

# A Game Theoretic Analysis of the RSU 9 School Board's Weighted Voting System

By: Amanda Gulley, Rockhurst University  
Faculty Mentor: Dr. Andrew Windle



# Counties and Towns of the Mt. Blue Regional School District (RSU 9) in Maine

## Franklin County

- Farmington
- Wilton
- Chesterville
- Industry
- New Sharon
- New Vineyard
- Temple
- Weld

## Somerset County

- Starks

## Kennebec County

- Vienna



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- Stems from the **one-person, one-vote policy**
- Farmington has **5 directors**, Wilton has **3 directors**, and the remaining 8 towns each have **1 director**.
- Together, Farmington and Wilton claim **half the directors** and **about 64% of the votes**.

## The Shapley-Shubik Power Index

- Formulated by Lloyd Shapley and Martin Shubik in 1954
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  - In a weighted system, voters cast "yea" or "nay" votes
- Uses the assumption that votes are cast one at a time
- Measures how often a voter turns a losing coalition into a winning one. This notion is called pivotal.



## The Shapley-Shubik Power Index

**Example:** Consider the weighted voting system of  $[4; 3,2,1]$  where **voter A has 3 votes**, **voter B has 2 votes**, and **voter C has 1 vote**. Since there are 3 voters, we have  $3!$  orderings of the voters:

ABC ACB  
BAC BCA  
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## The Shapley-Shubik Power Index

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ABC ACB  
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To calculate each voter's Shapley-Shubik power index we take the number of times a voter is pivotal divided by the total number of orderings.

$$\varphi = \frac{1}{6}(4, 1, 1) = \left(\frac{2}{3}, \frac{1}{6}, \frac{1}{6}\right)$$

# Analysis of the RSU 9 Weighted Voting System

Towns in the RSU 9 School District	Population of Towns	Cumulative Percentage of Votes Held by Director(s)
Farmington	7,760	42
Wilton	4,116	22.2
Chesterville	1,352	7.4
Industry	929	5
New Sharon	1,407	7.6
New Vineyard	757	4.1
Temple	528	2.9
Weld	419	2.3
Starks	640	3.5
Vienna	570	3.1

## Analysis of the RSU 9 Weighted Voting System

Towns in the RSU 9 School District	Population of Towns	Cumulative Percentage of Votes Held by Director(s)	Per Capita Percentage of Vote
Farmington	7,760	42	0.005412
Wilton	4,116	22.2	0.005394
Chesterville	1,352	7.4	0.005473
Industry	929	5	0.005382
New Sharon	1,407	7.6	0.005402
New Vineyard	757	4.1	0.005416
Temple	528	2.9	0.005492
Weld	419	2.3	0.005489
Starks	640	3.5	0.005468
Vienna	570	3.1	0.005439

## Analysis of the RSU 9 Weighted Voting System

Towns in the RSU 9 School District	Number of Directors each Town has on the Board	Percentage of the Vote Held by Each Director
Farmington	5	8.4
Wilton	3	7.4
Chesterville	1	7.4
Industry	1	5.0
New Sharon	1	7.6
New Vineyard	1	4.1
Temple	1	2.9
Weld	1	2.3
Starks	1	3.5
Vienna	1	3.1

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Starks	1	3.5
Vienna	1	3.1

- This school board's weighted voting system can be modeled as such:

[501; 84, 84, 84, 84, 84, 74, 74, 74, 74, 50, 76, 41, 35, 29, 31, 23]

- Roughly 1000 votes are distributed among the directors and our quota is 501.

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- This school board's weighted voting system can be modeled as such:

[501; 84, 84, 84, 84, 84, 74, 74, 74, 74, 50, 76, 41, 35, 29, 31, 23]

- Roughly 1000 votes are distributed among the directors and our quota is 501.
- There are 16! (which is **over 20 trillion**) number of orderings.
- With short cuts we have the number of orderings listed below:

$$\frac{16!}{4!5!} = 7,264,857,600$$

# Analysis of the RSU 9 Weighted Voting System

Towns in the RSU 9 School District	Number of Directors each Town has on the Board	Percentage of the Vote Held by Each Director	Shapley-Shubik Power Index of Each Director
Farmington	5	8.4	8.61
Wilton	3	7.4	7.4
Chesterville	1	7.4	7.4
Industry	1	5.0	5.01
New Sharon	1	7.6	7.6
New Vineyard	1	4.1	3.97
Temple	1	2.9	2.9
Weld	1	2.3	1.75
Starks	1	3.5	3.25
Vienna	1	3.1	2.92



## Analysis of the RSU 9 Weighted Voting System

Towns in the RSU 9 School District	Number of Directors each Town has on the Board	Percentage of the Vote Held by Each Director	Shapley-Shubik Power Index of Each Director	Per Capita Percentage of Vote	Per Capita Percentage of the Shapley-Shubik Power Indices
Farmington	5	8.4	8.61	0.005412	0.005546
Wilton	3	7.4	7.4	0.005394	0.00539
Chesterville	1	7.4	7.4	0.005473	0.00547
Industry	1	5.0	5.01	0.005382	0.005397
New Sharon	1	7.6	7.6	0.005402	0.005384
New Vineyard	1	4.1	3.97	0.005416	0.005247
Temple	1	2.9	2.9	0.005492	0.005486
Weld	1	2.3	1.75	0.005489	0.004185
Starks	1	3.5	3.25	0.005468	0.005084
Vienna	1	3.1	2.92	0.005439	0.005124

## Analysis of the RSU 9 Weighted Voting System When Considering Bloc Voting

- A group of people who have something in common can form a voting bloc which combines all their votes, and this group then votes as one voter.

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## Analysis of the RSU 9 Weighted Voting System When Considering Bloc Voting

- A group of people who have something in common can form a voting bloc which combines all their votes, and this group then votes as one voter.
- We are assuming a 3-player game. The 3 players (groups) will be Farmington, Wilton, and the remaining 8 towns on the board.
- Each of these groups has 2 choices, to organize as a voting bloc or to NOT organize as a voting bloc. This leaves us with  $2^3 = 8$  possible scenarios (**voting systems**) that could play out.

## Analysis of the RSU 9 Weighted Voting System When Considering Bloc Voting

Possible Scenario: **Wilton** is the only group to not organize a voting bloc

This voting system can be modeled as such: [501; 420, 74, 74, 74, 359]

## Analysis of the RSU 9 Weighted Voting System When Considering Bloc Voting

Possible Scenario: **Wilton** is the only group to not organize a voting bloc

This voting system can be modeled as such: [501; 420, 74, 74, 74, 359]

Towns/Groups	Number of votes for each town/group	Shapley-Shubik power index for each town/group
Farmington	420	0.3
Wilton	74	0.133
	74	0.133
	74	0.133
Remaining 8 towns	359	0.3

## Analysis of the RSU 9 Weighted Voting System When Considering Bloc Voting

### Remaining 8 towns organize Wilton

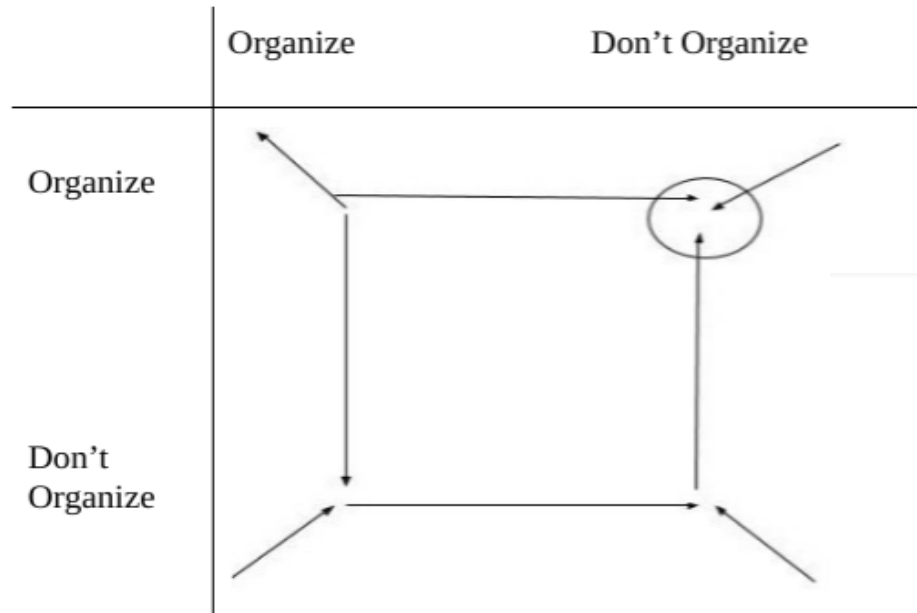
	Organize	Don't Organize
<u>Farmington</u>	(.333, .333, .333)	(.3, .4, .3)
Don't Organize	(.381, .143, .476)	(.278, .167, .556)

### Remaining 8 towns do not organize Wilton

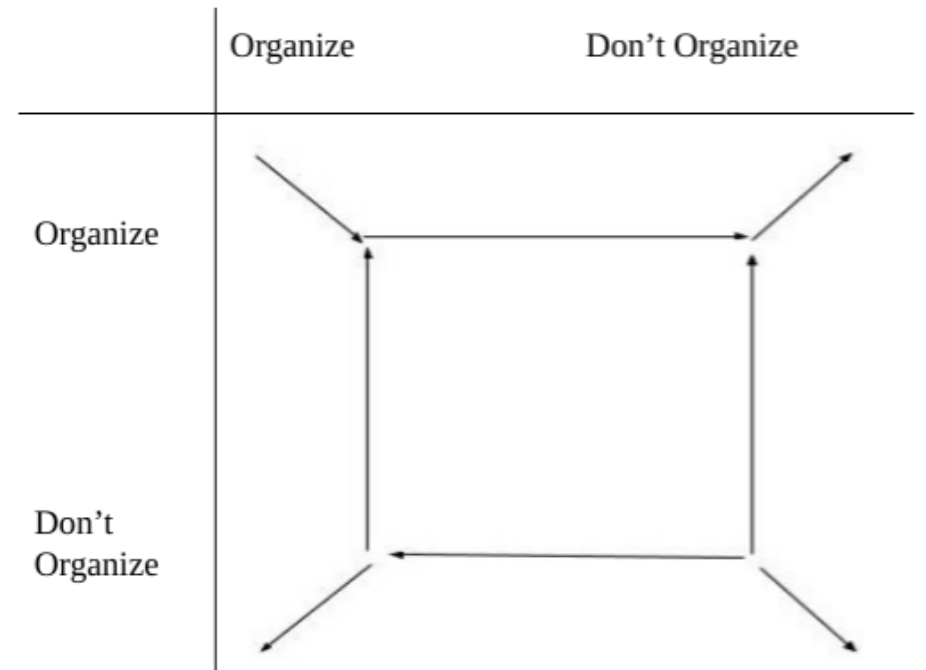
	Organize	Don't Organize
<u>Farmington</u>	(.556, .095, .349)	(.63, .127, .242)
Don't Organize	(.424, .282, .294)	(.43, .222, .348)

# Analysis of the RSU 9 Weighted Voting System When Considering Bloc Voting

Remaining 8 towns organize  
Wilton



Remaining 8 towns do not organize  
Wilton



Farmington



## Analysis of the RSU 9 Weighted Voting System When Considering Bloc Voting

- Original Weighted Voting System before introducing bloc voting:

[501; 84, 84, 84, 84, 84, 74, 74, 74, 74, 50, 76, 41, 35, 29, 31, 23]

- Ordered triple associated with this outcome: (.43, .222, .348)

- Natural Outcome: [501; 420, 74, 74, 74, 359]

- Ordered triple associated with this outcome: (.3, .4, .3)

## Conclusion

- Analysis with the Shapley-Shubik power index has confirmed that the directors of Farmington and Wilton do hold a high majority of the voting power.

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- For bloc voting, if all groups play rationally, the natural outcome would make the remaining 8 towns lose voting power.

## Conclusion

- Analysis with the Shapley-Shubik power index has confirmed that the directors of Farmington and Wilton do hold a high majority of the voting power.
- For bloc voting, if all groups play rationally, the natural outcome would make the remaining 8 towns lose voting power.
- However, the per capita percentage of voting power is roughly equal across all the towns in the school district.

**Thanks For  
Listening!**

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