Simulating Nonlinear Waves on Vortex Rings in Ideal Fluids

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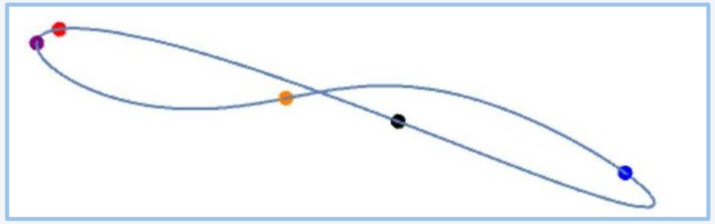
Introduction

Nonlinear waves on vortex filaments are fundamental for energy transfer in fluid dynamics and are crucial in the understanding of how quantum turbulent structures settle in nature.

Building the Geometric Vortex Lines

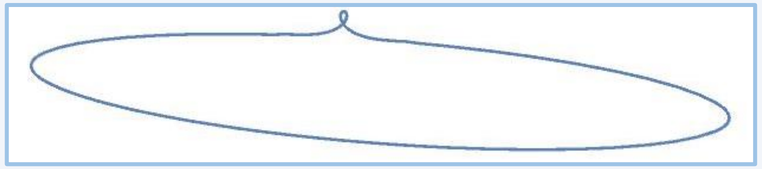
Jacobi Elliptic functions to create a deformed ring:

$$x(t, R, A, N, m) = Rcn(t, m) + Acn(Nt, m)cn(t, m)$$
$$y(t, R, A, N, m) = Rsn(t, m) + Acn(Nt, m)sn(t, m)$$
$$z(t, R, A, N, m) = -Asn(Nt, m)$$



Modify our vortex line using equations from Hasimoto[1]:

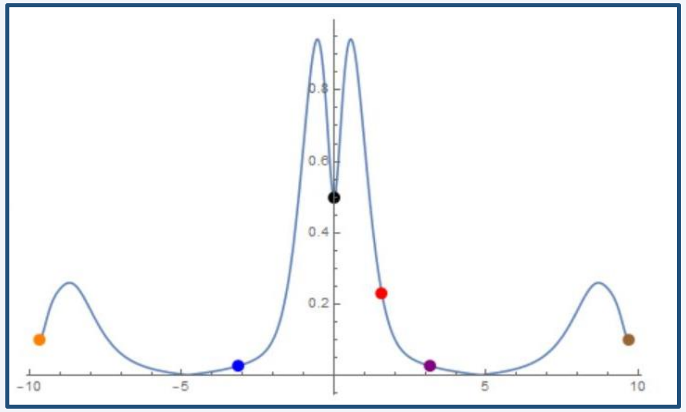
$$x = \frac{2\mu(N)}{\nu} sech(\eta(t, N))cos(\theta(t))$$
$$y = t - \frac{2\mu(N)}{\nu} tanh(\eta(t, N))$$
$$z = \frac{2\mu(N)}{\nu} sech(\eta(t, N))sin(\theta(t))$$



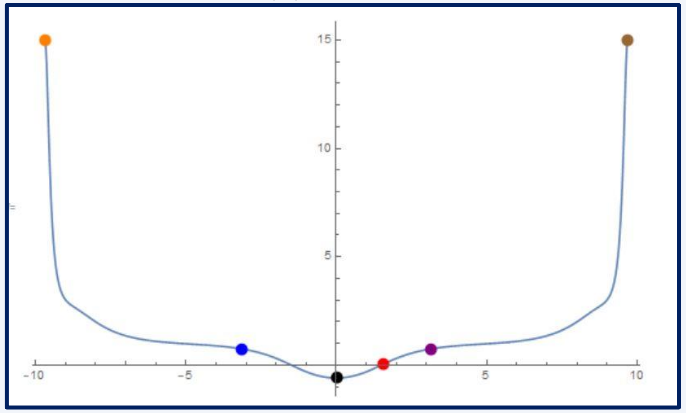
Torsion and Curvature

The curvature and torsion of the vortex lines assists in building intuition in the static case to better simulate waves that could persist in nature.

Curvature (κ) of Vortex Line

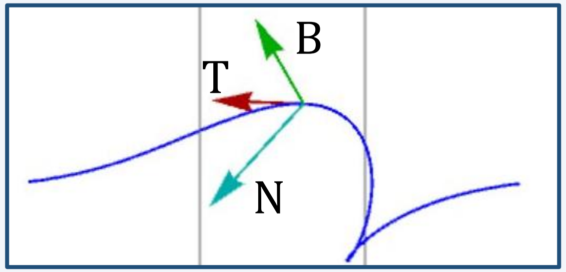


Torsion (τ) of Vortex Line



Knowledge of the curvature and torsion give insight to the curve's parameterization, via the Fernet-Serret equation:

$$\begin{bmatrix} T' \\ N' \\ B' \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix}$$



Simulations

Exploration of the space curve dynamics can be used to simulate the velocity the vortex induces. The induced velocity field from a vortex is represented by the Biot-Savart law leading to curve evolution via the curvature and binormal direction.

$$\frac{\partial \gamma}{\partial t} = \kappa \mathrm{B}$$

References

[1] Hasimoto, Hidenori. “A Soliton on a Vortex Filament.” *Journal of Fluid Mechanics*, vol. 51, no. 3, 1972, pp. 477–485., doi:10.1017/S0022112072002307.