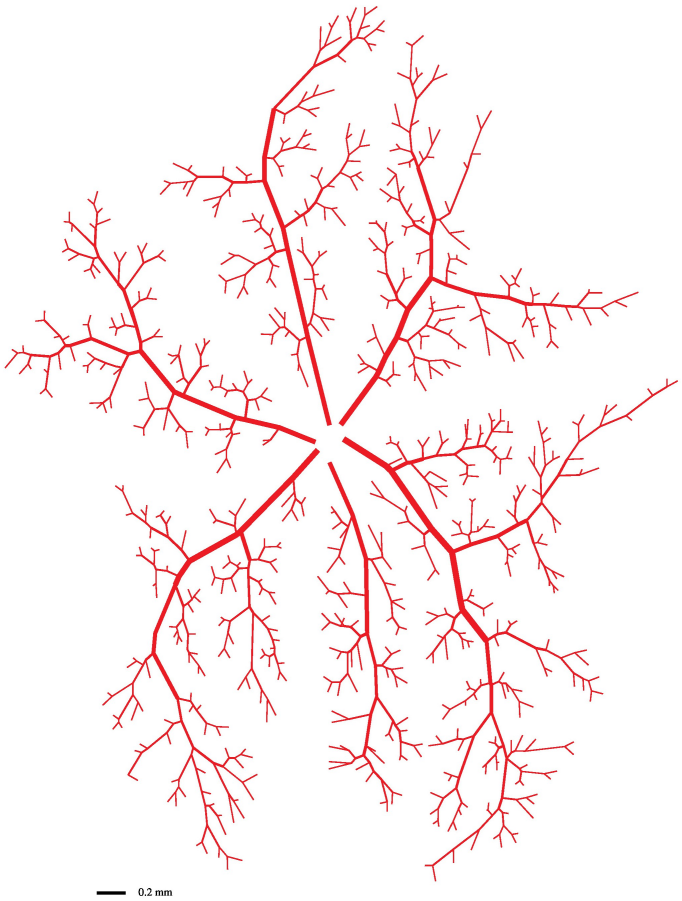



Figure: Arteriolar Network in the Mouse Retina

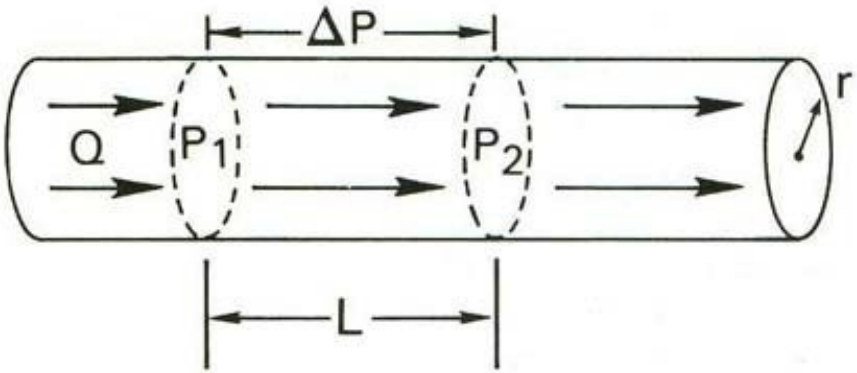
# Research Goals

## Objective

To develop a theoretical model of the human retinal arterioles based on oximetry data to predict retinal blood and tissue oxygenation

# Hemodynamics

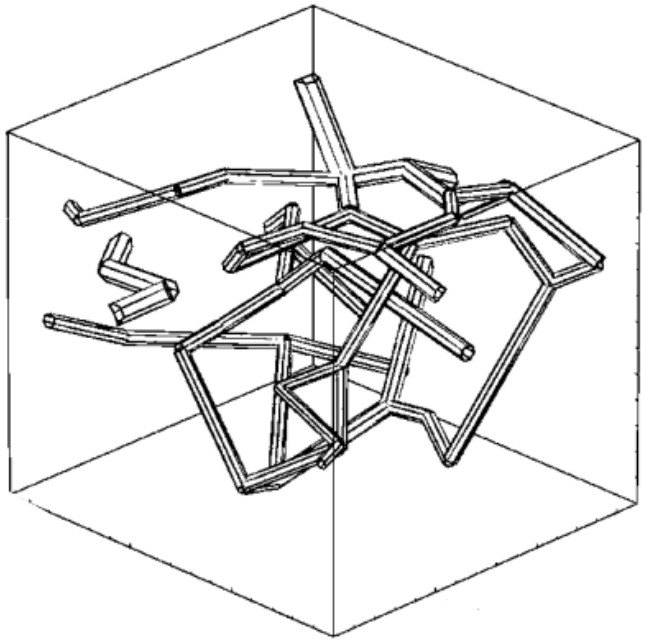
# Blood Flow




Poiseuille's Law

$$Q = \frac{\pi(\Delta P)D^4}{128L\mu}$$

# Oxygen Transport - Green's Function Model



*Source: Secomb et al. 2004*

- This more realistic model of oxygen transport accounts for the diffusion of oxygen from multiple blood vessels
- Vessel points are considered oxygen sources and tissue points are considered oxygen sinks

# Green's Function Model

- By the conservation of mass, oxygen diffusion is equal to oxygen consumption,

$$D\alpha\nabla^2 P = M(P) \tag{1}$$

$P$ - tissue $PO_2$
$D\alpha$ - rate of oxygen diffusion
$M(P)$ - consumption rate

- The Green's function $G(x, x^*)$ represents the $PO_2$ at a point $x$, resulting from a unit point source at $x^*$, and is the solution to

$$D\alpha\nabla^2 G = -\delta_3(x - x^*) \tag{2}$$

- Oxygen levels at each tissue point are calculated by summing the oxygen fields from these sources,

$$P = \Sigma G(x, x^*)q(x^*) \tag{3}$$

$q(x^*)$ - strength of oxygen source

# Heterogeneous Model

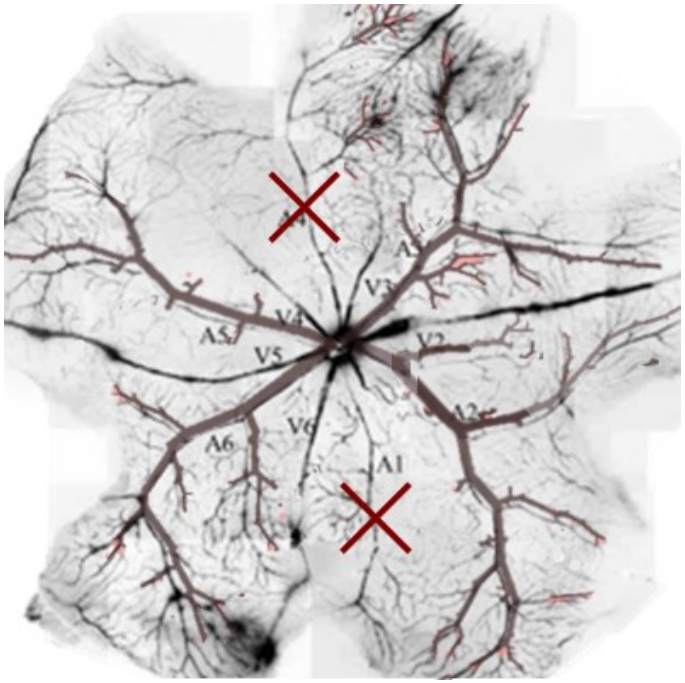
# Vascular Networks

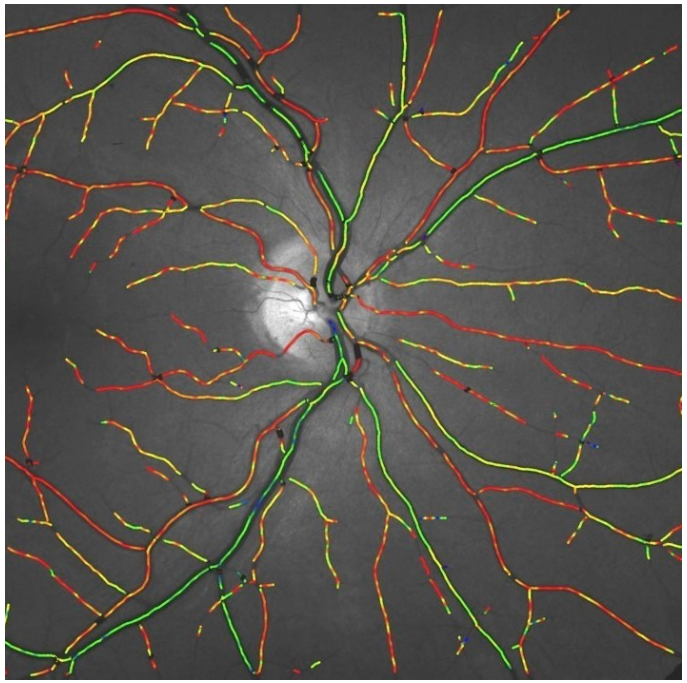
## Mouse Vasculature

## Human Vasculature

Superior

Temporal



Nasal

Inferior

*Source: Ganessan et al. 2010*

*Source: Stefánsson et al. 2019*

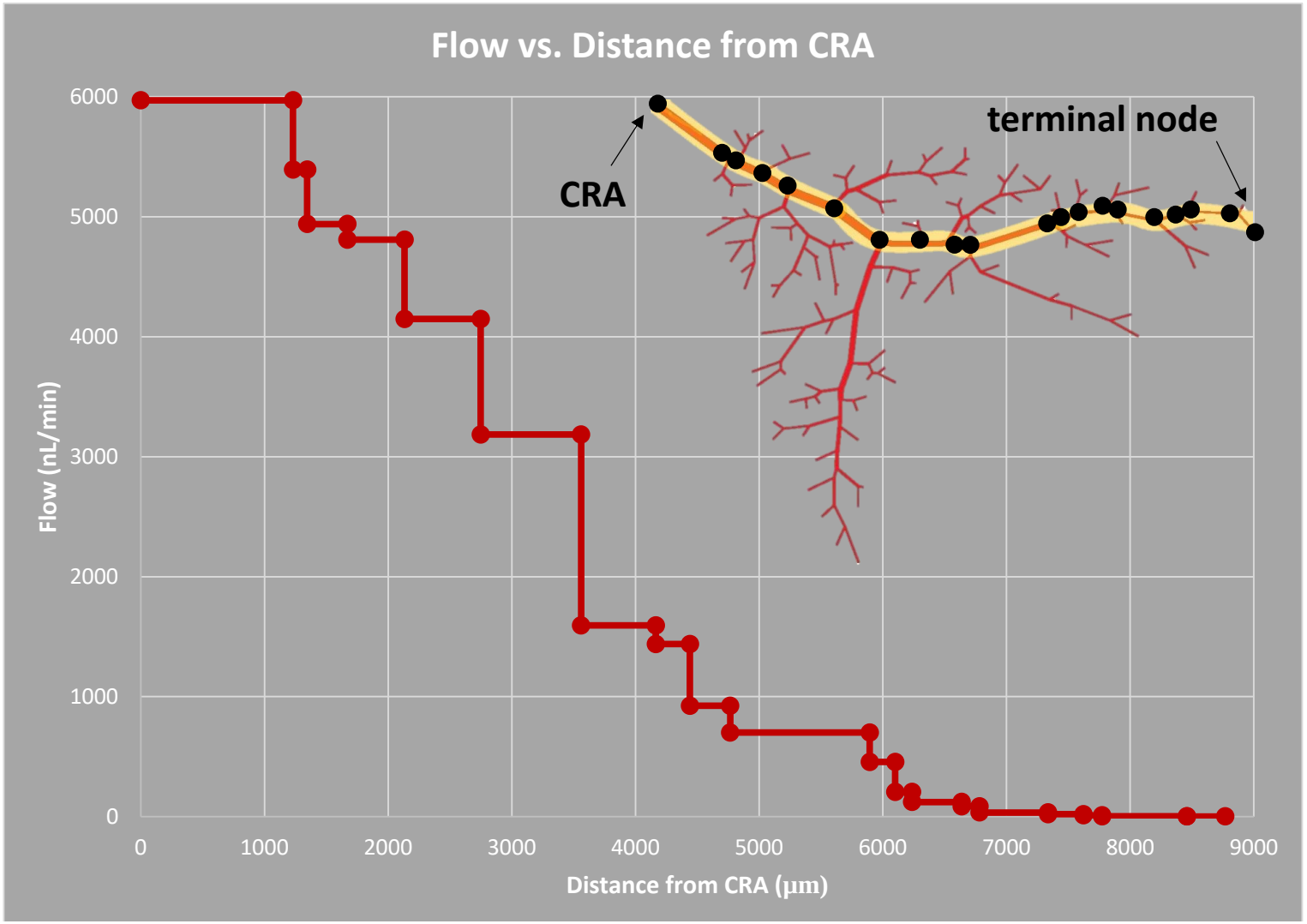
# Mouse to Man Scaling Factor

| Blood Vessel Diameters (µm) | | | | |
|---|---|---|---|---|
| Branch | Inf. Nasal | Sup. Nasal | Sup. Temporal | Inf. Temporal |
| **Mouse** | 30.42 | 26.94 | 24.02 | 28.48 |
| **Human** | 94 | 97 | 111 | 117 |

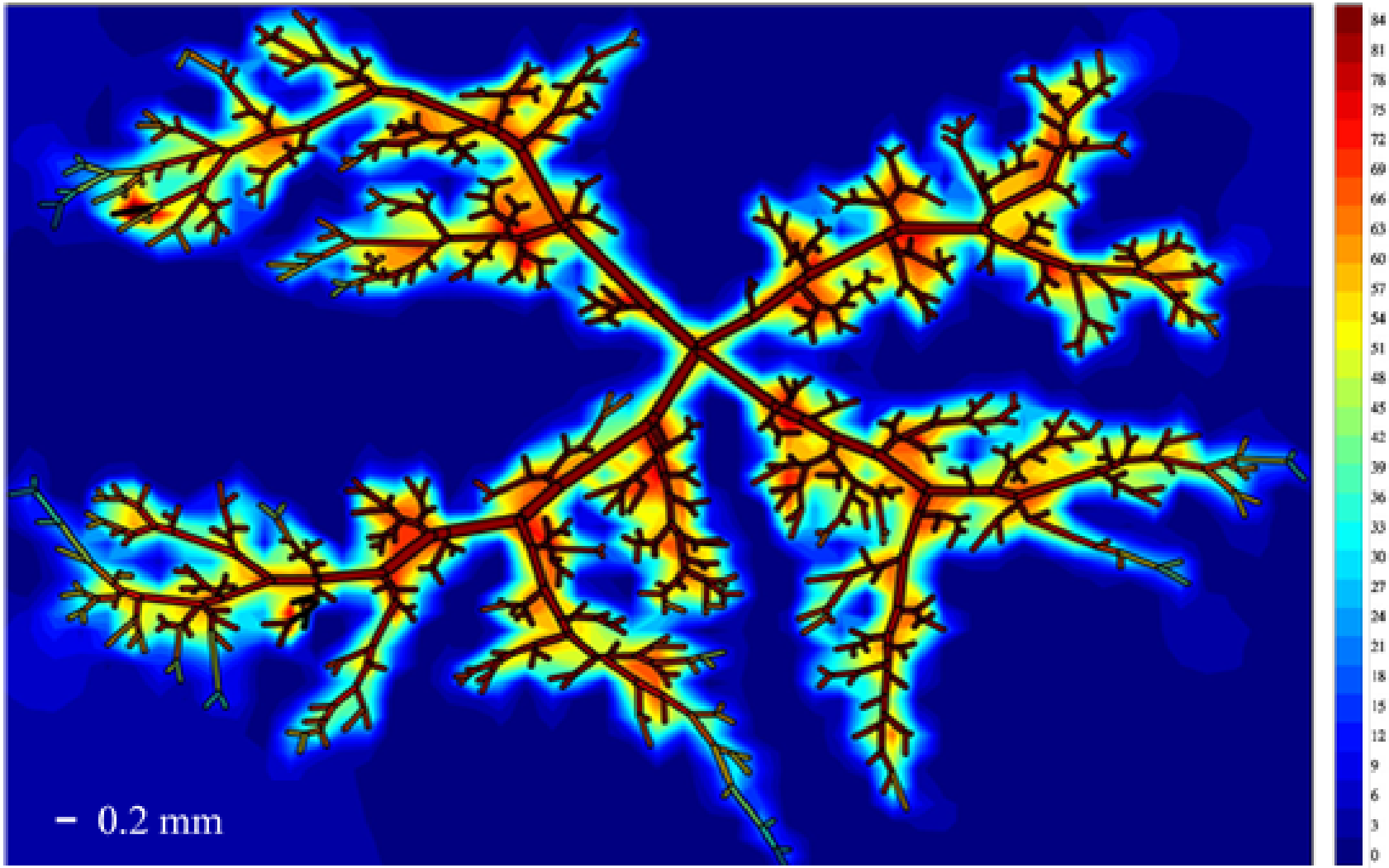
- A scaling factor of **3.8** was determined by comparing diameter values for the four main arterial branches

# Results

# Blood Flow Computed along Single Vessel Pathway



# Network Oxygenation



# Future Work

- Continue adapting the model to represent the human retinal blood vessels
- Develop a hybrid model of the retinal microvasculature that connects the capillaries and veins to the arterial network
- Relate our understanding of retinal blood flow to visual function and glaucoma-related damage

# Acknowledgments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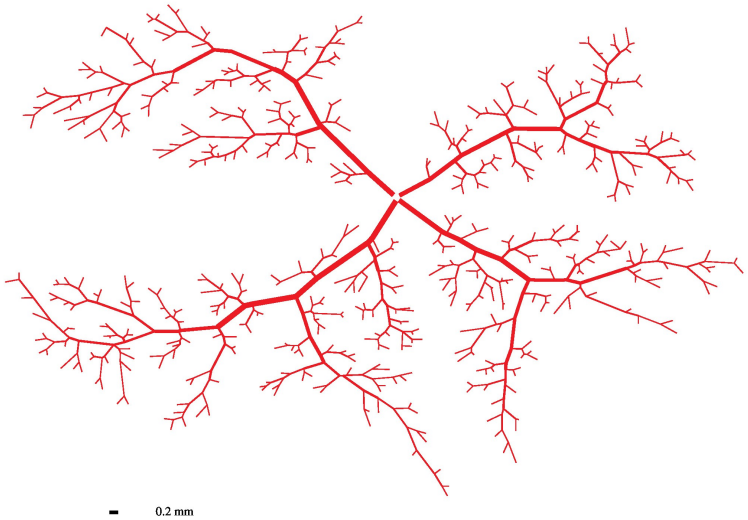
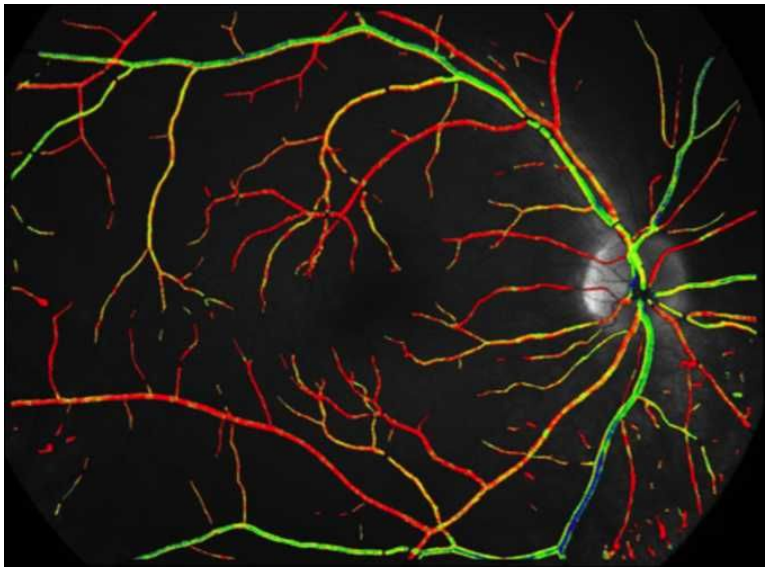
# Questions



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