Resilience of Ocean Circulation to Changes in Temperature and Salinity in a Classical Box Model

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Introduction

Due to climate change, the oceans have constantly been increasing in temperature, causing the melting of icecaps, changing marine ecosystems, and the possible disruption of ocean currents. These changing currents can be modeled mathematically to predict their trends. Generally, these models are too complex to analyze mathematically, and a much manageable alternative is to use simplified models, such as Stommel's two-box model[1].

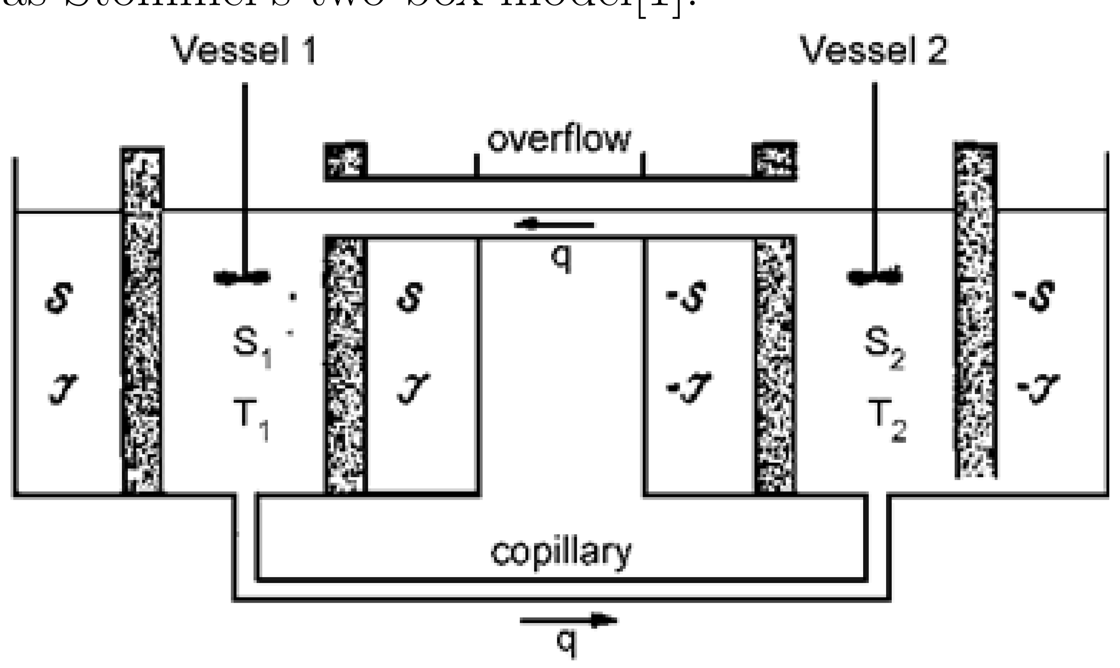

Figure 1:Two box model presented in Stommel's paper[1].

Stommel's model consists of two vessels, one with high temperature and salinity and one with low temperature and salinity [1]. Both vessels are surrounded by tanks with constant salinity S^* and temperature T^*, separated by porous walls. The system is defined by four differential equations, one for each of the salinity and temperature variables. These can be simplified by considering the solutions $S_1 = -S_2 = S$ and $T_1 = -T_2 = T$ and then nondimensionalizing them. R represents the ratio of the change in salinity and the change in temperature between the two vessels, and f represents the direction and the intensity of the flow between the two vessels.

Mathematical Section

$$\frac{dx}{d\tau} = \delta(1-x) - \frac{x}{\lambda}|y - Rx|$$

$$\frac{dy}{d\tau} = 1 - y - \frac{y}{\lambda}|y - Rx|$$

$$R = \frac{\beta S^*}{\alpha T^*}$$

By solving the differential equations, with y as the dimensionless temperature and x as the dimensionless salinity, three equilibrium solutions were found for the original values of $\delta = \frac{1}{6}$, $\lambda = \frac{1}{5}$, and R=2[1], with two stable solutions at $f = -1.1$ and $f = 0.23$ and an unstable solution at $f = -0.30$.

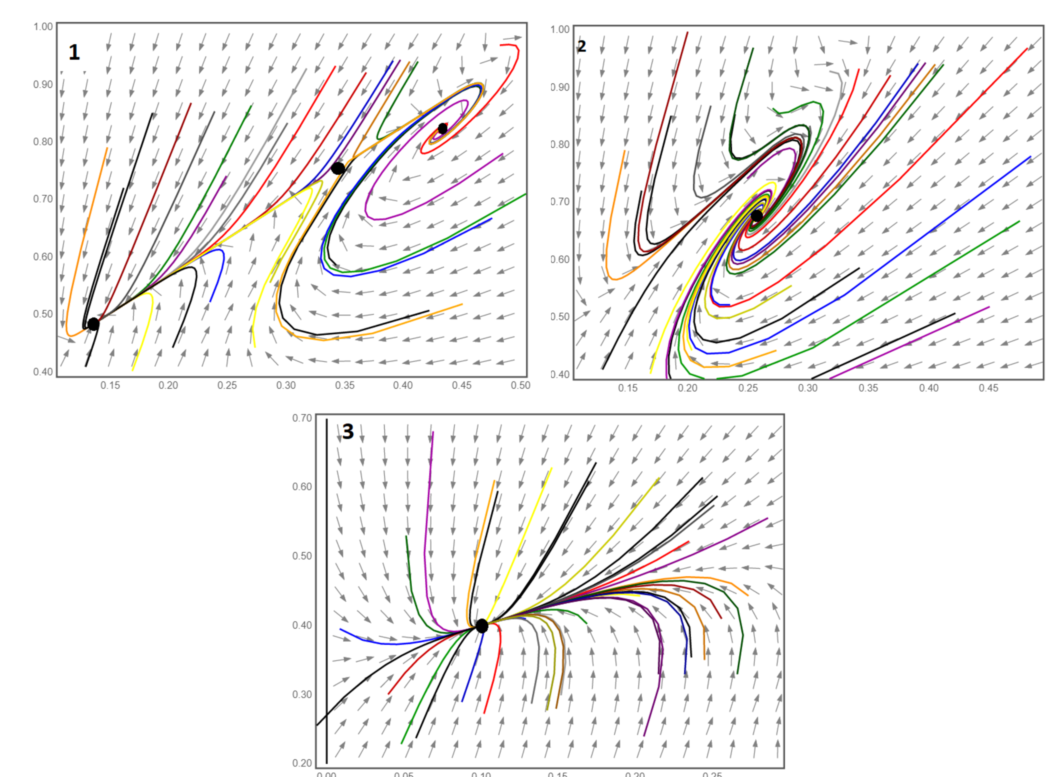

Figure 2:Vector field of the system at different R-values, showing the equilibrium solutions for (1)R=2 , (2)R=3, and (3)R=1.

For each new R-value, a new slope field was made to examine the solutions as stable or unstable. For $R = 1$ and $R = 3$, there is only one stable solution remaining.

Resilience

Resilience is the characteristic that systems can absorb an imposed change and keep working, even if in a different basin of attraction from a different equilibrium state[2]. A way to impose change is to start from one of the stable equilibrium solutions and change the parameters of the system for just enough time to force the original system to change to the basin of attraction of the other stable equilibrium solution [3]. Using this model, the new systems with varying values of R were compared back to the original scenario of R=2, and the time measured is the minimum time necessary for a perturbation that changes the parameter R to cause a shift between equilibrium states. This procedure was repeated for several R values ranging from $R = 6$ to $R = 2.5$.

Results

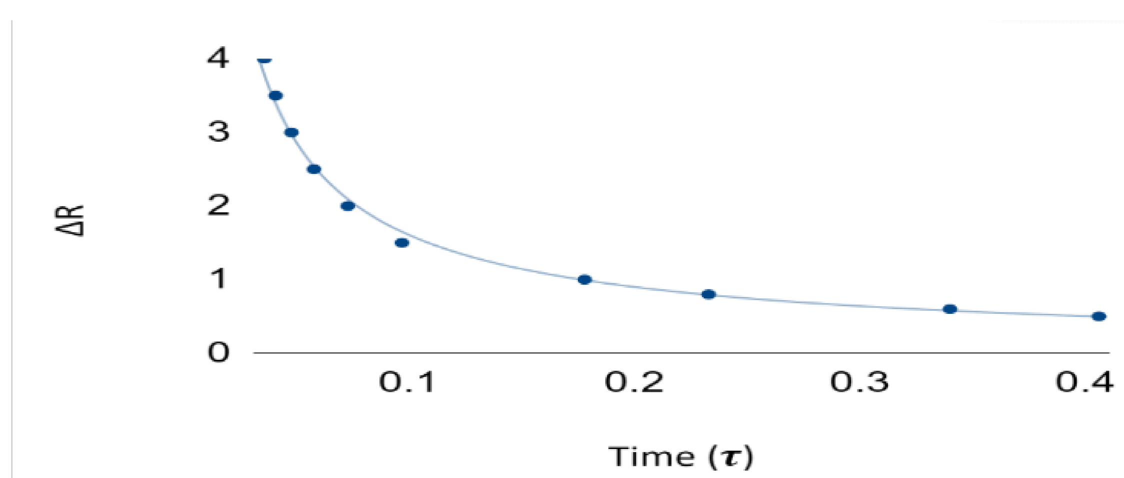

Figure 3:The minimum time length of a perturbation required to switch equilibrium solutions from its difference from $R = 2$.

Figure 3 shows how the results obtained matches with the information found by Cessi, which also showed the resilience of the system to be in the form of a power function [3]. The larger disturbances, or variations, need much shorter times to cause a shift between solutions.

Conclusion

The data indicates that the value of R has begun to decrease since temperatures are increasing due to global warming. The slowing of the ocean currents leads to less heat being carried to the North Atlantic region, causing a cooling of the region and a heating of the equatorial region[4]. Although this model does not lend itself to quantification of actual ocean circulation, it illustrates the urgency of mitigating the factors leading to climate change to avoid the possibility of tipping the Earth's climate system to a different state from which it will be difficult to return.

References

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