The Finite Blocking Problem on Cyclic Covers of the Regular Octagon

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Background: Finite Blocking on a Square Billiard Table

Definition: Translation Surface

A **translation surface** is a class of polygons in the plane with edge identification such that each edge is paired with exactly one other parallel edge of the same length.

Two points on a translation surface are finitely blocked if there is a finite collection of points such that any straight line from one point to the other passes through this set of points. The finite blocking problem asks whether a given surface is finitely blocked.

To understand the blocking problem, let us examine a square billiard table with pockets at each corner. For any pair of corners, there is a billiard trajectory from one to the other.

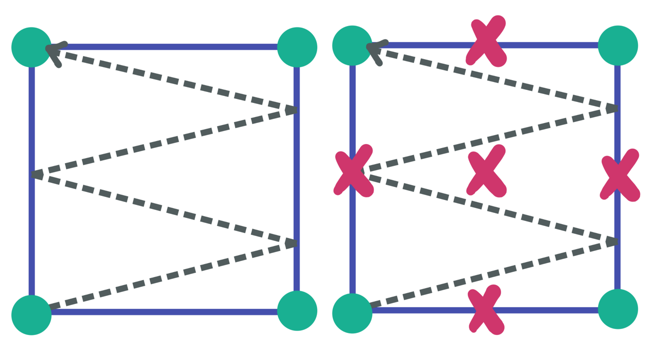

Fig.: Can we still go from any corner to any other corner if we introduce obstacles?

We can unfold the square billiard table into a square torus.

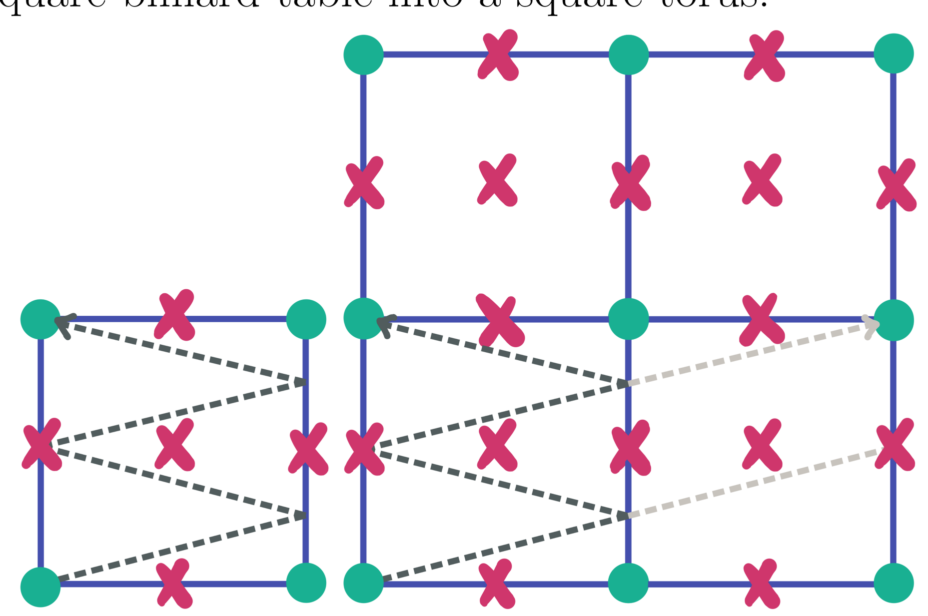

Fig.: This produces a translation surface.

One can see that the corners of the above image have the finite blocking property – there is no way to get from one corner to another without passing through an obstacle.

Definition: Auto-Blocking Property

We say a translation surface is **Auto-Blocking** if every point is finitely blocked from itself.

Goal: The Finite Blocking Problem on n-Covers of the Octagon

What do we currently know about this problem?

Theorem

For a square-tiled surface, any pair of points are blocked from each other.

Theorem (Apisa-Wright)

The only surfaces with infinitely many pairs of finitely blocked points are square-tiled surfaces and half-translation covers.

Target: n-Covers of the Octagon

We seek to examine the blocking problem on cyclic covers of the regular octagon surface branched over the cone point and the Weierstrass points.

Since these covers are only branched over Weierstrass points, the covers of a set degree n are specified by a finite collection of integers.

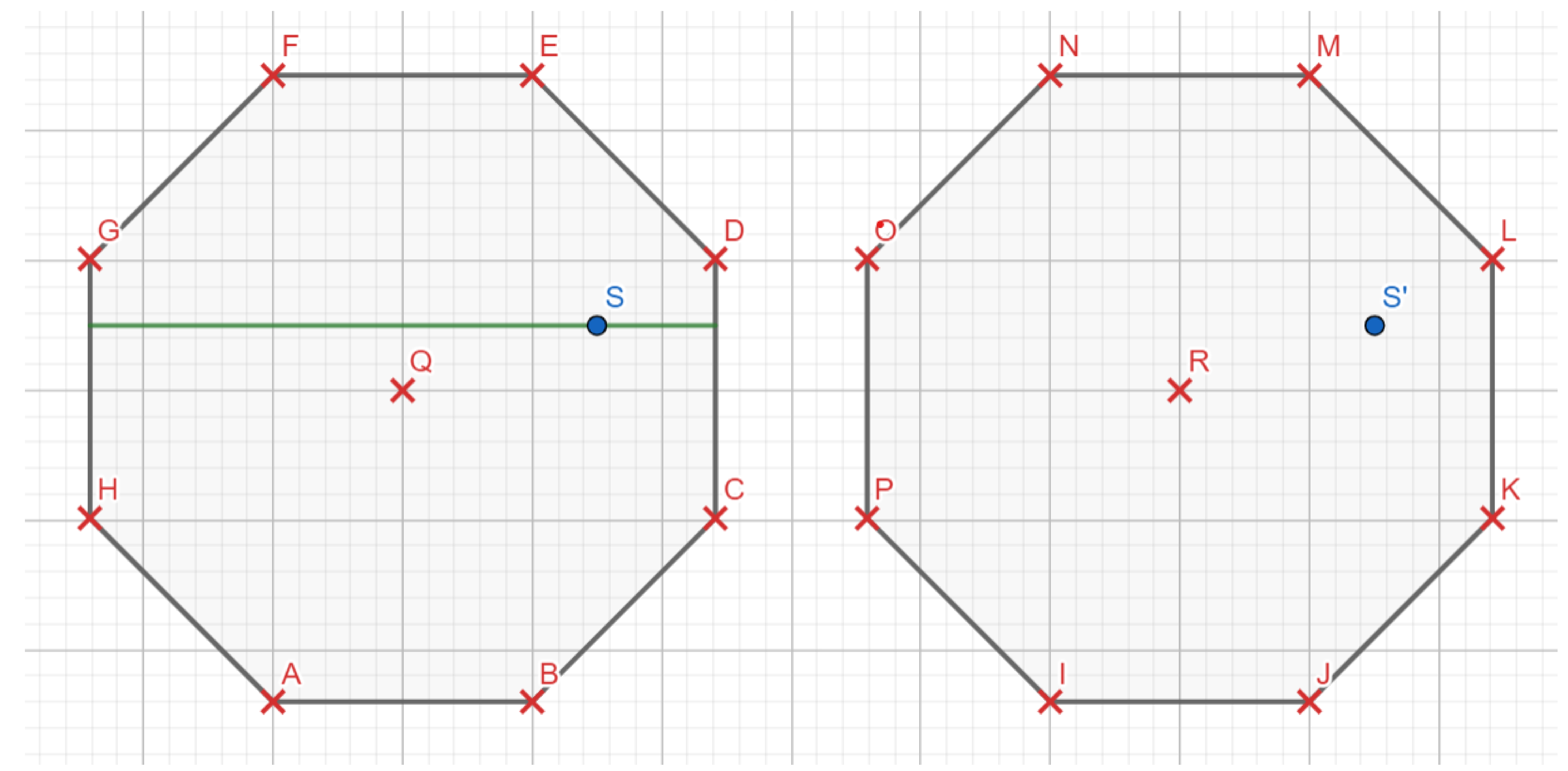

Fig.: Double cover of the octagon with trivial holonomy.

The above double cover of the octagon is an example of two points that are finitely blocked on the green line, since it hits S but not S'.

Algorithm

Step 1: Fix a cover. Straight line flow permutes the points in a fiber. Given a cylinder direction and a point in the base, compute this permutation.

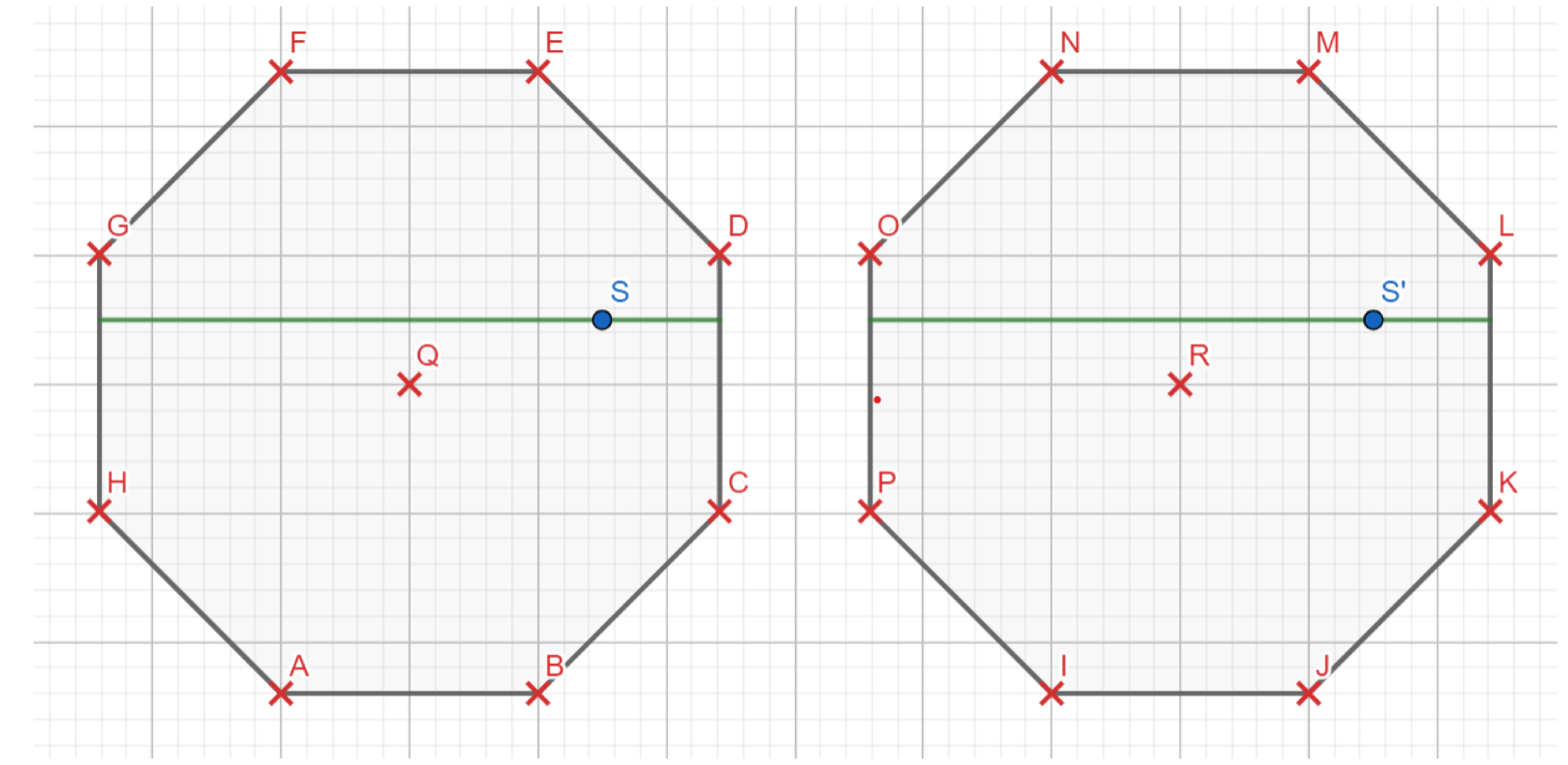

Fig.: Double cover of the octagon with nontrivial holonomy.

Step 2: If the permutation has fixed points, then the cover does not have the Auto-Blocking Property.

Step 3: Use a computer program to check every possible n-cover of the octagon.

Progress and Results

Degree	#{Cyclic Covers}	# w/ Auto-Blocking Property
2	512	1
3	19683	≤ 12
4	262144	≤ 29

We can verify that the one candidate degree 2 cyclic cover that the computer identified does in fact have the Auto-Blocking Property.

Next Steps

- Certify whether candidate surfaces of degree 3 and 4 have the Auto-Blocking Property
- Analyze covers of higher degree

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