## The Game of Cycles: Extended

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**Motivation**

Graph theory is a field of mathematics that can be utilized to model a variety of processes and thus knowing more about them increases our ability to solve a broad range of applications.

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**Definitions**

- **Planar graph**: A collection of vertices and edges such that no two edges cross.
- **Directed graph**: A graph with arrows drawn on some or all edges indicating direction.
- **Sink**: A vertex of a directed graph with all edges connected to that vertex directed towards that vertex.
- **Source**: A vertex of a directed graph with all edges connected to that vertex directed away from that vertex.
- **Cycle cell**: An enclosed area of a graph with nothing inside of it such that every edge enclosing the area is directed the same direction.
- **Involutive Symmetry**: A board has involutive symmetry if there is a non-trivial involution, τ, on the set of vertices of the board such that for any vertex v, τ(τ(v)) = v.
- **Self-involutive edge**: An edge where the involution of an edge is itself.
- **Nowhere-involutive edge**: An edge where no edge of the cell has its partner fixed by the involution.

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**Goal**

Our research team aims to answer questions about the Game of Cycles on various families of planar graphs.

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**Related Theorems**

**Theorem 6**: [2] Let G be a board with an involution such that each cell is either self-involutive or nowhere-involutive. If there is no self-involutive edge, then Player 2 has a winning strategy. If there is exactly one self-involutive edge whose vertices are not fixed by the involution, then Player 1 has a winning strategy using the mirror-reverse strategy.

**Mirror-Reverse Strategy**

- If possible to win by completing a cycle, do so.
- If that is not possible, mirror other player's strategy by observing the player's most recent move and playing / \ j, the partner edge of j with its arrow reversed.

**Main Results**

**Theorem**: Let G be a board containing two cycles, C_m and C_n, such that C_m and C_n are connected by a single vertex. If m = 3 and n = 3, then Player 2 has a winning strategy. If m = 3 and n > 3, then Player 1 has a winning strategy.

**Related Results**

- **Cycle Graph C_4 and C_7**
- **Cycle Graphs C_7 and C_9**
- **Complete Graph K_4**
- **Cycle Graph C_6**
- **Cycle Graph with a chord**

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**Other Results**

**Theorem**: In a two player game played on the 3-Prism and 5-Prism graphs, Player 1 has a winning strategy.

**Future Research**

- Can we determine an overarching strategy for m connected cycle graphs comprised of n odd cycles and \( p \) even cycles?

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**References**