Simulating Nonlinear Waves on Vortex Rings in Ideal Fluids
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Introduction
Nonlinear waves on vortex filaments are fundamental for energy transfer in fluid dynamics and are crucial in the understanding of how quantum turbulent structures settle in nature.

Building the Geometric Vortex Lines
Jacobi Elliptic functions to create a deformed ring:
\[ x(t, R, A, N, m) = Rcn(t, m) + Acn(Nt, m)cn(t, m) \]
\[ y(t, R, A, N, m) = Rsn(t, m) + Acn(Nt, m)sn(t, m) \]
\[ z(t, R, A, N, m) = -Asn(Nt, m) \]

Modify our vortex line using equations from Hasimoto\(^1\):
\[ x = \frac{2\mu(N)}{v} \text{sech}(\eta(t, N))\cos(\theta(t)) \]
\[ y = t - \frac{2\mu(N)}{v} \tanh(\eta(t, N)) \]
\[ z = \frac{2\mu(N)}{v} \text{sech}(\eta(t, N))\sin(\theta(t)) \]

Knowledge of the curvature and torsion give insight to the curve’s parameterization, via the Fernet-Serret equation:
\[
\begin{bmatrix}
T' \\
n' \\
B'
\end{bmatrix} = \begin{bmatrix}
0 & \kappa & 0 \\
-\kappa & 0 & \tau \\
0 & -\tau & 0
\end{bmatrix}
\begin{bmatrix}
T \\
n \\
B
\end{bmatrix}
\]

Simulations
Exploration of the space curve dynamics can be used to simulate the velocity the vortex induces. The induced velocity field from a vortex is represented by the Biot-Savart law leading to curve evolution via the curvature and binormal direction.
\[ \frac{\partial y}{\partial t} = \kappa B \]

References