An ODE model of yaws elimination in Lihir Island, Papua New Guinea

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Introduction

- Yaws is a bacterial infection caused by *Treponema pallidum* ssp. *pertenue*.
- The World Health Organization (WHO) attempted to eradicate the disease in the 1950's and 60's, yet the disease has had recent resurgence.
- An oral dosage of azithromycin was approved in 2012 as treatment and thus, brought a new interest of eradication.
- The WHO coined their new treatment strategies as the Morges strategy which includes two forms of mass drug administration.
  - Total Community Treatment (TCT) in which all individuals in a population are treated.
  - Total Targeted Treatment (TTT) in which only individuals that show visible sign of infection and their close contacts are treated.
- We develop and analyze a deterministic ODE model of yaws to test the effectiveness of the different treatment strategies.

Yaws Overview

- Yaws is transmitted through skin to skin contact through a cut or abrasion in the skin and mainly affects children.
- After exposure the disease progresses through five main stages: primary yaws, primary latency, secondary yaws, secondary latency, and tertiary yaws.
- Only primary and secondary yaws stages are infectious.
- Tertiary yaws is a noninfectious but detrimental stage and may result in massive necrotic tissue destruction and only occurs in up 10% of all untreated cases.

Scheme of Yaws Transmission

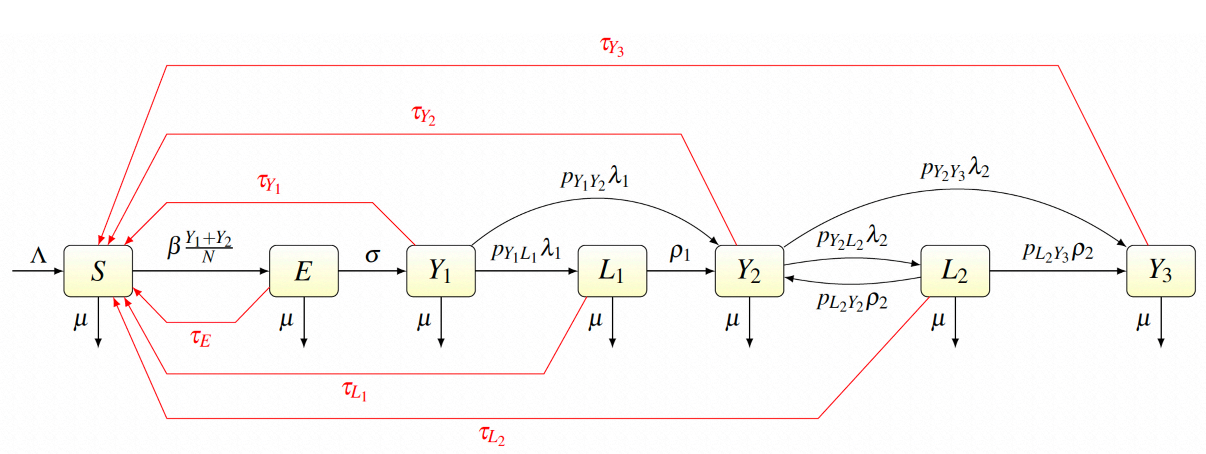

Fig. 1: Susceptible (S), Exposed (E), Primary Yaws (Y_1), Primary Latency (L_1), Secondary Yaws (Y_2), Secondary Latency (L_2), Tertiary Yaws (Y_3). The rates are explained in Table 1.

ODE Model of Yaws Transmission

$$\frac{dS}{dt} = \Lambda + \sum_{I\in\{E,Y_1,Y_2,Y_3,L_1,L_2\}} \tau_I I - \left(\beta\frac{Y_1+Y_2}{N} + \mu\right) S$$

$$\frac{dE}{dt} = \beta\frac{Y_1+Y_2}{N}S - (\sigma+\tau_E+\mu)E$$

$$\frac{dY_1}{dt} = \sigma E - (\lambda_1+\tau_{Y_1}+\mu)\,Y_1$$

$$\frac{dY_2}{dt} = p_{Y_1Y_2}\lambda_1 Y_1 + \rho_1 L_1 + p_{L_2Y_2}\rho_2 L_2 - (\lambda_2+\tau_{Y_2}+\mu)Y_2$$

$$\frac{dY_3}{dt} = p_{Y_2Y_3}\lambda_2 Y_2 + p_{L_2Y_3}\rho_2 L_2 - (\mu+\tau_{Y_3})Y_3$$

$$\frac{dL_1}{dt} = p_{Y_1L_1}\lambda_1 Y_1 - (\rho_1+\tau_{L_1}+\mu)L_1$$

$$\frac{dL_2}{dt} = p_{Y_2L_2}\lambda_2 Y_2 - (\rho_2+\tau_{L_2}+\mu)\,L_2.$$

Symbol	Meaning	Value
Λ	Birth rate	$\frac{27.2}{12*1000}$
μ^{-1}	Expected life span	$65*12$
β	Transmission rate	0.0166
σ^{-1}	Length of the incubation period	$\frac{21}{30}$
λ_1^{-1}	Length of primary yaws	3
λ_2^{-1}	Length of secondary yaws	3
ρ_1^{-1}	Length of latency after primary yaws	1.5
ρ_2^{-1}	Length of second latency	30
$p_{Y_1Y_2}$	Probability of immediate secondary yaws infection after primary yaws	0.12
$p_{Y_1L_1}$	Probability of latency period after primary yaws	$1-p_{Y_1Y_2}$
$p_{Y_2Y_3}$	Probability of immediate tertiary yaws infection after secondary yaws	0.0001
$p_{Y_2L_2}$	Probability of latency period after secondary yaws	$1-p_{Y_2Y_3}$
$p_{L_2Y_2}$	Probability of relapsing to secondary yaws during latent period after secondary yaws	0.9999
$p_{L_2Y_3}$	Probability of developing tertiary yaws during latent period	$1-p_{L_2Y_2}$
τ_I	Treatment rates	varies

Table 1: Parameters of Yaws dynamics. Time in months, rates per month. See preprint for the references.

Results - Equilibria of the Dynamics

	Disease-free	Endemic
S^*	$\frac{\Lambda}{\mu}$	$\frac{\Lambda}{\mu R_0}$
E^*	0	$\left(\frac{\Lambda}{\mu}\right)\left(1-\frac{1}{R_0}\right)\left(\frac{1}{1+k_{Y_1}+k_{Y_2}+k_{Y_3}+k_{L_1}+k_{L_2}}\right)$
I^*	0	$k_I E^*$, for $I\in\{Y_1,Y_2,Y_3,L_1,L_2\}$

$$k_{Y_1} = \frac{\sigma}{v_{Y_1}},\ k_{L_1} = \frac{p_{Y_1L_1}\lambda_1\sigma}{v_{Y_1}v_{L_1}},\ k_{Y_2} = \frac{(p_{Y_1Y_2}\lambda_1 k_{Y_1} + \rho_1 k_{L_1})v_{L_2}}{v_{Y_2}v_{L_2} - p_{L_2Y_2}\rho_2 p_{Y_2L_2}\lambda_2},$$

$$k_{L_2} = \frac{p_{Y_2L_2}\lambda_2 k_{Y_2}}{v_{L_2}},\ k_{Y_3} = \frac{p_{Y_2Y_3}\lambda_2 k_{Y_2} + p_{L_2Y_3}\rho_2 k_{L_2}}{v_{Y_3}}.$$

Table 2: Two different equilibria of the Yaws dynamics. The disease-free equilibrium is (globally asymptotically) stable when $R_0<1$. The disease-free equilibrium is unstable if $R_0>1$.

$$R_0 = \left(\frac{\beta\sigma}{v_E v_{Y_1}}\right)\left(1+\left(\frac{\lambda_1 v_{L_2}}{v_{L_1}}\right)\left(\frac{p_{Y_1L_1}\rho_1 + p_{Y_1Y_2}v_{L_1}}{v_{L_2}v_{Y_2} - p_{Y_2L_2}\lambda_2 p_{L_2Y_2}\rho_2}\right)\right)$$

where v_I denote the sum of all total rates out of the compartment I.

Results - Effects of Treatment

- We simulated two rounds of initial TCT and followed by subsequent rounds of TTT.
- 20.5 years to achieve a thousandfold decrease in cases with the typical Morges strategy.
- Continuous application of TCT strategy every six months can achieve the same results in about 3.5 years
- Using TTT, the longer the duration of latency and the shorter the duration of secondary yaws, the longer elimination takes.

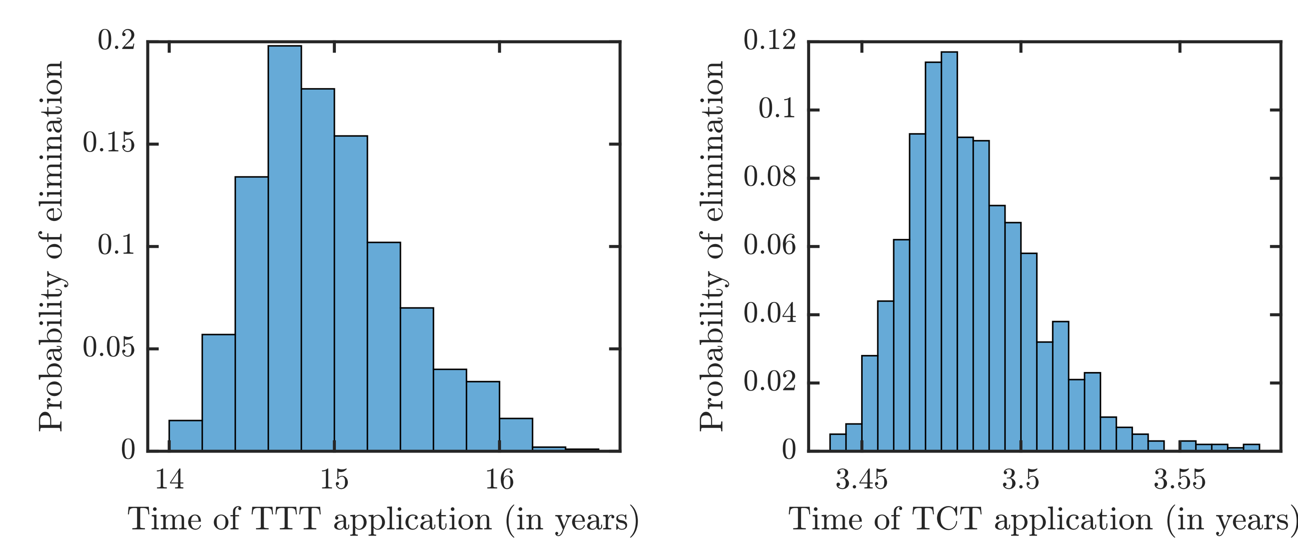

Fig. 2: Distribution of times needed to decrease yaws cases thousand times using TTT (left) or TCT (right)

Conclusions

- We found that due to the high prevalence of latent infections, it is very hard to eliminates yaws by using TTT.
- There are many other additional benefits of using TCT, such as it requires no active surveillance.
- There is a natural simplicity in the ODE models that allows for an easy estimation of the basic reproduction number. Even with different model parameters, we do not necessarily have to rerun the simulations to be able to predict the model outcomes.
- Economics plays a key role in the feasibility of yaws eradication. Underdeveloped areas are more prone to transmission and are harder to screen for active infections.

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