Exploring the garden of Petaluma knots

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Knots and Knot Diagrams

- A closed curve in 3-dimensional space
- Typically visualized using knot diagrams
  - 2-D regular projection
- Knots are classified by *Crossing Number*
Knot Equivalence

- Knots are malleable
  - Not all knot diagrams are nice

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- That is why crossing number is the least amount of crossing of any diagram

0₁
Petaluma Model

• All crossings in the center
• Classified by number of “petals”
• Impossible to tell which strands go over and under
• Heights assigned by a string of numbers (1,2,...,n)
How it works

• Generate a string of numbers 1 to (# of petals)
  • Ex. For a 5-petal knot, (1,4,2,5,3)
• Assign a counterclockwise orientation
• Assign heights using the string of numbers
  • 1 is the highest (on top)
  • n is the lowest (on bottom)
• See what type of knot we get
Example: 5 petals with (1,4,2,5,3) heights

Assign Heights Counter-Clockwise          Top View          Side View
Example: 5 petals with (1,4,2,5,3) heights

Assign Heights Counter-Clockwise  Top View  Side View
Petaluma Properties

• Cyclic invariance
  • Ex. 14253=42531=25314 etc.
  • This allows us to start every height at 1 (Heights =\{1,2,\ldots,n\})

• For a diagram with n petals there are \((n-1)!\) petal diagrams we can generate

<table>
<thead>
<tr>
<th>Petal Number</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
<th>11</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Petal Diagrams</td>
<td>2</td>
<td>24</td>
<td>720</td>
<td>40,320</td>
<td>3,628,800</td>
<td>479,001,600</td>
</tr>
</tbody>
</table>
Project Goals

• Classify all 5, 7, 9, 11, and 13 petal knots
• Separate the knots into 5 categories
  • Unknots
  • Alternating
  • Non-Alternating
  • Composite
  • Unknown

• Study relationships within a petal number
• Study relationships across petal numbers
Knot Categories

- Unknot
- Composite Knots
- Prime Knots
Alternating vs. Non-Alternating

$5_1$  

$8_{20}$
Unknots

• All knots have been classified through 16 crossings
• Unknots have 17 or more

So in our data:
• Unknots have crossing number 0
• Alternating have crossing number $\geq 3$
• Composite have crossing number $\geq 6$
• Non-alternating have crossing number $\geq 8$
• Unknots have crossing number $\geq 17$
Alternating Knots by Crossing Number

- Alternating knots favor odd crossing numbers
Non-Alternating Knots by Crossing Number

• Non-Alternating knots favor even crossing numbers
Totals for All Petal Numbers

[Bar chart showing percentages for different petal numbers, with categories labeled: Unknots, Alt, NonAlt, Composite, Unknowns]
Category Percentages by Crossing Number

11 Petals

13 Petals

Legend:
- Unknots
- Alt
- NonAlt
- Composite
- Unknowns
Future Work

• Tabulate 15 Petals
  • Computationally heavy – use properties to thin out how many need to be generated

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<td>479,001,600</td>
<td>87,178,291,200</td>
</tr>
</tbody>
</table>

• Study/create other knot models
Acknowledgments

• Dr. Eric Rawdon
• Dr. Jason Parsley and Grace Yao (Wake Forest University)
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Thank You