Exploring Why The Conway Knot Is Not Slice

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Abstract

In 2018, Lisa Piccirillo wrote a proof about the famous 11-crossing Conway knot. This knot, named for mathematician John Horton Conway who passed away in 2020 due to Covid-19, had been stumping knot theorists for more than fifty years. Thanks to Piccirillo's proof, we now know more about this strange knot and its properties. This poster context and history of the Conway Knot as well as some of the concepts used in Piccirillo's proof.

Slice - A property of knots

There are two types of slice - topologically slice, and smoothly slice. The Conway knot is topologically slice; it's status as smoothly slice was the one in question for so long.

A knot is slice if it bounds a smooth disk in the 4-ball.

Here is one way to visualize the process of going into 4-D space to determine sliceness:

John Conway and his legacy

John Horton Conway (1937 - 2020) was a British mathematician who studied in the fields of group theory, number theory, algebra, geometric topology, theoretical physics, combinatorial game theory, and knot theory! Most notably he created the “Game of Life” and was most proud of his discovery of the surreal numbers. Both the Conway polynomial and the Conway knot are named for him.

Lisa Piccirillo’s proof

In 2018, Lisa Piccirillo attended a conference as a grad student, and heard about this Conway knot problem. After one week she proved the Conway knot is not slice. She used a mutant knot, knot traces, invariants, and many other advanced topological ideas to prove this. Since then she earned her PhD in mathematics from the University of Texas at Austin in 2019 and became a professor at MIT in July 2020.

The Conway Knot

The Conway knot has been contested over for years because knot theorists were not able to prove it’s sliceness. Piccirillo used a knot with the same trace as the Conway knot to prove it is not slice.

Resources


