Effects of Mars’ Chaotic Obliquity on Ice Cover: A Budyko Approach

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Figure: Variables of A Planet’s Orbit [https://earthobservatory.nasa.gov]
Mars’ Chaotic Obliquity

**Figure:** Obliquity at the north pole surface of Mars at the summer solstice. Time Axis: −20Myr to 10Myr in increments of 5Myr [7]

**Figure:** Change in Mars’ Obliquity
Energy Balance: Earth as an Example

Temperature Change = Energy In - Energy Out

Figure: A: Energy Balance Diagram on Earth
Budyko Equation Model

\[ R \frac{\partial T}{\partial t} = Q_s(y)(1 - \alpha(y, \eta)) - (A + BT(y, t)) - C (T(y, t) - \bar{T}(t)) \]

\( Q_s \) is absorbed insolation
\( A + BT \) is emitted longwave radiation
\( C \) is energy transport across latitudes

Energy Balance: Temperature change = energy in - energy out
R denotes heat capacity of surface layer
Absorbed Insolation: Finding the Annual Average Q and Distribution

\[ Q_s(y)(1 - \alpha(y, \eta)) \]

**Figure:** Finding Solar Constant (I) (Irradiance per Area) for a Planet [nasa.gov]

**Figure:** S-plot representing the distribution function of Q across the sine of latitude.
**Albedo**

*Figure:* Image produced by combining data from instruments from NASA’s Mars Global Surveyor depicting the Martian north polar ice cap.[https://mars.nasa.gov/]

\[ Q_s(y)(1 - \alpha(y, \eta)) \]

*Figure:* Super-black material named Vantablack created by Surrey NanoSystems

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Albedo Function

\[ Q_s(y)(1 - \alpha(y, \eta)) \]

\(\alpha(y, \eta)\) represents the albedo \((\text{reflectivity})\) dependent on sine of latitude and the position of ice line.

**Figure:** Contrasting Reflectivity levels on different colored surfaces with different albedos
Emitting Longwave Radiation and Latitudinal Transport

\[
(A + BT(y, t)) \quad \text{and} \quad C(T(y, t) - \bar{T}(t))
\]

Figure: A: Energy Balance Diagram of Earth

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Dynamic Ice Lines

Figure: Ice Cap Diagram: $\eta_e$ and $\eta_p$ on the interval $[-1, 1]$ with condition that $\eta_e \leq \eta_p$.

\[
\frac{d\eta_e}{dt} = \rho(T_c - T(\eta_e, t)),
\]
\[
\frac{d\eta_p}{dt} = \rho(T(\eta_p, t) - T_c).
\]

▶ $T_c =$ highest temp. ice is present year-round
▶ $\rho$ determines rate of change of ice line relative to temp. change
▶ $T(\eta_e, t) > T_c$ implies degradation (poleward) of $\eta_e$
▶ $T(\eta_e, t) < T_c$ implies formation (equatorward) of $\eta_p$
Ice Cover

Figure: Different Ice Regime Scenarios. Top Left: Ice Belt around Equator, Top Right: Polar Ice Caps, Bottom Left: Ice Free, Bottom Right: Full Ice Cover (Snowball State)

Figure: Depiction of Ice Regimes on Mars at Different Obliquities. Low Obliquity shows a Large stable Ice Cap. High Obliquity shows one Ice Cap.

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The final non-dimensionalized version of our model is as follows:

\[ T^* (y) = \left( \frac{q}{1 + \delta} \right) \sigma_6 (y, \beta) (\alpha^* (y, \alpha)) + \delta (1 - \alpha (S(\beta))) \]  

(3)

where

\[ \alpha^* (y, \alpha) = \begin{cases} 
1, & y < \eta_e, y > \eta_p \\
1 - \alpha/2, & y = \eta_e, y = \eta_p \\
1 - \alpha, & \eta_e < y < \eta_p 
\end{cases} \]  

(4)

and

\[ \frac{d\eta_e}{d\tau} = \lambda (T^*(\eta_e) - 1) \quad \frac{d\eta_p}{d\tau} = -\lambda (T^*(\eta_p) - 1) \]  

(5)
**Fillipov System**

\[
\frac{dx}{dt} = -(x + 1/2) \cdot (x - 1/2) \cdot (x - (1 - 1.1 \cdot \sin t))
\]

**Figure:** Simple Fillipov System (blue) with a sine function (yellow)
### Table: Nondimensionalized Parameter Values for Mars

<table>
<thead>
<tr>
<th>Parameter</th>
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<th>Formula</th>
<th>Earth Value</th>
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<td>$\delta_1$, $\delta_2$, $\delta_3$</td>
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<td>$C/B$</td>
<td>1.6</td>
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<td>$q$</td>
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- $\alpha$: contrast between albedos and the extent of the ice albedo
- $q$: radiative forcing of the planet
- $\delta$: efficiency of the horizontal heat transport term
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Observations: Looking at $\alpha$

**Figure:** Stable Ice Cap at $\alpha = 0$ for $\delta_1$ using the sine function

**Figure:** Stable Ice Cap at $\alpha = 0$ for $\delta_2$ using the sine function

**Figure:** Stable Ice Cap at $\alpha = 0$ for $\delta_2$ using Laskar data
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Constant ice cap with small values of albedo contrast ($\alpha$) with oscillations
Observations: Comparing Laskar Data and the sine Function for $q$

**Figure**: Ice Cap Regimes for $q = [1.2,1.55]$ for $\delta_2$ using Laskar Data

**Figure**: Ice Cap Regimes for $q = [1.3,1.4]$ for $\delta_2$ using the sine Function
Observations: Comparing $\delta_2$ and $\delta_3$ for $q$

Figure: Ice Cap Regimes for $q = [1.3,1.4]$ for a larger heat efficiency ($\delta_2$) using the sine Function

Figure: Ice Cap Regimes for $q = [1.15,1.5]$ for a lower heat efficiency ($\delta_3$) using the sine Function
Observations: Looking at $q = [1.6-1.95]$

Figure: Ice Cap Regimes for $q = [1.15,1.5]$. Oscillations between Stable Partial Ice Cover and Ice Free Regimes
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Figure: Ice Cap Regimes for $q = [1.15,1.5]$. Oscillations between Stable Partial Ice Cover and Ice Free Regimes

Presence of oscillations between stable ice cover and ice free regimes.
Albedo Contrast and Horizontal Heat Efficiency

**Figure:** Plot of Local Minimum Point Values of Solutions

**Figure:** Plot of Local Maximum Point Values of Solutions
Overall Discussion Ideas

- Constant ice cap with oscillations with small values of $\alpha$ (albedo contrast).
- Specific interval in the $q$ (radiative forcing) parameter space where stable oscillations are seen. Oscillations between stable ice cover and ice free regimes seen with higher values of $q$.
- Lower magnitudes of $\delta$ (efficiency of horizontal heat transport) show more instances of stable ice caps.
- No presence of stable ice belt (poles ice free).
Constant ice cap with oscillations with small values of $\alpha$ (albedo contrast)

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Next Steps

Looking at longer time scale data of 5Gyr
Accounting for diffusion of solutions in our sinusoidal function.
How mean obliquity and standard deviation increase over 5Gyr
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References


Jacques Laskar, A.C.M. Correia, Mickael Gastineau, Fr´ed´eric Joutel, Benjamin Levrard, et al.. Long term evolution and chaotic diffusion of the insolation quantities of Mars.. 2004. ¡hal-00000860¡


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Thank You!