Octahedral Knots

Overview

Given a knot K we define $d_8(K)$, the octahedral number of K, to be the minimum number of octahedra, connected face to face, required to construct a closed loop that represents K.

Numerical Results

- The Minimal Step number of a knot K, $m(K)$, is number of integer lattice steps to construct K.

Knot (K)	Conway	$m(K)$	$d_8(K)$
Unknot	∞	4	8
3_1	3	24	36
4_1	22	30	48
5_1	5	34	52
5_2	32	36	52
6_1	42	40	64
6_2	312	40	60
6_3	2112	40	64
7_1	7	44	68
7_2	52	46	68
7_3	43	44	68
7_4	313	44	72
7_5	322	46	72
7_6	2212	46	76
7_7	21112	44	72
8_{19}	3,3,2-	42	76

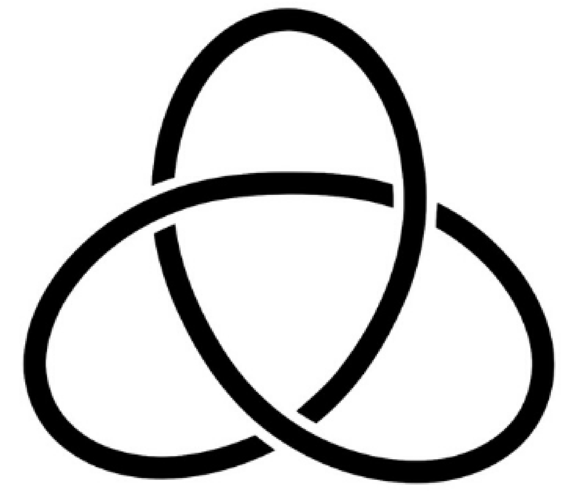
- Knot: Alexander-Briggs Notation of a knot
- Conway: Conway Notation of a knot
- $m(K)$: Minimal Step Number of a knot
- $d_8(K)$: Octahedral Number of a knot

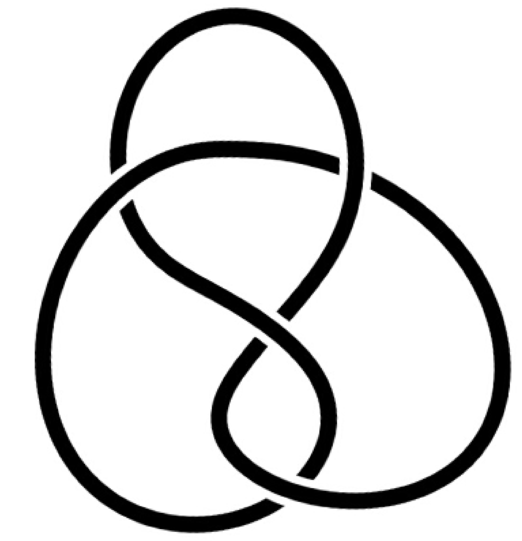
Conjecture

- We have observed that an upper bound of the Octahedral Number for any given knot ranges between 1.5 and 2 times the Minimal Step number of a knot.

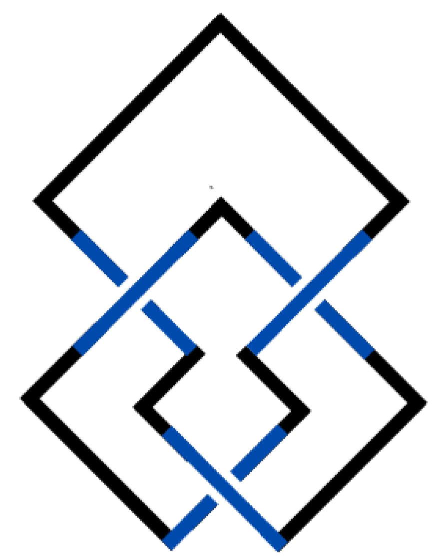
$$d_8(N_1) = \begin{cases} 12 + 8N, & \text{N is even} \\ 16 + 8N, & \text{N is odd} \end{cases}$$

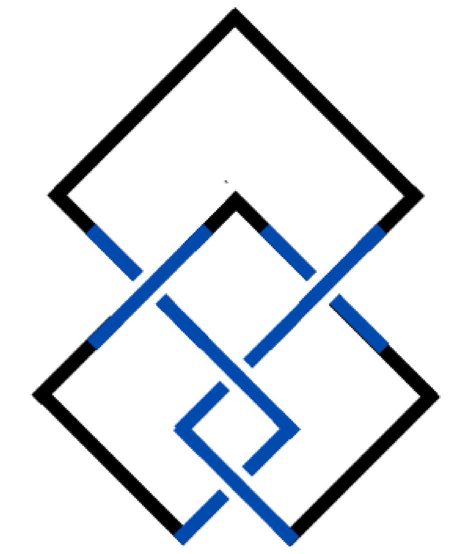
Knot Assembly Process



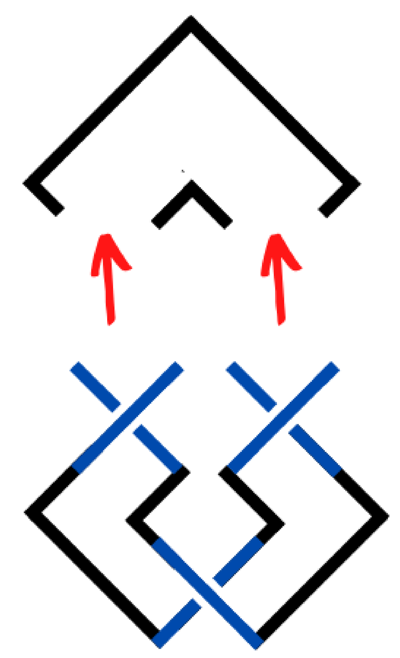


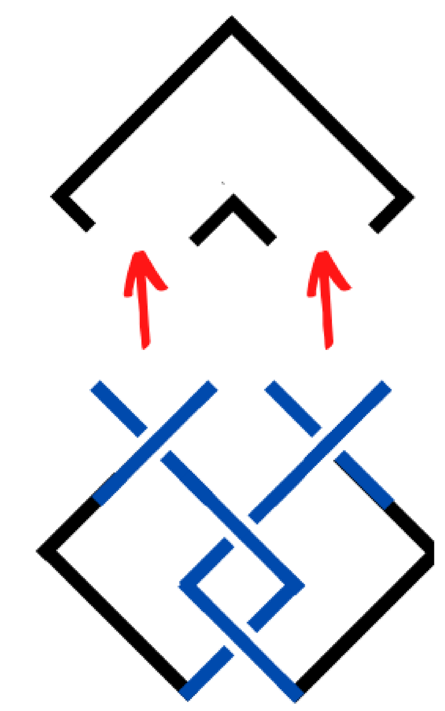
- Knots 3_1 (left) and 4_1 (right).



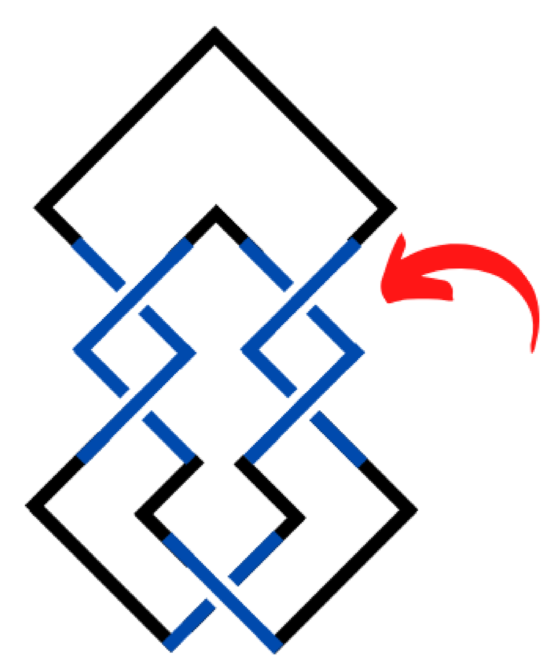


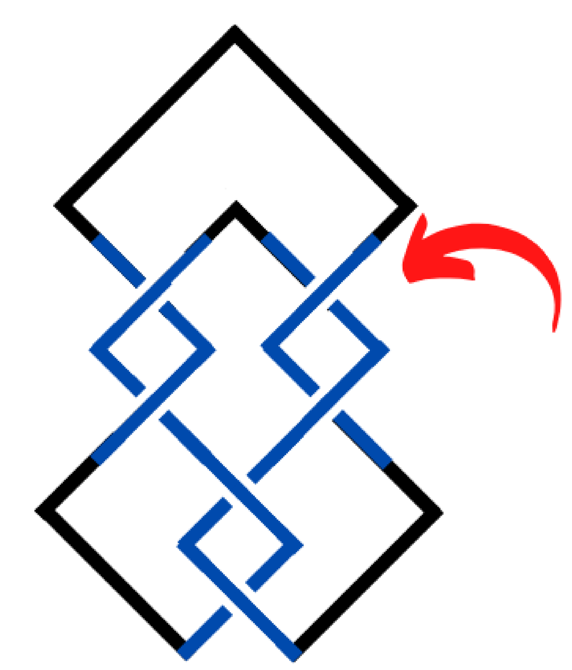
- Knots re-drawn to show parallel crossings.



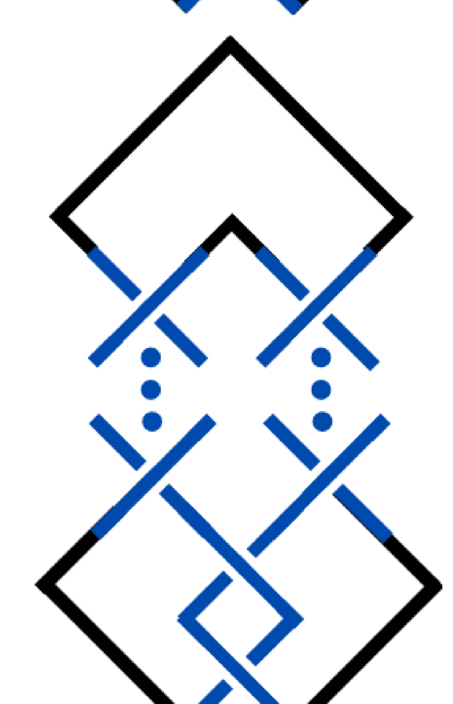


- Knot expansion in order to create other knots.





- The insertion of crossings to produce 5_1 (left) and 6_1 (right).



- The separation generalization.

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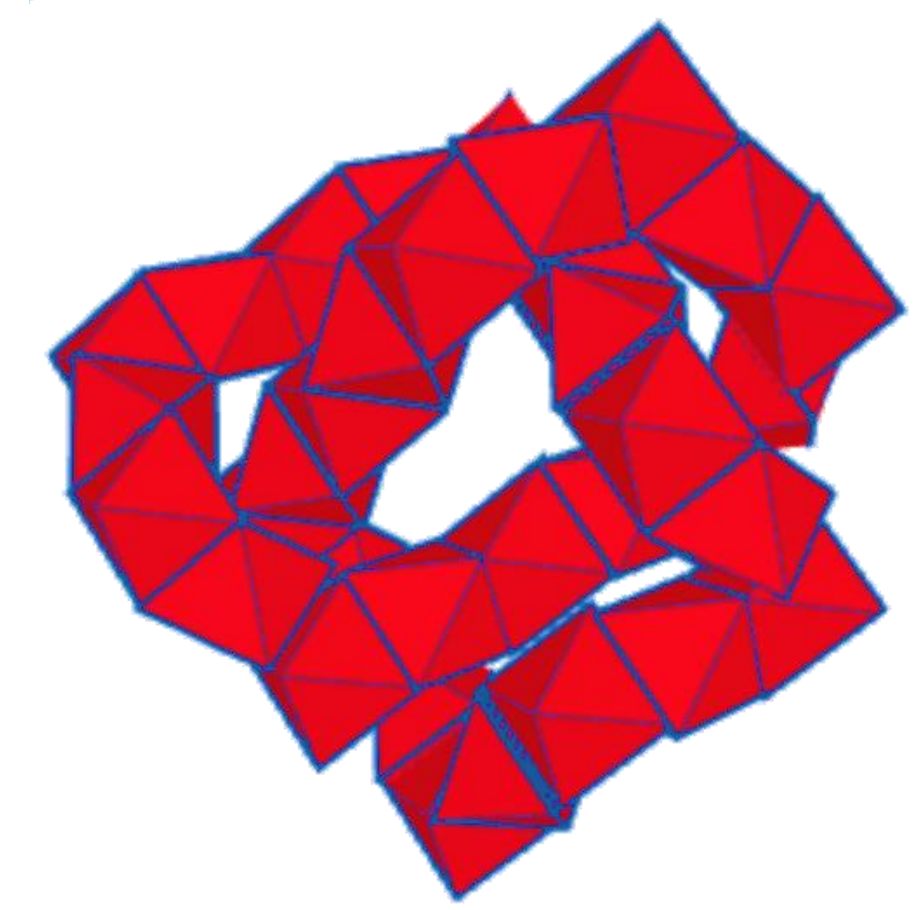
3D Model



- This represents the 3_1 knot created using 36 octahedra.

Future Research

- We intend to generalize our findings from the knots presented in this poster to a formula for which bounds can be determined from the Conway Notation of a given knot.
- Additionally, we intend to compare bounds created for the Conway Notation of a given knot to the bounds formed from a braid word of that given knot.

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