Abstract

To analyze the abundance of multidimensional data, tensor-based frameworks have been developed. Traditional matrix-based frameworks extract the most relevant features of vectorized data using the matrix-SVD. However, we may lose crucial high-dimensional relationships in this process. To facilitate efficient multidimensional feature extraction, we propose a projection-based classification algorithm using the t-SVD, a tensor-based extension of the matrix-SVD. We apply our algorithm to the StarPlus fMRI dataset.

Motivation - Matrix vs. Tensor

Matrix Method

- Uses matrix Singular Value Decomposition (SVD)
- Widely used in image processing
- Cannot identify relationships in higher dimensions

Tensor Method

- Better representation of high-dimensional structure
- Flexibility in choosing a transformation $M$

Background

- The mode-$k$ product $\otimes$ refers to the multiplication of a matrix $M$ along the $k$th dimension of the tensor.
- $\star_{M}$ product: $\star$ Given tensors $A \in \mathbb{R}^{n_1 \times n_2 \times \ldots \times n_M}$, $B \in \mathbb{R}^{m_1 \times m_2 \times \ldots \times m_M}$, and an invertible $M \in \mathbb{R}^{n_1 \times m_1}$, $C = A \star B = (A \Delta B) \times_M M^{-1}$ where $C \in \mathbb{R}^{n_2 \times m_2 \times \ldots \times n_M}$.
- Figure 2 shows the t-SVD of a tensor $A$.

Classification via Local t-SVD

We extend the algorithm in [7] to higher-order tensors and the $\star_M$-product.

Preprocessing

- Split training data $A$ into $c$ distinct classes: $A_1, A_2, \ldots, A_c$.
- For each class $i$, compute t-SVD and store first $k$ basis elements: $A_i = U_i s_i S_i V_i^T$, $U_i, s_i, V_i$.

Classifying a Test Image $\mathbf{T}$

- Project $\mathbf{T}$ onto space spanned by each class basis: $P_i = U_i s_i U_i^T \mathbf{T}$, for $i = 1, \ldots, c$.
- Categorize $\mathbf{T}$ as the class whose projection was "closest" to the original image: $i^* = \arg\min_{i} \| P_i - \mathbf{T} \|_F$.

To measure the performance of our algorithm, accuracy = $\frac{\# \text{correctly classified images}}{\# \text{images}}$.

Intuition - MNIST [6]

Tensor Method

Figure 3: Illustration of classifying two digits of the MNIST Dataset using the local t-SVD algorithm. Bases $U_0$, $U_1$ are generated by digits from class 0 and class 1, respectively. We project $\mathbf{T}$ onto the spaces spanned by $U_0$ and $U_1$ and obtain $P_0$ and $P_1$, respectively.

Power of Tensor Representations

Figure 4: Test accuracy with respect to number of basis elements for various choices of $\star_M$-product.

- Traditional matrix method overlooks the intrinsic characteristics of fMRI images as brain slices over time are very interconnected.
- Tensor method outperforms matrix method in test accuracy with:
  - appropriate choice of transformation matrix $M$
  - small number of basis elements

Impact of Brain Regions

We also experiment with an ROI-dependent $M$ calculated from the most prominent ROIs in each trial.

Conclusions and Future Work

- Local t-SVD classification approach outperforms the equivalent matrix-based approach.
- The most important brain regions for classification vary depending on the human subject.
- Explore applications in disease prevention and diagnosis by utilizing other fMRI datasets.
- Compare to other tensor-based frameworks such as Higher-Order SVD [5].

Reference


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