Rainwater Harvesting Potential on Geneseo's Campus

Kayla Kenny

Advisor Dr. Ahmad Almomani

Mathematics Department

Abstract

One of the best ways that Geneseo can become more sustainable is by introducing a rainwater harvesting system. The first steps to introducing such an innovation to our campus is to start with a smaller scale project to set a measure for the costs, planning, and overall direct benefits. For this reason, the following poster focuses on what would be needed in order to maximize the harvesting potential by increasing the collecting area to provide for the campus's gardens from April to October each year.

This project shall focus on creating two aboveground tanks that will be used from April to October to provide water for the school's gardens during the summer. As these will only be used for plants rather than human consumption, the filtration needed for these is minimal to none; the most that could be needed is a mesh screening over the gutters to prevent large debris. One tank shall be placed near Brodie Hall while the other shall be placed between Newton Hall and the Integrated Science Center—since these latter two buildings are connected, it would make the most sense to use both to provide for one tank.

What is Rainwater Harvesting?

Rainwater harvesting is the process in which the water that gathers on rooftops is guided through the gutters and down into a cistern where it will then be sent through the building's pipes for gardening usage. Mesh coverings are typically placed along the gutters to stop large debris from entering the system.

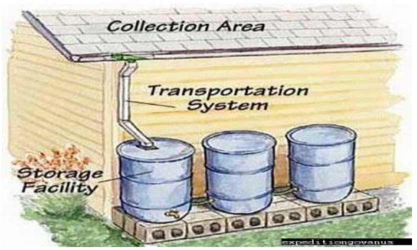

Figure 1: A Diagram of Rainwater Harvesting [5]

Collection Potential for Summer Gardens

Using the estimated roof areas of each building [3] and the average precipitation during the months when the tanks will be in use, we can estimate how much water these tanks can potentially collect. Brodie Hall has a roof area of 62,347 square-feet, Newton Hall has a roof area of 38,611 square-feet, and the Integrated Sciences Center has a roof area of 44,651 squared-feet. According to the statistics provided by [1], the Geneseo area receives about 21.95 inches of precipitation from April to October. When inputting the roof sizes in conjunction with the amount of precipitation during this time period into [2], we learn the harvesting potential of these buildings to be the following: Brodie could collect 852,585.87 gallons, Newton could collect 527,999.63 gallons, and the ISC could collect 610, 595.73 gallons. This means that the tanks can collect an average total of 1,991,181 gallons of water during this time period, which is equivalent to 68.91% of the total water that the gardens currently use.

Geneseo uses approximately 14,447,651.90 gallons of water each year with the summer gardens accounting for about 20% (or 2,889,530 gallons) of the total usage. This results in the gardens costing the school about $13,496.99 a year. With the amount of water we can save annually by harvesting our own, the school only needs to take approximately 898,349 gallons of water from the county, the equivalent of $4,196.19. This means that with a water harvesting system, the school can save $9,300.80 annually.

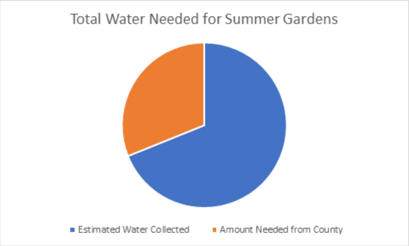

Figure 2: Pie Chart of Water Collected

Creating a Harvesting Tank

Since this is a dynamic system the volume of the tanks does not have to perfectly equate to the amount of water that could be potentially harvested over the whole time period. The best-case scenarios would be that these tanks are in a constant median range rather than always at full or low capacity. As such, simple mathematical equations can be used to derive the specifications needed for our tanks' volumes, diameters, and outlet diameters; these will all be calculated using the formulas from [6]. For these calculations we will also consistently work under the assumptions that the cistern will be about 9.84 feet tall, that it already holds approximately 3.28 feet of rain within it, and that a heavy rain event will produce a maximum of 0.08 feet of rain.

Diameter and Volume

In order to calculate the diameter of the tanks needed, we must first derive the amount of water that will be collected during a heavy rain event. A heavy rain event will be the time when the most water will enter the system at a certain time, meaning that the tanks will have to be ready for such a maximum intake at any time. To find the amount of water collected during such a storm, the area of the roofs must be multiplied by the height of the precipitation. Brodie could collect approximately 4,987.76 cubic feet of water while Newton and the ISC can collect 3,088.88 and 3,572.08 cubic feet of water, respectively. Since Newton and the ISC will be sharing a single tank, their maximum intact will be combined to be 6,660.96 cubic feet.

In order to find the diameters, we first set the volume of the heavy rain event equal to the formula for the volume of a cylinder, where in which the height will be the height of the empty space within the tank. From there we can modulate the equation until we have a formula for the diameter of the tanks.

$$V_{max} = \pi \times (height_{tank} - height_{water}) \times \frac{{D_t}^2}{2}$$

$$\frac{{D_t}^2}{2} = \frac{V_{max}}{\pi \times (9.84 - 3.28)}$$

$$\frac{D_t}{2} = \sqrt{\frac{V_{max}}{\pi \times 6.56}}$$

$$D_t = 2 \times \sqrt{\frac{V_{max}}{\pi \times 6.56}} \quad (1)$$

Using formula (1) we can conclude that Brodie will need a diameter of 31.11 feet and the Newton-ISC tank will need a diameter of 35.96 feet. Now that we have the diameters needed for each system, the volumes needed for each tank can be calculated. This is done with the standard equation for a cylinder, where in which our height will be the total height of the tank itself. Thus, we discover that the Brodie tank could withstand a volume of 7,479.71 cubic feet (or 55,952.12 gallons) while the Newton-ISC could withstand a volume of 9,993.66 cubic feet (or 74,757.77 gallons).

Diameter of the Outflow Orifice

In the case of an emergency, the cistern must have an outlet orifice for the water to drain out. To estimate the diameter of this outlet (d_{or}), a scenario is proposed in which the water within the tank is at a height, h, of 3.28 feet and must be drained in time, t, of 6 hours. For this we will also use our newly discovered diameters for each tank, d_{tank}, is 7.94 feet for Newton and 34.78 feet for Brodie.

The first step to calculating the outlet orifice, the time it takes for the water to drain. This can be done with a basic rate of change formula where:

$$Rate\ of\ Change = Rate\ In - Rate\ Out. \quad (1)$$

For this scenario, our rate of change, $\frac{dh}{dt}$, is multiplied by the area of a circle, $\pi \times \frac{{d_{tank}}^2}{4}$, to obtain "mass water balance" [6]. Since such an emergency would shut off all intake, the rate in will be 0. The rate out will be a modification of Torricelli's Law, that results in the following equation:

$$Rate\ Out = O_a \times d_{coeff} \times \sqrt{2 \times g \times h} \quad (2)$$

Where O_a is the area of the outlet, d_{coeff} is the discharge coefficient (1.97 feet), g is gravity (32.17 ft/s²), and h is the height of the tank (9.84 feet). The components to the rate of change formula are then combined to create the following:

$$\pi \times \frac{{d_{tank}}^2}{4} \times \frac{dh}{dt} = 0 - O_a \times d_{coeff} \times \sqrt{2 \times g \times h}. \quad (3)$$

Our goal here is to get O_a on its own—from there we can plug the resulting number into the formula for the outlet area and find the diameter. First, we must separate and integrate our variables in order to get rid of $\frac{dh}{dt}$. When we separate the variables, we get:

$$\frac{1}{\sqrt{h}} dh = -\frac{4 \times O_a \times d_{coeff} \times \sqrt{2 \times g}}{\pi \times {d_{tank}}^2} dt. \quad (4)$$

We then integrate equation (4) on both sides. The $\frac{1}{\sqrt{h}}$ will be integrated from 3.28 to 0 to represent the change in the water's height, in feet, as it drains. On the right side dt will be integrated from 0 to 21,600 to represent the change in time in seconds. Since gravity is measured in seconds, we translate our time from hours so the units align. When we integrate the equation, we end up with the following:

$$\int_{3.28}^{0} \frac{1}{\sqrt{h}} dh = -\frac{4 \times O_a \times d_{coeff} \times \sqrt{2 \times g}}{\pi \times {d_{tank}}^2} \times \int_{0}^{21,600} dt$$

$$-\int_{0}^{3.28} \frac{1}{\sqrt{h}} dh = -\frac{4 \times O_a \times d_{coeff} \times \sqrt{2 \times g}}{\pi \times {d_{tank}}^2} \times \int_{0}^{21,600} dt$$

$$\int_{0}^{3.28} \frac{1}{\sqrt{h}} dh = \frac{4 \times O_a \times d_{coeff} \times \sqrt{2 \times g}}{\pi \times {d_{tank}}^2} \times \int_{0}^{21,600} dt$$

$$2 = \frac{86,000 \times O_a \times d_{coeff} \times \sqrt{2 \times g}}{\pi \times {d_{tank}}^2} \quad (5)$$

Equation (5) is then rearranged so that O_a is on its own:

$$O_a = \frac{\pi \times {d_{tank}}^2}{43,200 \times d_{coeff} \times \sqrt{2 \times g}}. \quad (6)$$

The equation for the area of an outlet orifice is $o_a = \pi \times \frac{{d_{or}}^2}{4}$. Therefore, to find the diameter of the outlet orifice for each building we set equation (6) equal to the equation of the area of an outlet orifice:

$$\pi \times \frac{{d_{or}}^2}{4} = \frac{\pi \times {d_{tank}}^2}{43,200 \times d_{coeff} \times \sqrt{2 \times g}}. \quad (7)$$

The variables are separated once more so that d_{or} is on its own, giving the final equation needed to find the diameter of the outlet orifice:

$$d_{or} = \sqrt{\frac{{d_{tank}}^2}{10,800 \times d_{coeff} \times \sqrt{2 \times g}}}. \quad (8)$$

We plug in our variables for the discharge coefficient, gravity, and the respective tank diameters. This results in Brodie needing an outlet orifice with a diameter of 0.075 feet while the Newton-ISC tank requires one of 0.087 feet.

Cost

Since these tanks will be used strictly during the warmer season and only used for gardening purposes, many of the typical costs for rainwater harvesting systems are void. Instead, the major cost that is to be focused on is for the tank itself. An aboveground tank with a maximum volume of 50,000 gallons will be considered as the ones chosen for this experiment. While this number may be much lower than those found earlier, they would still be perfectly amenable for the uses needed.

Two aboveground, 50,000-gallon tanks bought from and installed by those from [4] costs about $64,000. The time that it would take to pay back the costs of these tanks can be found by simply dividing the total cost by the amount saved per year ($9,300.80). By this calculation, the school would be able to pay back this project in approximately 7 years.

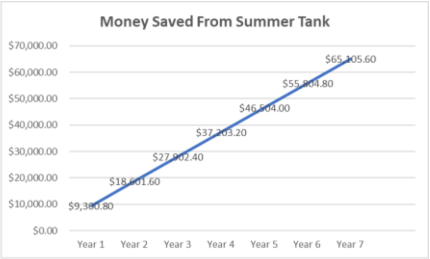

Figure 3:Line Graph of Money Saved

Conclusion

Overall, it would be highly beneficial both monetarily and, in our efforts, to be a more environmentally conscious campus to install a rainwater harvesting system. The minimum dimensions can be easily computed and the wide variety of buildings on campus lends itself to the possibility of creating an entire network of rainwater harvesting. The best outcome of this would be to have a system that will pay for itself in a handful of years and ultimately make the campus completely self-sufficient in terms of our water usage.

Acknowledgements

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