A two-bridge knot, or rational knot, has the form $b(p,q)$ such that $|\frac{p}{q}| = k + \frac{1}{k}$. Another notation for $b(p,q)$ is then $[k,a,b]$. This type of knot gives $k$-long, $a$-long, and $b$-long twists. While the minimality of rational knots for any $q$ up to $5$ is known, we want to study the minimality of knots with $q = 7$ of the form $b(7k \pm 3,7)$ (because the cases where $p = 7k \pm 1$ or $p = 7k \pm 2$ have been studied).

### Definition: Wirtinger Presentation

A knot is called **minimal** if its knot group admits epimorphisms onto the knot groups of only the trivial knots and itself. The $SL_2[\mathbb{C}]$ character of a fundamental group's representation is the trace of the representation. The $SL_2[\mathbb{C}]$ character variety is a space of equivalence classes of representations by character. The **Riley polynomial**, $R(x,z)$, which appears as a factor in the anti-diagonal entries of $WB - AW'$, is a polynomial in $y \in \mathbb{Z}$ with coefficients in $\mathbb{C}[y]$ given by $y - tr(BA)$. Now, $AW = WB \implies AW - WB = O$ which amounts to saying $R(x,z) = 0$. If the Riley polynomial is irreducible, then the character variety has only a single component. If the non-abelian character variety is irreducible, then the knot is minimal, as demonstrated by Nagasato et al in [3].

### Our work

We first study the presentation of the knot group. Next, we find its representation. Then, we calculate the discriminant of that factorization: $D_1 = (X - Y)^4 (X - Z)^6 B$ where $B$ has a repeated factor. We conjecture that when gcd$(a,\beta,\gamma,\delta) = 1$, then $R(x,z)$ is irreducible. Thus, we need to prove that $A$ has no repeated factor.

### Theorem in progress

For any integer $n \geq 0$, the following holds:
- The nonabelian character variety of the two-bridge knot $b(7(2n) + 3,7)$ is reducible iff $n \equiv 0 \pmod{3}$

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### References